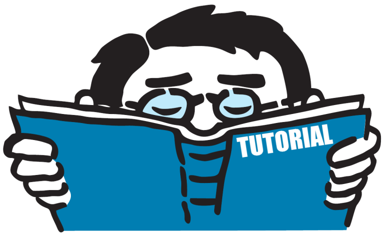

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How to

# Evaluate the Capacity Curve for Pushover Analysis

RFEM with RF-DYNAM Pro

**Am Zellweg 2**
**D-93464 Tiefenbach**
**Germany**

**Tel.: +49 9673 9203-0**
**Fax: +49 9673 9203-51**
**E-mail: info@dlubal.com**
**Web: www.dlubal.com**

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# 1 Introduction

The *Pushover Analysis* (POA) is a nonlinear static method for the seismic analysis of structures. A pre-determined lateral load pattern is applied onto the structure and steadily increased to identify yielding and plastic hinge formations and the load at which failure of the various structural components occurs. The *Capacity Curve* or *Pushover Curve* represents the nonlinear behavior of the structure and is a load-deformation curve of the base shear force versus the horizontal roof displacement of the building. Pushover analysis transforms a dynamic problem to a static problem.

In accordance to *FEMA 356* [1], at least two load patterns should be used in order to envelope the response. The most common load patterns are:

- Load pattern based on the fundamental mode shape or any other modes of interest. Therefore, the use of *RF-DYNAM Pro* is recommended and shown in the tutorial.
- Uniform load distribution (according to mass distribution)
- *FEMA* load distribution in accordance to *FEMA 356* [1]

The *Pushover Curve* is connected to the *Inelastic Response Spectrum* to evaluate a *Performance Point* which is the maximum displacement the building can cope with. There are different *Pushover Analysis* methods:

- *Capacity Spectrum Method* (CSM) applied in *ATC 40* [2]
- *N2-Method* introduced by *P.Fajfar* [3] and described in *EN 1998-1* [4]
- *Displacement Coefficients Method* (DCM)

Pushover analysis is based on the assumption that the structure has one dominant eigenvalue and mode shape. It is also assumed that this eigenvalue remains the same during the elastic and inelastic response. Detailed theoretical background of these *Pushover Methods* can be found in various literature (*i.e.*[5], [6], [3]). Multi-modal pushover analysis methods are described in [7] and [8].

The following tutorial shows how the *Capacity Curve* of a two-story steel frame can be evaluated with *RFEM*. The *Capacity Curve* is the basis for the pushover analysis employing one of the methods listed above. The following steps are performed in this tutorial:

- Definition of Plastic Hinges in accordance to *FEMA 356* [1]
- Definition of *Lateral Load Pattern* based on the fundamental eigenvalue
- Non-linear static calculation with incrementally increasing load
- Evaluation of the *Capacity Curve* and plastic hinge deformation

The structural system considered in this tutorial is a two-story steel frame. The structure and cross-sections are shown in Figure 1.1. The material used is steel *S 235*. When applying lateral loads to this system, the largest moments are expected at the joints. Therefore, plastic hinges are defined at the start and end of each beam. The structural system and the position of the plastic hinges represent a *strong-column / weak-beam* frame building where a beam-sway plastic mechanism is expected.

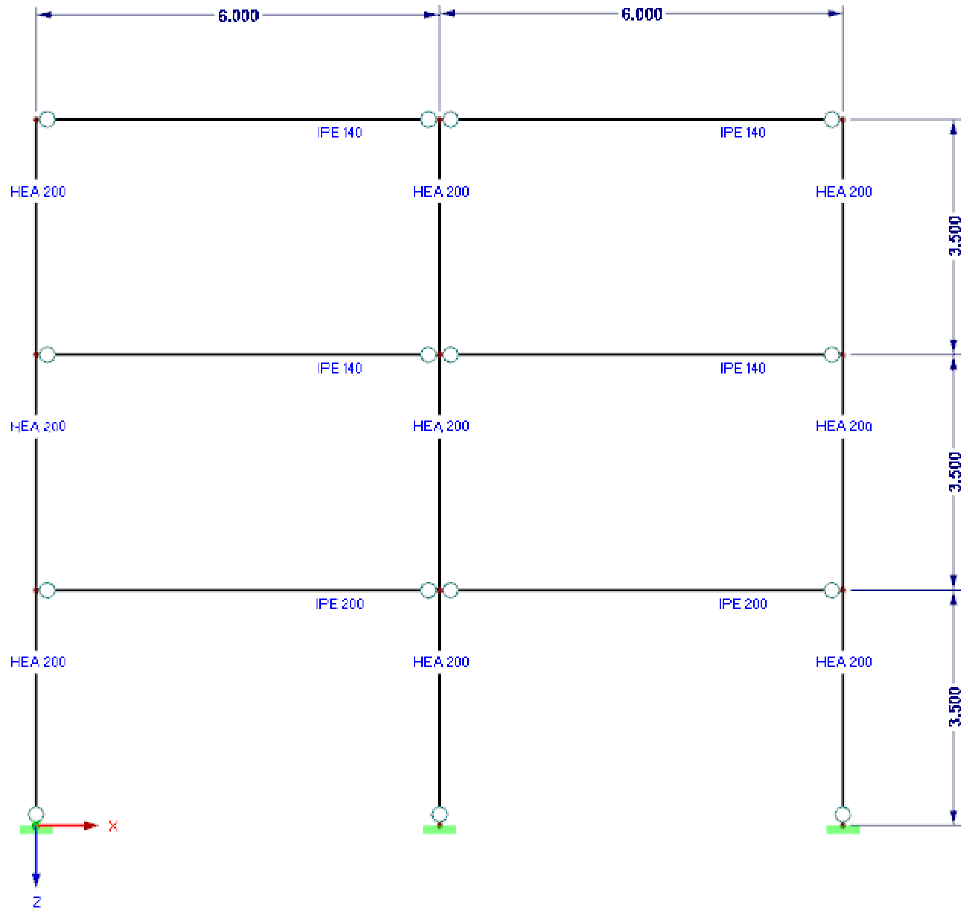


Figure 1.1: Two-story steel frame used to evaluate the *Pushover Curve*. The cross-sections and member lengths are labeled accordingly. The plastic hinges are illustrated as member end pins.

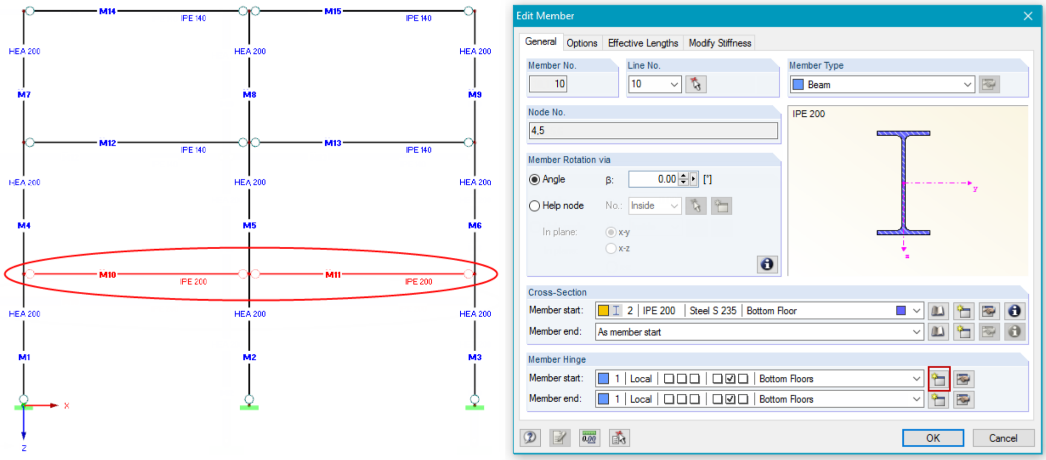
Plastic hinges are assigned to several members. Within RFEM, plastic hinges in accordance to the *FEMA 356, Chapter 5* [1] are available. The hinge diagram values and acceptance criteria are pre-defined for steel members. The yielding moments and yielding rotations are dependent on the cross-section and the material of the members. *RFEM* will set the yielding values automatically.

# 2 Plastic Hinges

It is recommended to create separate plastic hinge types for each member type and members of different lengths. All pre-defined hinge values are defined in the *Plastic Hinge* dialog box.

## 2.1 IPE 200 Hinges

1. Edit the first level *IPE 200* members, and define new hinges at the start and end of the member.



Edit Member

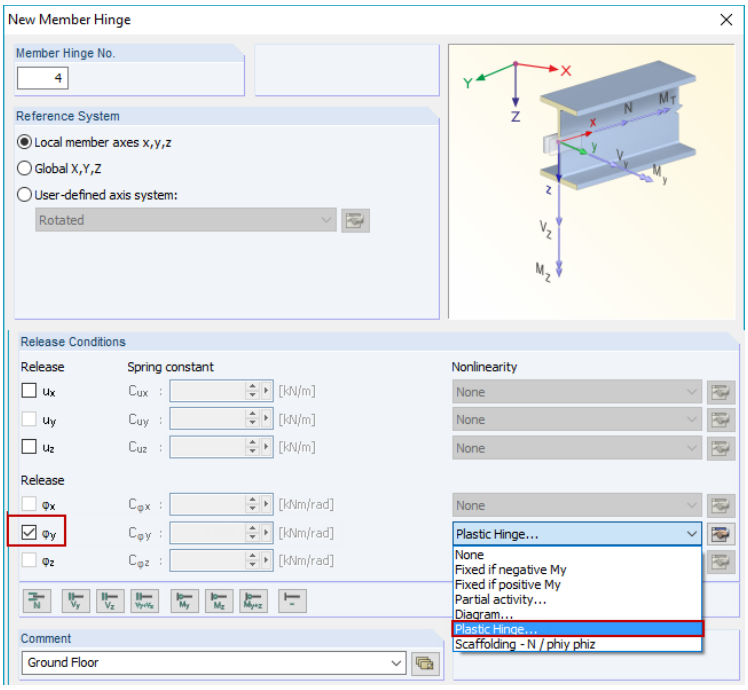
General | Options | Effective Lengths | Modify Stiffness

Member No.: 10
Line No.: 10
Member Type: Beam
Node No.: 4,5
Member Rotation via: ● Angle β: 0.00 [°]; ○ Help node No.: Inside; In plane: ○ x-y ○ x-z
IPE 200
Cross-Section
Member start: 2 | IPE 200 | Steel S 235 | Bottom Floor
Member end: As member start
Member Hinge
Member start: 1 | Local | ☐☐☐ | ☐☑☐ | Bottom Floors
Member end: 1 | Local | ☐☐☐ | ☐☑☐ | Bottom Floors

OK | Cancel

Figure 2.1: Edit the members with cross-section *IPE 200* and add new member hinges at the start and end of the members.

2. Define a moment hinge and choose the nonlinearity *Plastic Hinge*.



New Member Hinge

Member Hinge No.: 4

Reference System
● Local member axes x,y,z
○ Global X,Y,Z
○ User-defined axis system: Rotated

Release Conditions

| Release | Spring constant | | Nonlinearity |
|---|---|---|---|
| ☐ ux | Cux : | [kN/m] | None |
| ☐ uy | Cuy : | [kN/m] | None |
| ☐ uz | Cuz : | [kN/m] | None |
| Release | | | |
| ☐ φx | Cφx : | [kNm/rad] | None |
| ☑ φy | Cφy : | [kNm/rad] | Plastic Hinge... |
| ☐ φz | Cφz : | [kNm/rad] | |

None
Fixed if negative My
Fixed if positive My
Partial activity...
Diagram...
Plastic Hinge...
Scaffolding - N / phiy phiz

Comment: Ground Floor

Figure 2.2: Selection of the nonlinearity *Plastic Hinge* for the moment hinge.

3. When you enter the dialog *Nonlinearity - Plastic Hinge - My*, the hinge type *FEMA 356 Rigid-Plastic Automatic* is pre-set. The hinge diagram and the acceptance criteria in accordance to *FEMA 356 Table 5.6 - Beam Flexure* cannot be adjusted. For the cross-section *IPE 200*, an interpolation between *Line a* and *Line b* of *FEMA 356 Table 5.6 - Beam Flexure* is not required

and the values of *Line a* are used. A selection can be made between the *Primary* and *Secondary* acceptance criteria.

The yield limits are calculated in accordance to *FEMA 356 Equation 5-1 and 5-6*. For the *IPE 200* with *S 235* material the yield moment $M_{y,yield}$ and yield rotation $\varphi_{y,yield}$ of the hinge are determined with the following equations.

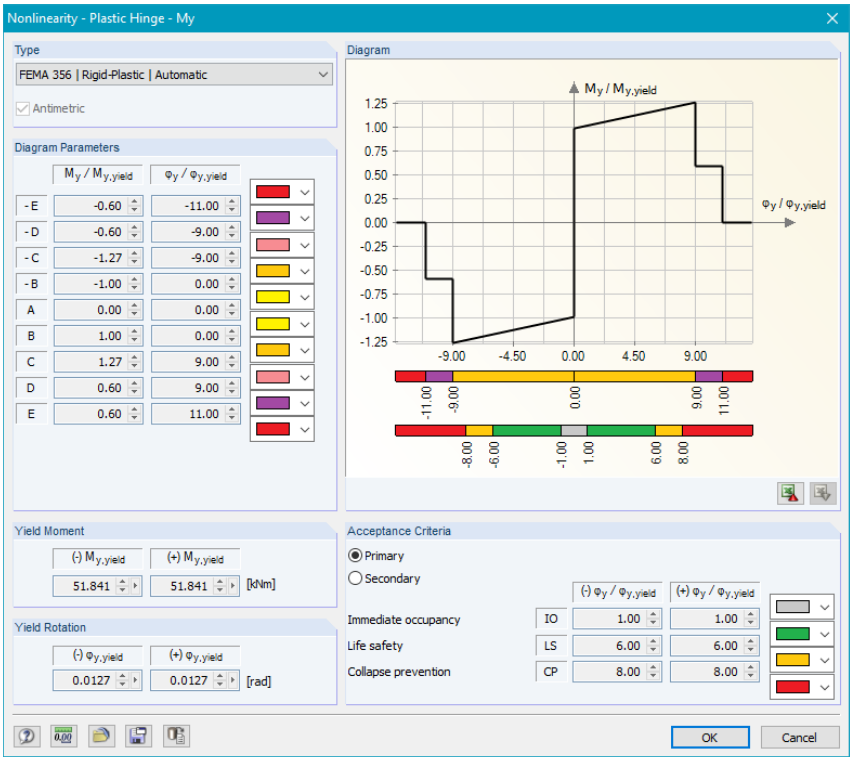
$$\begin{aligned} M_{y,yield} &= Z_y \cdot f_{yield} = 220{,}6\,\mathrm{cm}^3 \cdot 23{,}5\,\mathrm{kN/cm}^2 \\ M_{y,yield} &= 51{,}84\,\mathrm{kNm} \end{aligned} \tag{2.1}$$

$$\begin{aligned} \varphi_{y,yield} &= \frac{Z_y \cdot f_{yield} \cdot l_b}{6EI_y} = \frac{220{,}6\,\mathrm{cm}^3 \cdot 23{,}5\,\mathrm{kN/cm}^2 \cdot 600\,\mathrm{cm}}{6 \cdot 21\,000\,\mathrm{kN/cm}^2 \cdot 1\,943\,\mathrm{cm}^4} \\ \varphi_{y,yield} &= 0{,}0\,127\,\mathrm{rad} \end{aligned} \tag{2.2}$$

The following definitions are used in Equations 2.1 and 2.2.

$Z_y$ : Plastic section modulus
$f_{yield}$ : Yield strength of *S 235* (EN 10025-2:2004-11)
$l_b$ : Length of the beam
E : Modulus of elasticity
$I_y$ : Moment of inertia

In this tutorial, all default values are used. However, values can be adjusted in the dialog box to account for various scenarios (*i.e.* connections, interaction with normal forces, etc.).



Nonlinearity - Plastic Hinge - My

Type: FEMA 356 | Rigid-Plastic | Automatic

☑ Antimetric

Diagram Parameters

| | $M_y / M_{y,yield}$ | $\varphi_y / \varphi_{y,yield}$ | |
|---|---|---|---|
| -E | -0.60 | -11.00 | 🟥 |
| -D | -0.60 | -9.00 | 🟪 |
| -C | -1.27 | -9.00 | 🟥 |
| -B | -1.00 | 0.00 | 🟧 |
| A | 0.00 | 0.00 | 🟨 |
| B | 1.00 | 0.00 | 🟨 |
| C | 1.27 | 9.00 | 🟧 |
| D | 0.60 | 9.00 | 🟥 |
| E | 0.60 | 11.00 | 🟪 |
| | | | 🟥 |

Yield Moment

| (-) $M_{y,yield}$ | (+) $M_{y,yield}$ | |
|---|---|---|
| 51.841 | 51.841 | [kNm] |

Yield Rotation

| (-) $\varphi_{y,yield}$ | (+) $\varphi_{y,yield}$ | |
|---|---|---|
| 0.0127 | 0.0127 | [rad] |

Acceptance Criteria

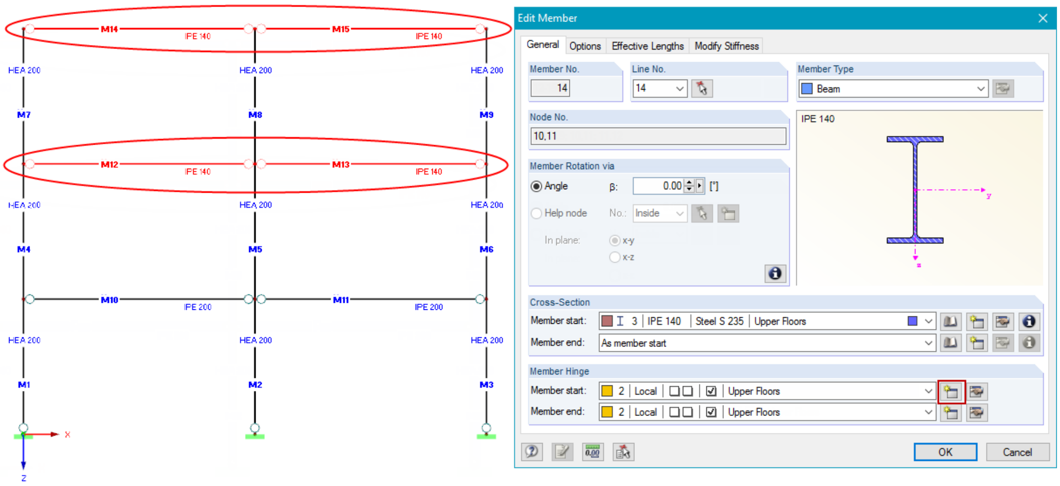
● Primary
○ Secondary

| | | (-) $\varphi_y / \varphi_{y,yield}$ | (+) $\varphi_y / \varphi_{y,yield}$ | |
|---|---|---|---|---|
| | | | | ⬜ |
| Immediate occupancy | IO | 1.00 | 1.00 | 🟩 |
| Life safety | LS | 6.00 | 6.00 | 🟨 |
| Collapse prevention | CP | 8.00 | 8.00 | 🟥 |

Figure 2.3: Plastic Hinge with the default *FEMA 356 | Rigid-Plastic | Automatic* hinge selected. The hinge diagram parameters are set automatically and can be found in *FEMA 356 Table 5.6*. The yield limits are pre-set for the *IPE 200* with *S 235* material and a member length of $l_b = 6$ m.

## 2.2 IPE 140 Hinges

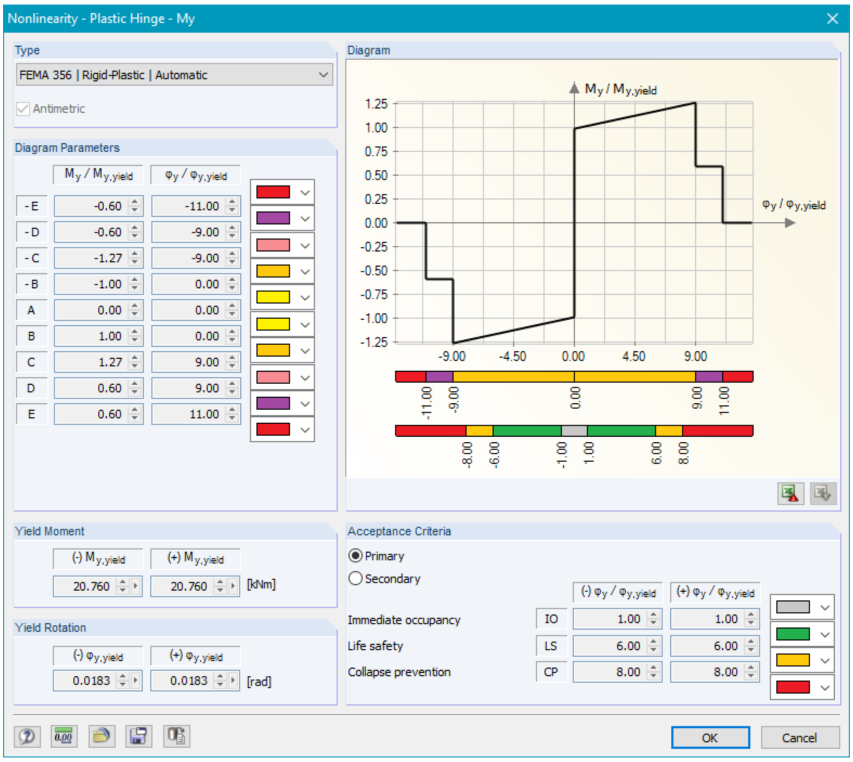
4. Define the plastic hinges for the upper level beams with *IPE 140* cross-sections.



Member No.: 14
Line No.: 14
Member Type: Beam
Node No.: 10,11
Member Rotation via
● Angle β: 0.00 [°]
○ Help node No.: Inside
In plane: ● x-y ○ x-z
Cross-Section
Member start: 3 | IPE 140 | Steel S 235 | Upper Floors
Member end: As member start
Member Hinge
Member start: 2 | Local | ☐☐ | ☑ | Upper Floors
Member end: 2 | Local | ☐☐ | ☑ | Upper Floors

Figure 2.4: Edit the members with cross-section *IPE 140* and add new member hinges at the start and at the end of the members.

The automatic plastic hinges are used. The diagram parameters and acceptance criteria are identical to the hinges defined for the *IPE 200* members (compare with Figure 2.3). A cross-section interpolation is not required for the *IPE 400*. The difference to the previously defined hinges include the yield limits calculated in Equations 2.1 and 2.2 using the *IPE 140* plastic section modulus $Z_y = 88{,}34$ cm$^3$ and the moment of inertia $I_y = 541{,}20$ cm$^4$. The resulting yielding limits are shown in Figure 2.5.



Type: FEMA 356 | Rigid-Plastic | Automatic
☑ Antimetric

Diagram Parameters

| | $M_y / M_{y,\text{yield}}$ | $\varphi_y / \varphi_{y,\text{yield}}$ | |
|---|---|---|---|
| -E | -0.60 | -11.00 | 🟥 |
| -D | -0.60 | -9.00 | 🟪 |
| -C | -1.27 | -9.00 | 🟥 |
| -B | -1.00 | 0.00 | 🟧 |
| A | 0.00 | 0.00 | 🟨 |
| B | 1.00 | 0.00 | 🟨 |
| C | 1.27 | 9.00 | 🟧 |
| D | 0.60 | 9.00 | 🟥 |
| E | 0.60 | 11.00 | 🟪 |
| | | | 🟥 |

Yield Moment

| $(-)\ M_{y,\text{yield}}$ | $(+)\ M_{y,\text{yield}}$ | |
|---|---|---|
| 20.760 | 20.760 | [kNm] |

Yield Rotation

| $(-)\ \varphi_{y,\text{yield}}$ | $(+)\ \varphi_{y,\text{yield}}$ | |
|---|---|---|
| 0.0183 | 0.0183 | [rad] |

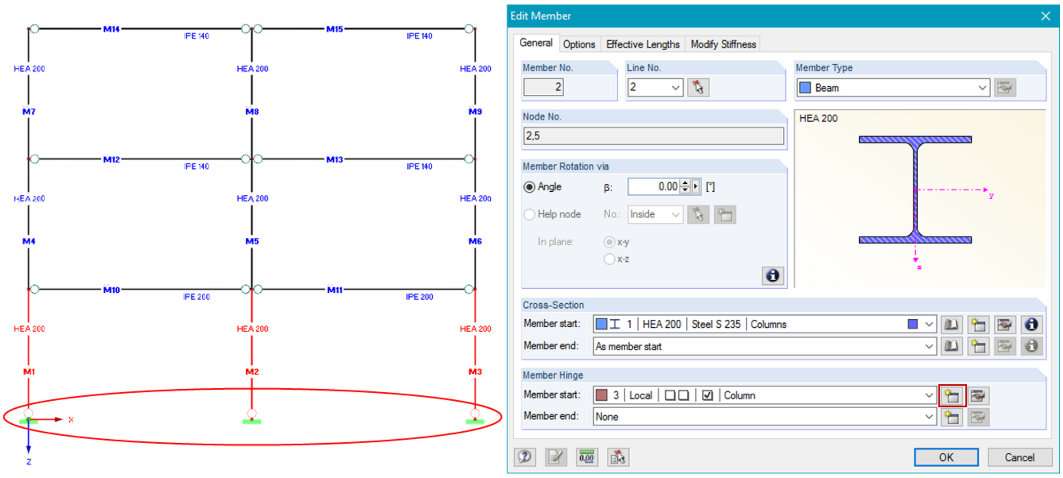
Acceptance Criteria
● Primary
○ Secondary

| | | $(-)\ \varphi_y / \varphi_{y,\text{yield}}$ | $(+)\ \varphi_y / \varphi_{y,\text{yield}}$ | |
|---|---|---|---|---|
| | | | | ⬜ |
| Immediate occupancy | IO | 1.00 | 1.00 | 🟩 |
| Life safety | LS | 6.00 | 6.00 | 🟨 |
| Collapse prevention | CP | 8.00 | 8.00 | 🟥 |

Figure 2.5: Plastic Hinge with the default *FEMA 356 | Rigid-Plastic | Automatic* hinge selected. The hinge diagram parameters are set automatically and can be found in *FEMA 356 Table 5.6*. The yield limits are pre-set for the *IPE 140* with *S 235* material and a member length of $l_b = 6$ m.

## 2.3 HEA 200 Hinges

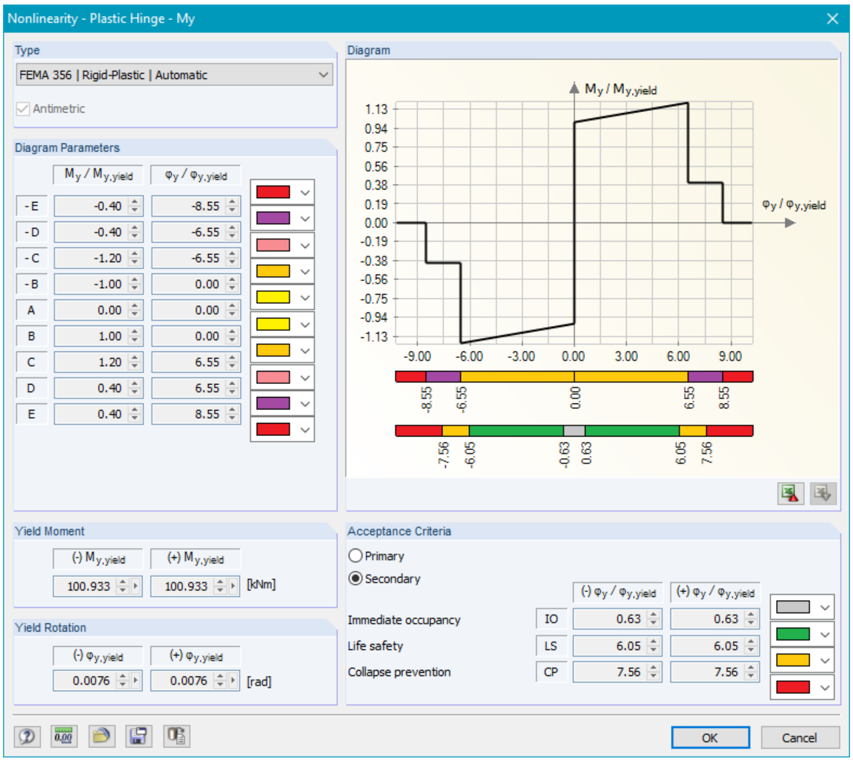
5. We define plastic hinges at the bottom of the columns to ensure final collapse of the steel frame.



Member No.: 2
Line No.: 2
Member Type: Beam
Node No.: 2,5
Member Rotation via: ● Angle β: 0.00 [°]
○ Help node No.: Inside
In plane: ● x-y ○ x-z
HEA 200
Cross-Section
Member start: 1 | HEA 200 | Steel S 235 | Columns
Member end: As member start
Member Hinge
Member start: 3 | Local | ☐☐ | ☑ | Column
Member end: None

Figure 2.6: Edit the members with cross-section *HEA 200* and add new member hinges at the start of the members.

6. Again, we choose the default automatic plastic hinge option for this cross-section. For the cross-section *HEA 200* an interpolation between *Line a* and *Line b* of *FEMA 356 Table 5.6 - Beam Flexure* (Columns - Flexure with $P/P_{CL} << 0.2$) [1] is required as the flange slenderness condition is not fulfilled for *Line a*. The plastic hinge dialog is shown in Figure 2.7. The diagram parameters are different to the previously defined hinges for the *IPE 200* and *IPE 400*.



Type: FEMA 356 | Rigid-Plastic | Automatic
☑ Antimetric

Diagram Parameters

| | $M_y / M_{y,yield}$ | $\varphi_y / \varphi_{y,yield}$ | |
|---|---|---|---|
| | | | 🟥 |
| -E | -0.40 | -8.55 | 🟪 |
| -D | -0.40 | -6.55 | 🟥 |
| -C | -1.20 | -6.55 | 🟧 |
| -B | -1.00 | 0.00 | 🟨 |
| A | 0.00 | 0.00 | 🟨 |
| B | 1.00 | 0.00 | 🟧 |
| C | 1.20 | 6.55 | 🟥 |
| D | 0.40 | 6.55 | 🟪 |
| E | 0.40 | 8.55 | 🟥 |

Yield Moment

| (-) $M_{y,yield}$ | (+) $M_{y,yield}$ | |
|---|---|---|
| 100.933 | 100.933 | [kNm] |

Yield Rotation

| (-) $\varphi_{y,yield}$ | (+) $\varphi_{y,yield}$ | |
|---|---|---|
| 0.0076 | 0.0076 | [rad] |

Acceptance Criteria
○ Primary
● Secondary

| | | (-) $\varphi_y / \varphi_{y,yield}$ | (+) $\varphi_y / \varphi_{y,yield}$ | |
|---|---|---|---|---|
| | | | | ⬜ |
| Immediate occupancy | IO | 0.63 | 0.63 | 🟩 |
| Life safety | LS | 6.05 | 6.05 | 🟨 |
| Collapse prevention | CP | 7.56 | 7.56 | 🟥 |

Figure 2.7: Plastic Hinge with the default *FEMA 356 | Rigid-Plastic | Automatic* hinge selected. The hinge diagram parameters are interpolated automatically to match the criteria for flange and web slenderness in accordance to *FEMA 356 Table 5.6* (Columns - Flexure with $P/P_{CL} << 0.2$). The secondary acceptance criteria is selected. The yield limits are set automatically for the *HEA 200* with *S 235* material and a member length of $l_b = 3{,}5$ m.

RFEM automatically calculates the slenderness checks and interpolates the correct values for diagram parameters and acceptance criteria as shown below.

$$\frac{b_f}{2t_f} \leq \frac{52}{\sqrt{f_{yield}}} \tag{2.3}$$

$$\frac{b_f}{2t_f} \geq \frac{65}{\sqrt{f_{yield}}} \tag{2.4}$$

with

$b_f$ : Width of the cross-section, $b_f = 200$ mm

$t_f$ : Thickness of the flange, $t_f = 10$ mm

$f_{yield}$ : *S 235* yield strength(EN 10025-2:2004-11), required in ksi,
$f_{yield} = 23{,}5\ \text{kN}/\text{cm}^2 = 34{,}08$ ksi

For the *HEA 200* the flange slenderness conditions are not fulfilled, and the hinge diagram values and acceptance criteria are linearly interpolated between *Line a* and *Line b* of *FEMA 356 Table 5.6*. The interpolated values are shown in Figure 2.7.

Web slenderness is not an issue for the *HEA 200*. This is checked with the following condition:

$$\frac{h}{t_w} \leq \frac{300}{\sqrt{f_{yield}}} \tag{2.5}$$

With

$h$ : Height of the cross-section between the flanges, $h = 170$ mm

$t_w$ : Thickness of the web, $t_w = 6{,}5$ mm

$f_{yield}$ : *S 235* yield strength (EN 10025-2:2004-11), required in ksi,
$f_{yield} = 23{,}5\ \text{kN}/\text{cm}^2 = 34{,}08$ ksi

This results in $26{,}15 \leq 51{,}39$ and no interpolation due to web slenderness needs to be done.

$$\frac{h}{t_w} \leq \frac{418}{\sqrt{f_{yield}}} \tag{2.6}$$

$$\frac{h}{t_w} \leq \frac{640}{\sqrt{f_{yield}}} \tag{2.7}$$

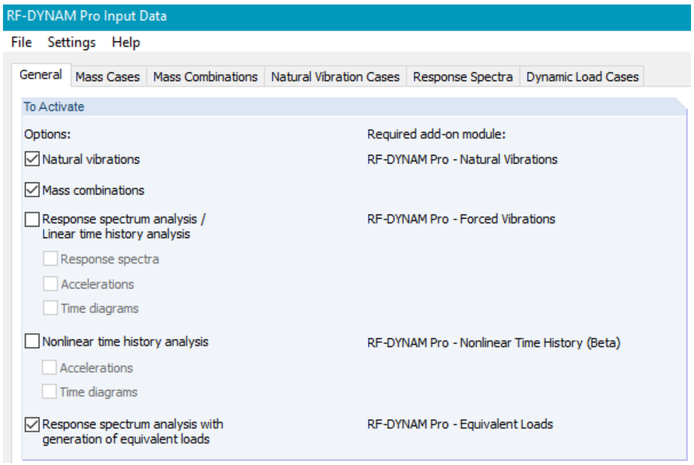
# 3 Load Pattern

Increasing lateral load patterns need to be applied to the structure up to the inelastic state. The goal is to monitor the progressive yielding of the structure. *FEMA 356* [1] and the *EN 1998-3* [9] recommends applying at least two load patterns in order to envelope the response. In this tutorial only one type of load pattern is shown.

The most common load distribution is in accordance to the structure's dominant mode shape. The eigenvalues and dominant mode shape can easily be determined in *RFEM* with the add-on module *RF-DYNAM Pro*. *RF-DYNAM Pro - Equivalent Loads*, a response spectrum analysis is performed and equivalent loads are exported into *Load Cases* within *RFEM*.

*RF-DYNAM Pro* evaluates eigenvalues, the mass distribution of the structure and the linear response spectrum in accordance to a various choice of building standards. This information is required for the *Pushover Analysis* when utilizing the *N2-Method* or the *Capacity Spectrum Method*.

1. Open the add-on module *RF-DYNAM Pro* and select the *Response spectrum analysis with generation of equivalent loads*.

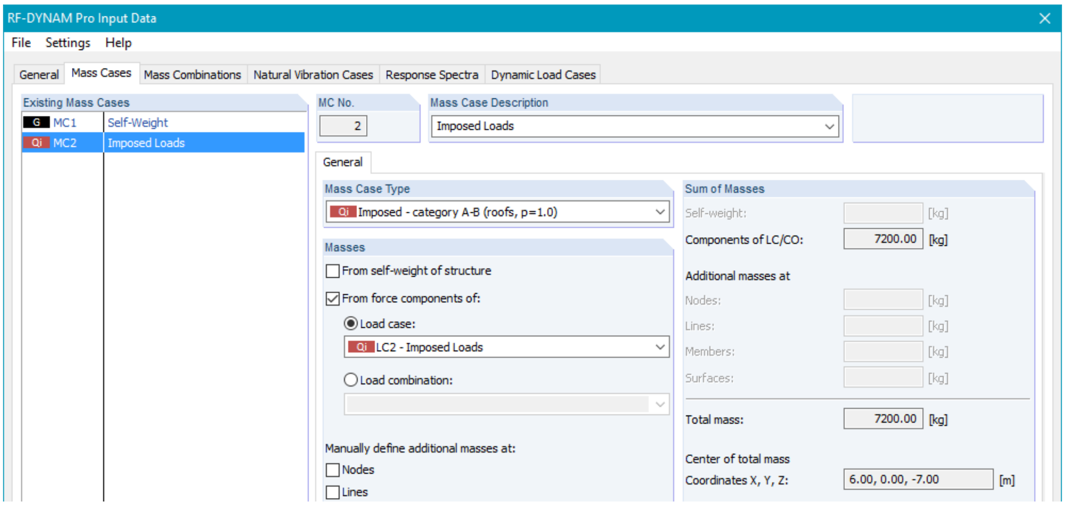


To Activate

| Options: | Required add-on module: |
|---|---|
| ☑ Natural vibrations | RF-DYNAM Pro - Natural Vibrations |
| ☑ Mass combinations | |
| ☐ Response spectrum analysis / Linear time history analysis | RF-DYNAM Pro - Forced Vibrations |
| ☐ Response spectra | |
| ☐ Accelerations | |
| ☐ Time diagrams | |
| ☐ Nonlinear time history analysis | RF-DYNAM Pro - Nonlinear Time History (Beta) |
| ☐ Accelerations | |
| ☐ Time diagrams | |
| ☑ Response spectrum analysis with generation of equivalent loads | RF-DYNAM Pro - Equivalent Loads |

Figure 3.1: Within the *General* tab of the add-on module *RF-DYNAM Pro*, the response spectra analysis with generation of equivalent loads is selected.

2. Masses are defined in the *Mass Case* and *Mass Combination* tab. Defined masses in this example are the self-weight of the structure and imposed loads. The masses are imported from *Load Cases* into the module *RF-DYNAM Pro* and are combined in *Mass Combinations*. Combination factors can be adjusted.



Existing Mass Cases

| | | |
|---|---|---|
| G | MC1 | Self-Weight |
| Qi | MC2 | Imposed Loads |

MC No.: 2

Mass Case Description: Imposed Loads

Mass Case Type: Qi Imposed - category A-B (roofs, p=1.0)

Masses

☐ From self-weight of structure

☑ From force components of:

● Load case: Qi LC2 - Imposed Loads

○ Load combination:

Manually define additional masses at:

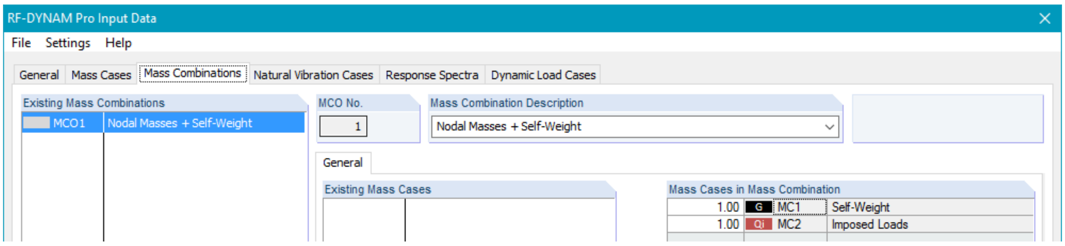
☐ Nodes

☐ Lines

Sum of Masses

| | | |
|---|---|---|
| Self-weight: | | [kg] |
| Components of LC/CO: | 7200.00 | [kg] |
| Additional masses at | | |
| Nodes: | | [kg] |
| Lines: | | [kg] |
| Members: | | [kg] |
| Surfaces: | | [kg] |
| Total mass: | 7200.00 | [kg] |
| Center of total mass Coordinates X, Y, Z: | 6.00, 0.00, -7.00 | [m] |

Figure 3.2: The self-weight and imposed loads imported from *LC2* are defined as separate mass cases.



Existing Mass Combinations

| | |
|---|---|
| MCO1 | Nodal Masses + Self-Weight |

MCO No.: 1

Mass Combination Description: Nodal Masses + Self-Weight

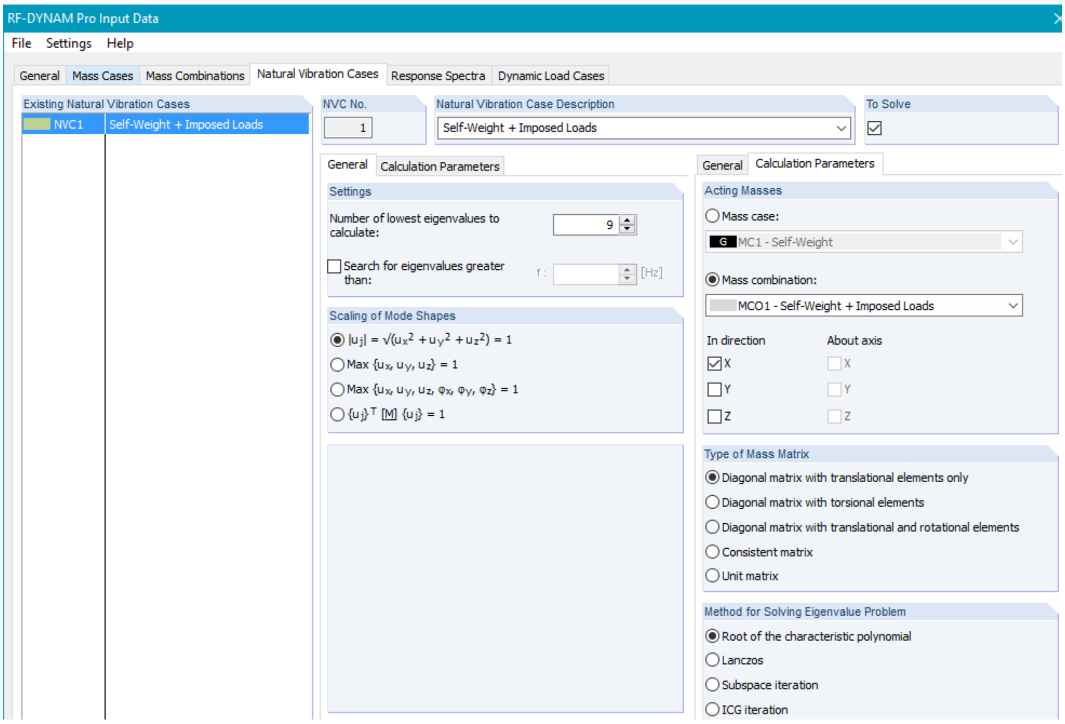
Existing Mass Cases

Mass Cases in Mass Combination

| | | | |
|---|---|---|---|
| 1.00 | G | MC1 | Self-Weight |
| 1.00 | Qi | MC2 | Imposed Loads |

Figure 3.3: The masses from self-weight and imposed loads are combined with a factor of 1,0.

3. The eigenvalues and mode shapes are calculated based on the defined masses. In this example, only the deformation in the X-direction is of interest. The masses are lumped at the structure's nodes. The settings of the *Natural Vibration Case* are illustrated in Figure 3.4.



Existing Natural Vibration Cases

| | |
|---|---|
| NVC1 | Self-Weight + Imposed Loads |

NVC No.: 1

Natural Vibration Case Description: Self-Weight + Imposed Loads

To Solve: ☑

Settings

Number of lowest eigenvalues to calculate: 9

☐ Search for eigenvalues greater than: f : [Hz]

Scaling of Mode Shapes

● $|u_j| = \sqrt{(u_X^2 + u_Y^2 + u_Z^2)} = 1$

○ Max $\{u_X, u_Y, u_Z\} = 1$

○ Max $\{u_X, u_Y, u_Z, \varphi_X, \varphi_Y, \varphi_Z\} = 1$

○ $\{u_j\}^T [M] \{u_j\} = 1$

Acting Masses

○ Mass case: G MC1 - Self-Weight

● Mass combination: MCO1 - Self-Weight + Imposed Loads

In direction: ☑ X ☐ Y ☐ Z

About axis: ☐ X ☐ Y ☐ Z

Type of Mass Matrix

● Diagonal matrix with translational elements only

○ Diagonal matrix with torsional elements

○ Diagonal matrix with translational and rotational elements

○ Consistent matrix

○ Unit matrix

Method for Solving Eigenvalue Problem

● Root of the characteristic polynomial

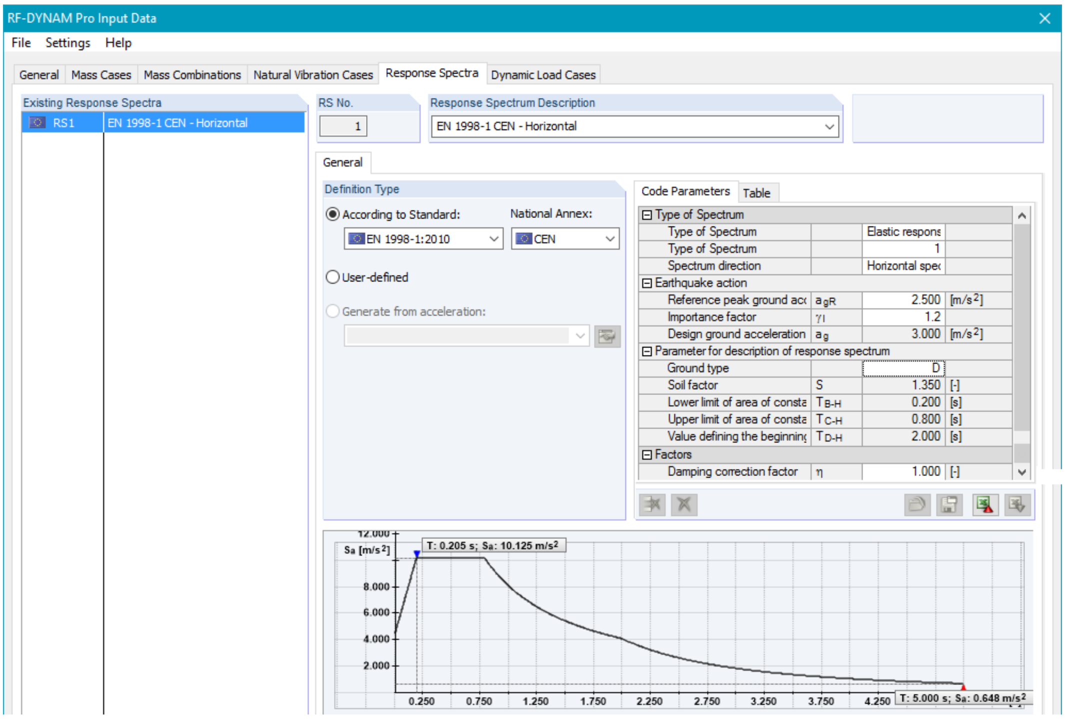
○ Lanczos

○ Subspace iteration

○ ICG iteration

Figure 3.4: The eigenvalues and mode shapes are calculated with the shown settings. The masses are lumped at nodes and act only in the X-direction.

4. The linear elastic response spectrum is defined in accordance to the *EN 1998-1* [4]. The chosen parameters are shown in Figure 3.5.



Existing Response Spectra: RS1 EN 1998-1 CEN - Horizontal

RS No.: 1

Response Spectrum Description: EN 1998-1 CEN - Horizontal

Definition Type

● According to Standard: EN 1998-1:2010 National Annex: CEN

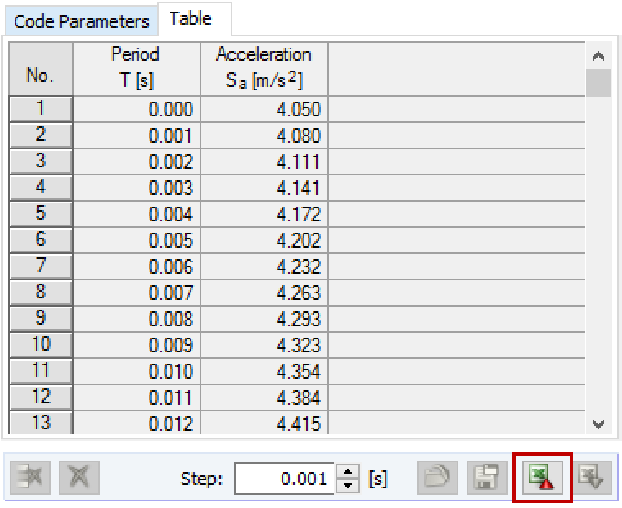
○ User-defined

○ Generate from acceleration:

| | | | |
|---|---|---|---|
| ⊟ Type of Spectrum | | | |
| Type of Spectrum | | Elastic respons | |
| Type of Spectrum | | 1 | |
| Spectrum direction | | Horizontal spec | |
| ⊟ Earthquake action | | | |
| Reference peak ground ac | $a_{gR}$ | 2.500 | [m/s$^2$] |
| Importance factor | $\gamma_I$ | 1.2 | |
| Design ground acceleration | $a_g$ | 3.000 | [m/s$^2$] |
| ⊟ Parameter for description of response spectrum | | | |
| Ground type | | D | |
| Soil factor | S | 1.350 | [-] |
| Lower limit of area of consta | $T_{B-H}$ | 0.200 | [s] |
| Upper limit of area of consta | $T_{C-H}$ | 0.800 | [s] |
| Value defining the beginnin | $T_{D-H}$ | 2.000 | [s] |
| ⊟ Factors | | | |
| Damping correction factor | $\eta$ | 1.000 | [-] |

Figure 3.5: Linear elastic response spectrum with applicable parameters in accordance to the *EN 1998-1* [4].

The acceleration values of the response spectrum are automatically calculated in accordance to the chosen standard. These tabulated values can be exported to *Excel* as shown in Figure 3.6. This is useful to convert the linear spectrum into the inelastic spectrum required for the *Pushover Analysis*.

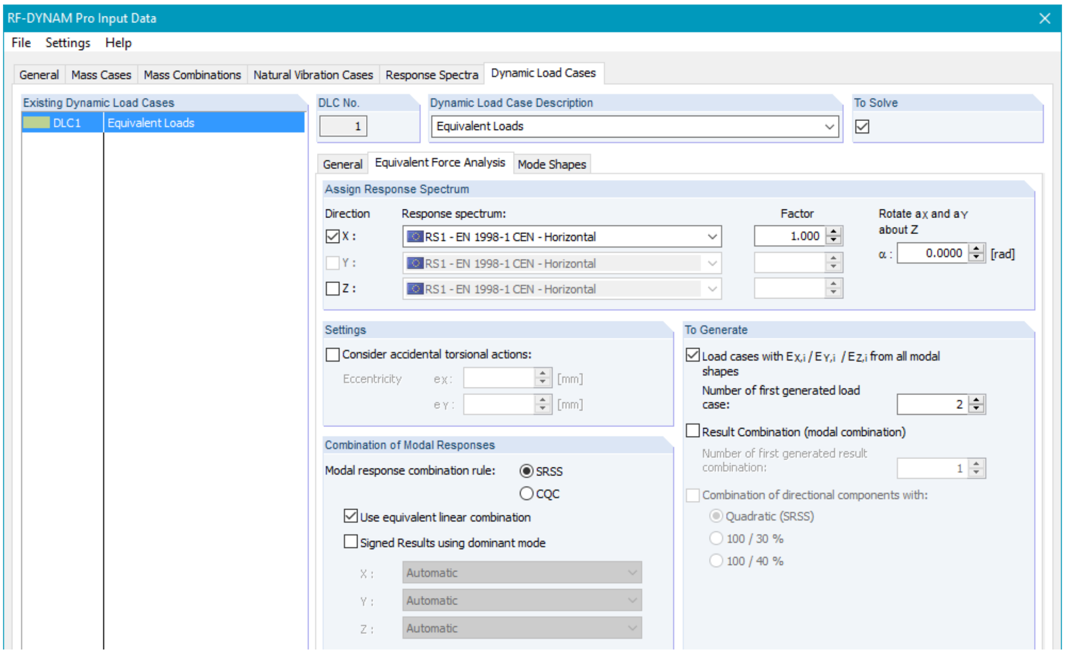


| No. | Period T [s] | Acceleration $S_a$ [m/s$^2$] |
|---|---|---|
| 1 | 0.000 | 4.050 |
| 2 | 0.001 | 4.080 |
| 3 | 0.002 | 4.111 |
| 4 | 0.003 | 4.141 |
| 5 | 0.004 | 4.172 |
| 6 | 0.005 | 4.202 |
| 7 | 0.006 | 4.232 |
| 8 | 0.007 | 4.263 |
| 9 | 0.008 | 4.293 |
| 10 | 0.009 | 4.323 |
| 11 | 0.010 | 4.354 |
| 12 | 0.011 | 4.384 |
| 13 | 0.012 | 4.415 |

Step: 0.001 [s]

Figure 3.6: Tabulated values of the linear elastic response spectrum can be exported to *Excel*.

5. The multi-modal response spectrum analysis can now be performed. The structure is only excited in the X-direction. The generated equivalent loads are exported into *Load Cases*. The settings for the response spectrum analysis are shown in Figure 3.7.



Existing Dynamic Load Cases: DLC1 Equivalent Loads

DLC No.: 1

Dynamic Load Case Description: Equivalent Loads

To Solve: ☑

Assign Response Spectrum

| Direction | Response spectrum: | Factor |
|---|---|---|
| ☑ X : | RS1 - EN 1998-1 CEN - Horizontal | 1.000 |
| ☐ Y : | RS1 - EN 1998-1 CEN - Horizontal | |
| ☐ Z : | RS1 - EN 1998-1 CEN - Horizontal | |

Rotate $a_X$ and $a_Y$ about Z
α : 0.0000 [rad]

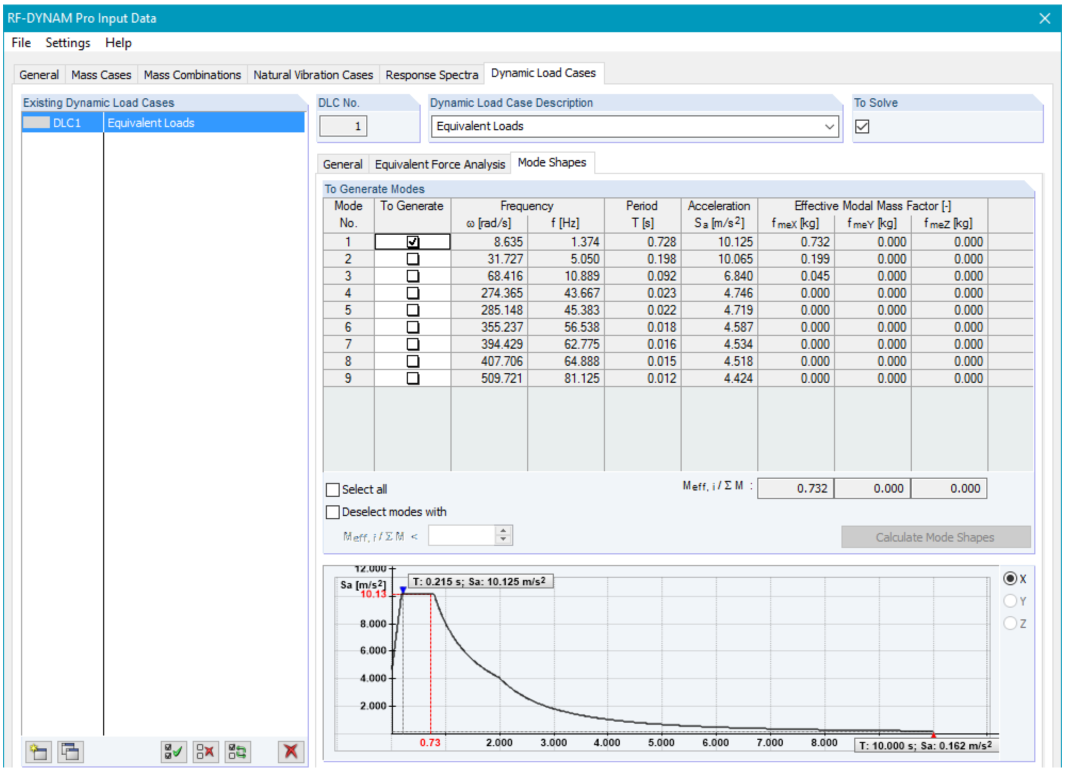
Settings
☐ Consider accidental torsional actions:
Eccentricity $e_X$ : [mm]
$e_Y$ : [mm]

Combination of Modal Responses
Modal response combination rule: ● SRSS ○ CQC
☑ Use equivalent linear combination
☐ Signed Results using dominant mode
X : Automatic
Y : Automatic
Z : Automatic

To Generate
☑ Load cases with $E_{X,i}$ / $E_{Y,i}$ / $E_{Z,i}$ from all modal shapes
Number of first generated load case: 2
☐ Result Combination (modal combination)
Number of first generated result combination: 1
☐ Combination of directional components with:
● Quadratic (SRSS)
○ 100 / 30 %
○ 100 / 40 %

Figure 3.7: Settings for the multi-modal response spectrum analysis with the export of equivalent loads.

In this tutorial, only the fundamental mode shape is evaluated. The selection of eigenvalues is shown in Figure 3.8. Other eigenvalues can easily be analyzed by selecting them in the *Mode Shapes* tab. The equivalent loads of each selected eigenvalue are exported into separate *Load Cases*.



Existing Dynamic Load Cases: DLC1 Equivalent Loads

DLC No.: 1

Dynamic Load Case Description: Equivalent Loads

To Solve: ☑

To Generate Modes

| Mode No. | To Generate | Frequency ω [rad/s] | Frequency f [Hz] | Period T [s] | Acceleration $S_a$ [m/s²] | Effective Modal Mass Factor [-] $f_{meX}$ [kg] | $f_{meY}$ [kg] | $f_{meZ}$ [kg] |
|---|---|---|---|---|---|---|---|---|
| 1 | ☑ | 8.635 | 1.374 | 0.728 | 10.125 | 0.732 | 0.000 | 0.000 |
| 2 | ☐ | 31.727 | 5.050 | 0.198 | 10.065 | 0.199 | 0.000 | 0.000 |
| 3 | ☐ | 68.416 | 10.889 | 0.092 | 6.840 | 0.045 | 0.000 | 0.000 |
| 4 | ☐ | 274.365 | 43.667 | 0.023 | 4.746 | 0.000 | 0.000 | 0.000 |
| 5 | ☐ | 285.148 | 45.383 | 0.022 | 4.719 | 0.000 | 0.000 | 0.000 |
| 6 | ☐ | 355.237 | 56.538 | 0.018 | 4.587 | 0.000 | 0.000 | 0.000 |
| 7 | ☐ | 394.429 | 62.775 | 0.016 | 4.534 | 0.000 | 0.000 | 0.000 |
| 8 | ☐ | 407.706 | 64.888 | 0.015 | 4.518 | 0.000 | 0.000 | 0.000 |
| 9 | ☐ | 509.721 | 81.125 | 0.012 | 4.424 | 0.000 | 0.000 | 0.000 |

☐ Select all
$M_{eff,i} / \Sigma M$ : 0.732 0.000 0.000
☐ Deselect modes with
$M_{eff,i} / \Sigma M$ <

Figure 3.8: Selection of the fundamental eigenvalue in the *Mode Shapes* tab.

6. Calculate the *RF-DYNAM Pro* with the button [OK & Calculate]. The load case *LC3* contains the equivalent loads in accordance to the dominant mode shape. The load distribution is shown in Figure 3.9. Due to the hinge difinition, two FE mesh points exist at each node. Consequently, two loads are exported for each node.

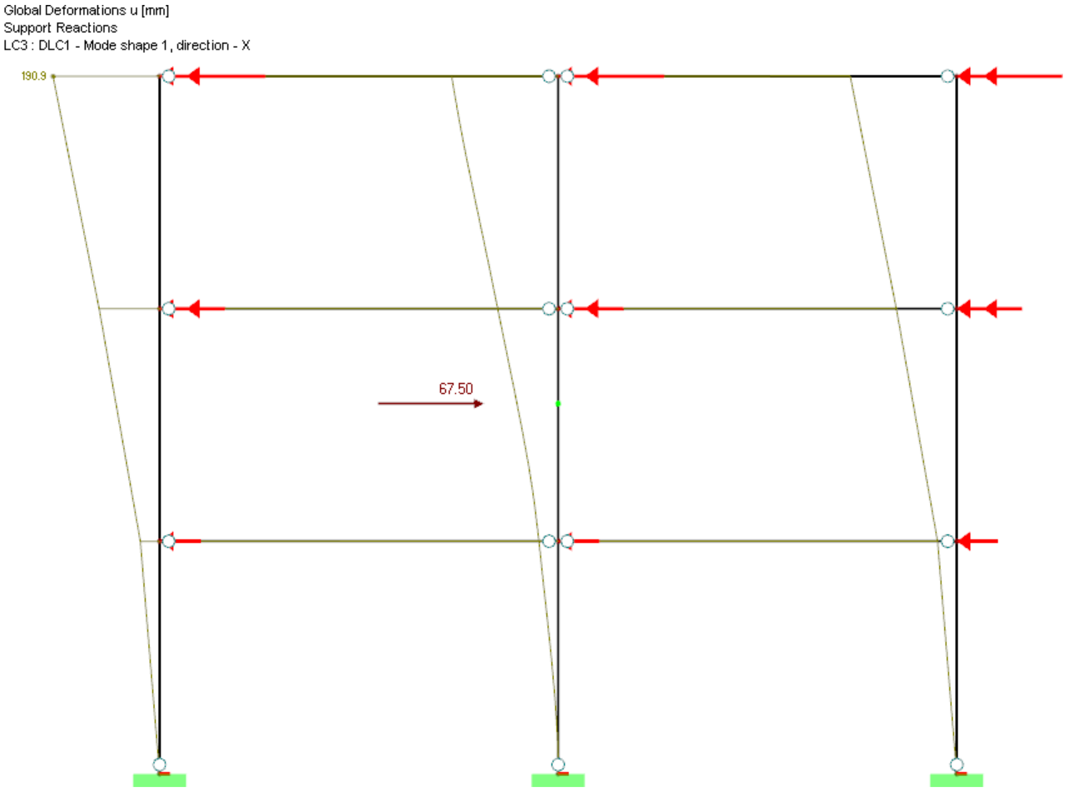
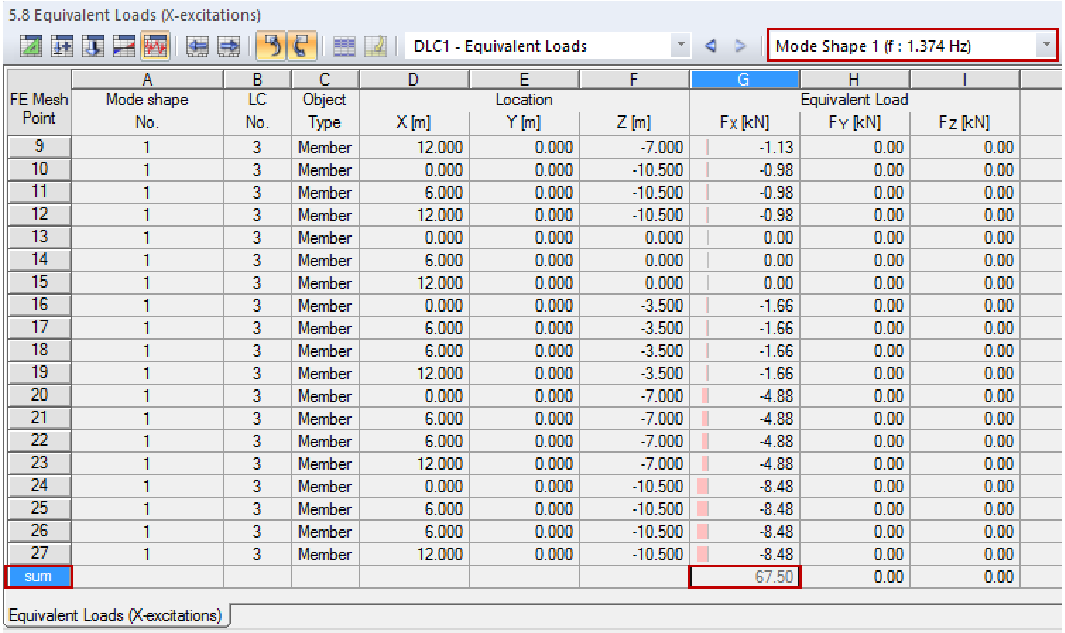


Figure 3.9: Load distribution in accordance to the dominant mode shape of the structure. The equivalent loads are exported from *RF-DYNAM Pro*.

7. The base shear force for the dominant mode is listed in *Table 5.8* as shown in Figure 3.10



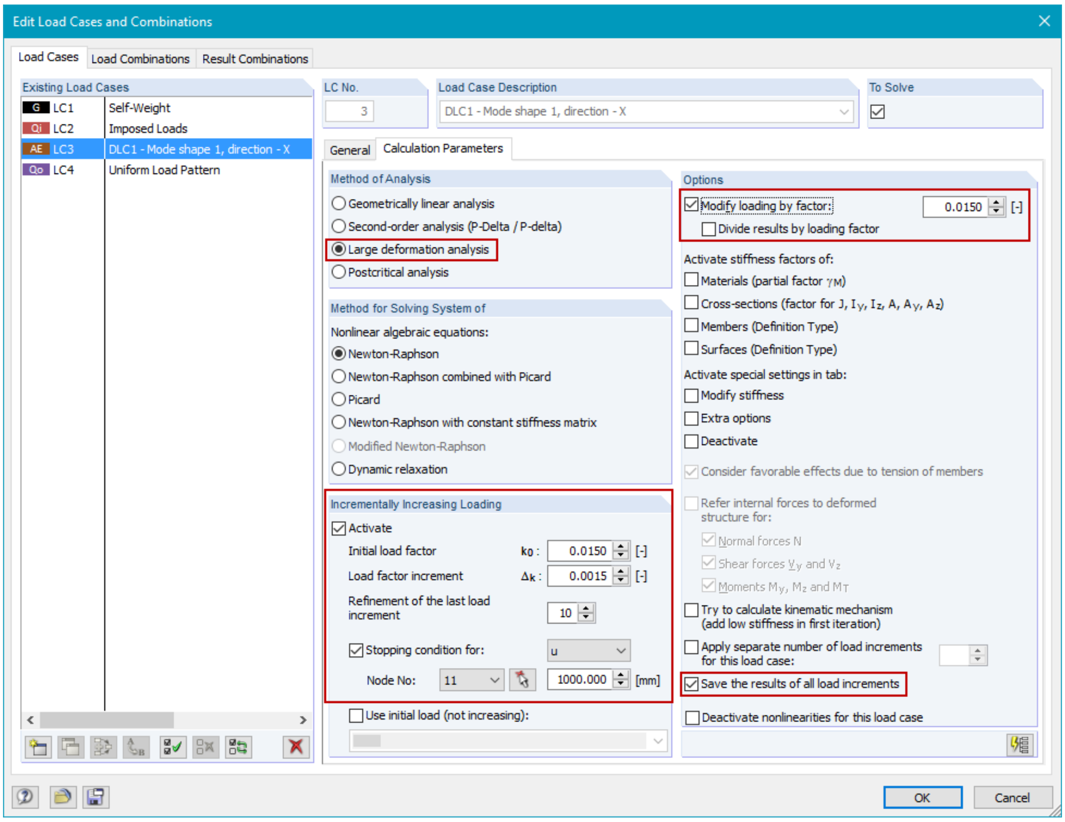
| | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| FE Mesh Point | Mode shape No. | LC No. | Object Type | Location X [m] | Y [m] | Z [m] | Equivalent Load $F_X$ [kN] | $F_Y$ [kN] | $F_Z$ [kN] |
| 9 | 1 | 3 | Member | 12.000 | 0.000 | -7.000 | -1.13 | 0.00 | 0.00 |
| 10 | 1 | 3 | Member | 0.000 | 0.000 | -10.500 | -0.98 | 0.00 | 0.00 |
| 11 | 1 | 3 | Member | 6.000 | 0.000 | -10.500 | -0.98 | 0.00 | 0.00 |
| 12 | 1 | 3 | Member | 12.000 | 0.000 | -10.500 | -0.98 | 0.00 | 0.00 |
| 13 | 1 | 3 | Member | 0.000 | 0.000 | 0.000 | 0.00 | 0.00 | 0.00 |
| 14 | 1 | 3 | Member | 6.000 | 0.000 | 0.000 | 0.00 | 0.00 | 0.00 |
| 15 | 1 | 3 | Member | 12.000 | 0.000 | 0.000 | 0.00 | 0.00 | 0.00 |
| 16 | 1 | 3 | Member | 0.000 | 0.000 | -3.500 | -1.66 | 0.00 | 0.00 |
| 17 | 1 | 3 | Member | 6.000 | 0.000 | -3.500 | -1.66 | 0.00 | 0.00 |
| 18 | 1 | 3 | Member | 6.000 | 0.000 | -3.500 | -1.66 | 0.00 | 0.00 |
| 19 | 1 | 3 | Member | 12.000 | 0.000 | -3.500 | -1.66 | 0.00 | 0.00 |
| 20 | 1 | 3 | Member | 0.000 | 0.000 | -7.000 | -4.88 | 0.00 | 0.00 |
| 21 | 1 | 3 | Member | 6.000 | 0.000 | -7.000 | -4.88 | 0.00 | 0.00 |
| 22 | 1 | 3 | Member | 6.000 | 0.000 | -7.000 | -4.88 | 0.00 | 0.00 |
| 23 | 1 | 3 | Member | 12.000 | 0.000 | -7.000 | -4.88 | 0.00 | 0.00 |
| 24 | 1 | 3 | Member | 0.000 | 0.000 | -10.500 | -8.48 | 0.00 | 0.00 |
| 25 | 1 | 3 | Member | 6.000 | 0.000 | -10.500 | -8.48 | 0.00 | 0.00 |
| 26 | 1 | 3 | Member | 6.000 | 0.000 | -10.500 | -8.48 | 0.00 | 0.00 |
| 27 | 1 | 3 | Member | 12.000 | 0.000 | -10.500 | -8.48 | 0.00 | 0.00 |
| sum | | | | | | | 67.50 | 0.00 | 0.00 |

Figure 3.10: A list of all generated equivalent loads together with the base shear force is provided in *Table 5.8* separate for each mode.

# 4 Non-Linear Static Calculation with Incrementally Increasing Load

The load case *LC3* exported from *RF-DYNAM Pro* includes the equivalent loads and is directly used for the non-linear static analysis for the *Pushover Curve*.

1. Open the *Calculation Parameters* for *LC3* and adjust the parameters. The final settings used in this tutorial are shown in Figure 4.1.
2. Perform a *Large Deformation Analysis* to enable the *Incrementally Increasing Loading* feature.
3. Modify the load with a factor to scale the equivalent loads exported from *RF-DYNAM Pro*. To ensure the load steps are small enough, the following factor is recommended:
$$\frac{1}{\sum F_i} = \frac{1}{67{,}50\,\mathrm{kN}} = 0{,}015 \tag{4.1}$$
using the base shear force $F_i$ as illustrated in Figure 3.9 and listed in Figure 3.10.
4. Activate the *Incrementally Increasing Loading* feature. Set the *Initial Load Factor* to 0,015 for the above discussed reasons. To increase the loads by $\sum \mathrm{F} = 0{,}1$ kN in each calculation step, the load factor increment must be set to 0,0015.
These settings highly depend on the structure and the expected deformation. The smaller the *Load Increment* value, the longer the calculation time. However, additional data points will be available in the *Pushover Curve* for a more exact *Pushover Analysis*. A *Load Increment* convergence study should be performed to find the ideal balance between data points and calculation time. A stopping condition is not necessarily required.
5. Save the result of all load increments.



Edit Load Cases and Combinations

Existing Load Cases

| | | |
|---|---|---|
| G | LC1 | Self-Weight |
| Qi | LC2 | Imposed Loads |
| AE | LC3 | DLC1 - Mode shape 1, direction - X |
| Qo | LC4 | Uniform Load Pattern |

LC No.: 3
Load Case Description: DLC1 - Mode shape 1, direction - X
To Solve: ☑

Method of Analysis
○ Geometrically linear analysis
○ Second-order analysis (P-Delta / P-delta)
● Large deformation analysis
○ Postcritical analysis

Method for Solving System of
Nonlinear algebraic equations:
● Newton-Raphson
○ Newton-Raphson combined with Picard
○ Picard
○ Newton-Raphson with constant stiffness matrix
○ Modified Newton-Raphson
○ Dynamic relaxation

Incrementally Increasing Loading
☑ Activate
Initial load factor $k_0$: 0.0150 [-]
Load factor increment $\Delta k$: 0.0015 [-]
Refinement of the last load increment: 10
☑ Stopping condition for: u
Node No: 11 1000.000 [mm]
☐ Use initial load (not increasing):

Options
☑ Modify loading by factor: 0.0150 [-]
☐ Divide results by loading factor
Activate stiffness factors of:
☐ Materials (partial factor $\gamma_M$)
☐ Cross-sections (factor for J, $I_y$, $I_z$, A, $A_y$, $A_z$)
☐ Members (Definition Type)
☐ Surfaces (Definition Type)
Activate special settings in tab:
☐ Modify stiffness
☐ Extra options
☐ Deactivate
☑ Consider favorable effects due to tension of members
☐ Refer internal forces to deformed structure for:
☑ Normal forces N
☑ Shear forces $V_y$ and $V_z$
☑ Moments $M_y$, $M_z$ and $M_T$
☐ Try to calculate kinematic mechanism (add low stiffness in first iteration)
☐ Apply separate number of load increments for this load case:
☑ Save the results of all load increments
☐ Deactivate nonlinearities for this load case

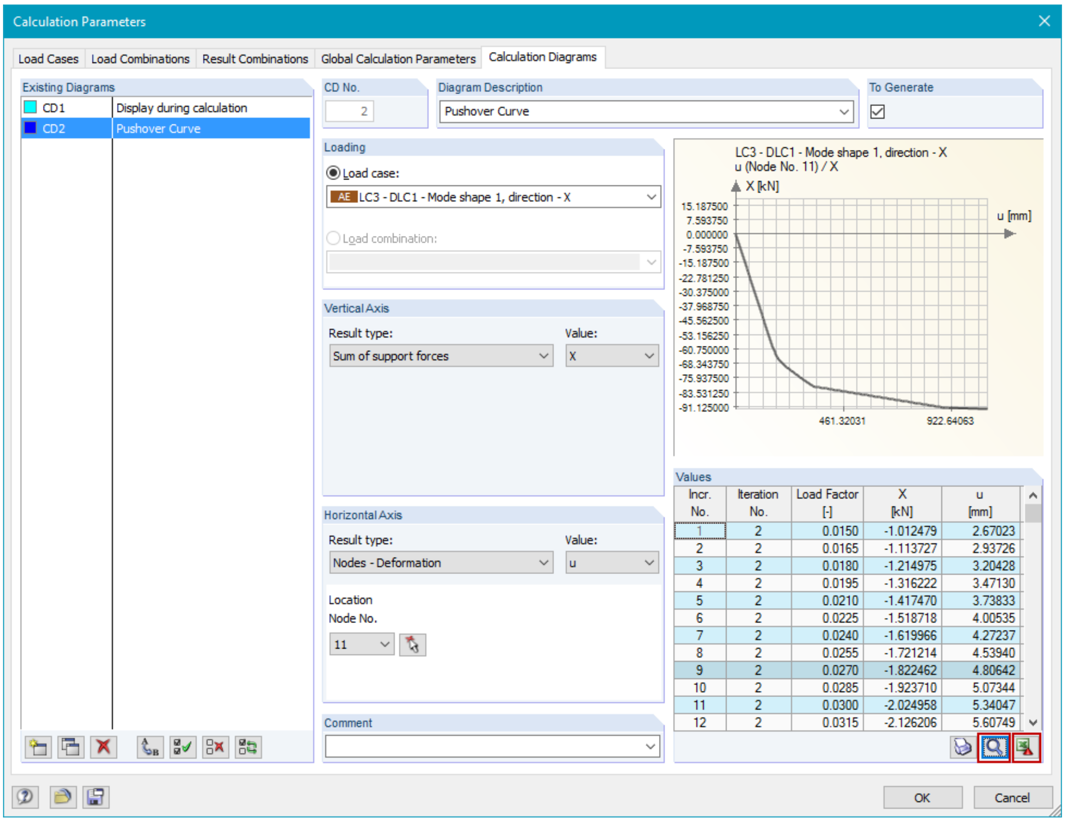
Figure 4.1: The calculation parameters of *LC3* to perform a non-linear static calculation.

6. Calculate the load case *LC3*

# 5 Capacity Curve

Once *LC3* is calculated, the *Pushover Curve* is available. This curve is required to evaluate the *Performance Point* using the *N2-Method* or the *Capacity Spectrum Method*, which is not included in this tutorial.

1. Go to the *Global Calculation Parameters* and select the *Calculation Diagram* tab.
2. Define a new *Calculation Diagram* with the settings shown in Figure 5.1.



Existing Diagrams

| | |
|---|---|
| CD1 | Display during calculation |
| CD2 | Pushover Curve |

CD No.: 2

Diagram Description: Pushover Curve

To Generate: ☑

Loading

● Load case: AE LC3 - DLC1 - Mode shape 1, direction - X

○ Load combination:

Vertical Axis

Result type: Sum of support forces  Value: X

Horizontal Axis

Result type: Nodes - Deformation  Value: u

Location  
Node No.: 11

Comment:

Values

| Incr. No. | Iteration No. | Load Factor [-] | X [kN] | u [mm] |
|---|---|---|---|---|
| 1 | 2 | 0.0150 | -1.012479 | 2.67023 |
| 2 | 2 | 0.0165 | -1.113727 | 2.93726 |
| 3 | 2 | 0.0180 | -1.214975 | 3.20428 |
| 4 | 2 | 0.0195 | -1.316222 | 3.47130 |
| 5 | 2 | 0.0210 | -1.417470 | 3.73833 |
| 6 | 2 | 0.0225 | -1.518718 | 4.00535 |
| 7 | 2 | 0.0240 | -1.619966 | 4.27237 |
| 8 | 2 | 0.0255 | -1.721214 | 4.53940 |
| 9 | 2 | 0.0270 | -1.822462 | 4.80642 |
| 10 | 2 | 0.0285 | -1.923710 | 5.07344 |
| 11 | 2 | 0.0300 | -2.024958 | 5.34047 |
| 12 | 2 | 0.0315 | -2.126206 | 5.60749 |

Figure 5.1: The definition of the *Calculation Diagram* to obtain the *Pushover Curve*. The sum of all lateral loads (base shear) is displayed on the vertical axis. The roof level deformation is displayed on the horizontal axis.

3. Zoom into the *Pushover Curve* using the button 🔍. Data values can be exported to Excel using the button. This is useful for the *Pushover Analysis*.

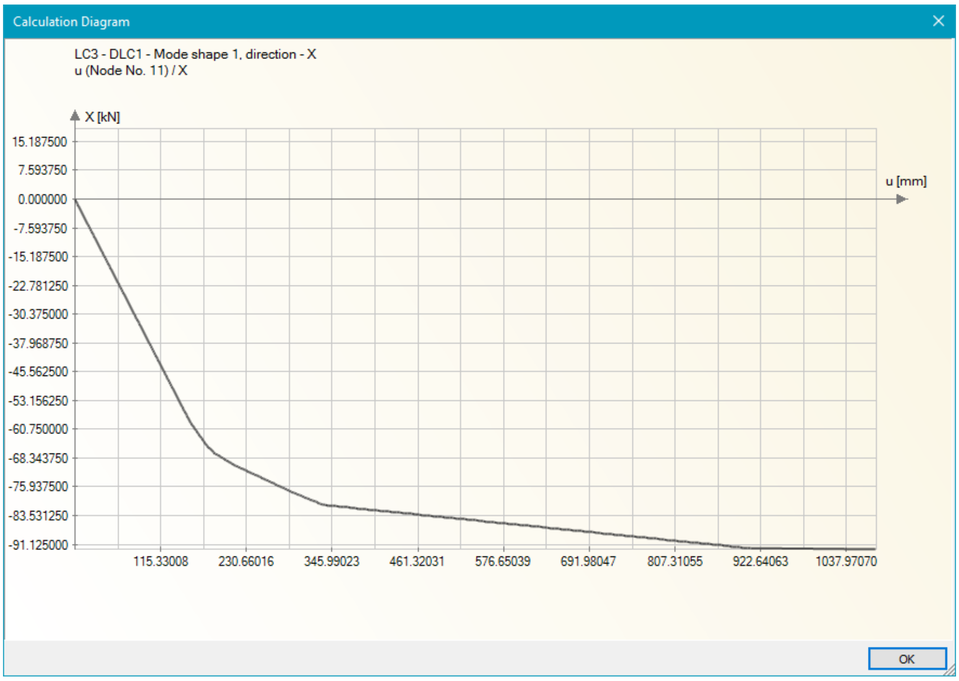
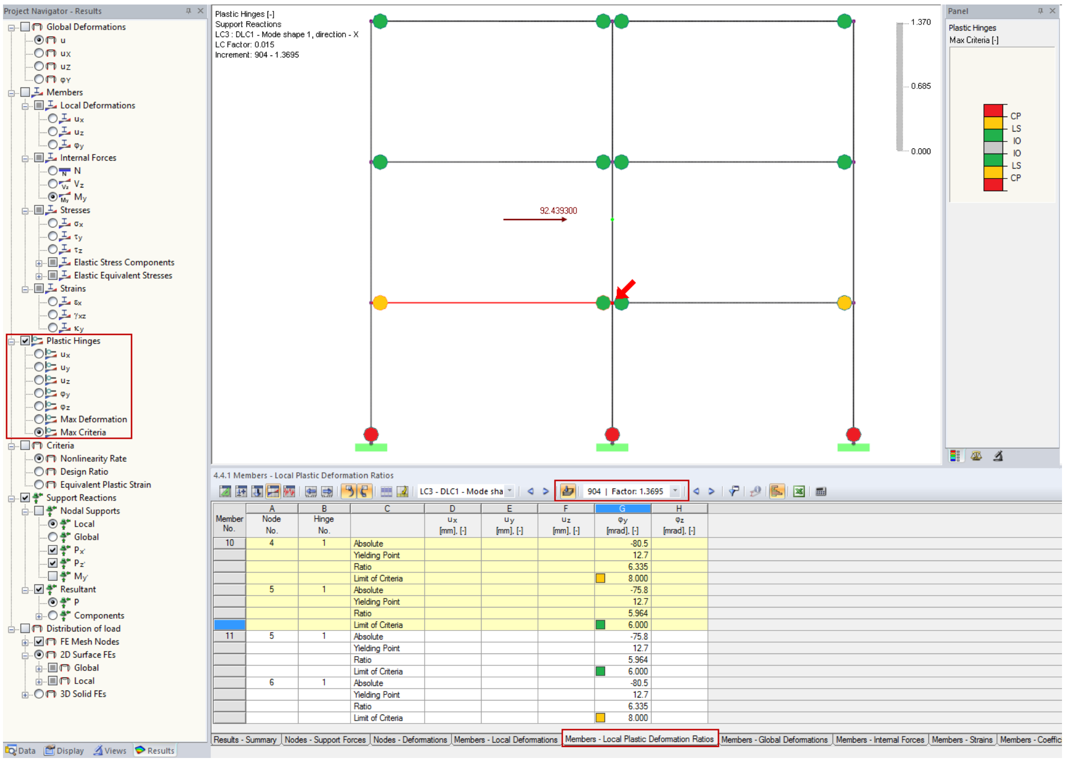


Figure 5.2: The *Pushover Curve* for the load pattern according to the dominant mode shape.

4. Each load increment can be viewed graphically. The displayed plastic hinge color depends on the defined acceptance criteria. An overview for these possibilities are shown in Figure 5.3.



Plastic Hinges [-]
Support Reactions
LC3 : DLC1 - Mode shape 1, direction - X
LC Factor: 0.015
Increment: 904 - 1.3695

4.4.1 Members - Local Plastic Deformation Ratios

| Member No. | A<br>Node No. | B<br>Hinge No. | C | D<br>ux [mm], [-] | E<br>uy [mm], [-] | F<br>uz [mm], [-] | G<br>φy [mrad], [-] | H<br>φz [mrad], [-] |
|---|---|---|---|---|---|---|---|---|
| 10 | 4 | 1 | Absolute | | | | -80.5 | |
| | | | Yielding Point | | | | 12.7 | |
| | | | Ratio | | | | 6.335 | |
| | | | Limit of Criteria | | | | 🟨 8.000 | |
| | 5 | 1 | Absolute | | | | -75.8 | |
| | | | Yielding Point | | | | 12.7 | |
| | | | Ratio | | | | 5.964 | |
| | | | Limit of Criteria | | | | 🟩 6.000 | |
| 11 | 5 | 1 | Absolute | | | | -75.8 | |
| | | | Yielding Point | | | | 12.7 | |
| | | | Ratio | | | | 5.964 | |
| | | | Limit of Criteria | | | | 🟩 6.000 | |
| | 6 | 1 | Absolute | | | | -80.5 | |
| | | | Yielding Point | | | | 12.7 | |
| | | | Ratio | | | | 6.335 | |
| | | | Limit of Criteria | | | | 🟨 8.000 | |

Figure 5.3: The internal moments together with colored plastic hinges are shown in the main graphic. Load increments are selected with the drop-down in the *Panel*. The plastic hinge color legend is in accordance to the acceptance criteria defined.

# Literature

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