Program: RFEM 5, RSTAB 8

Category: Large Deformation Analysis, Post-Critical Analysis, Isotropic Linear Elasticity, Member

Verification Example: 0045 – Snap-Through

0045 – Snap-Through

Description

A structure is made of two trusses, which are embedded into the hinge supports according to the **Figure 1**. The structure is loaded by the concentrated force F_z. The self-weight is neglected in this example. Determine the relationship between the loading force F_z and the deflection u_z considering large deformations generally. Determine the deflection under the loading force $F_z = 122000$ kN of the connection point of the trusses. The problem is described by the following set of parameters.

Material	Steel	Modulus of Elasticity	E	210000.000	MPa
		Poisson's Ratio	ν	0.300	–
Geometry	Structure	Truss Length	L_0	3.000	m
		Height	h	1.500	m
	Cross-Section	Width	a	100.000	mm
Load		Force	F_z	122000.000	kN

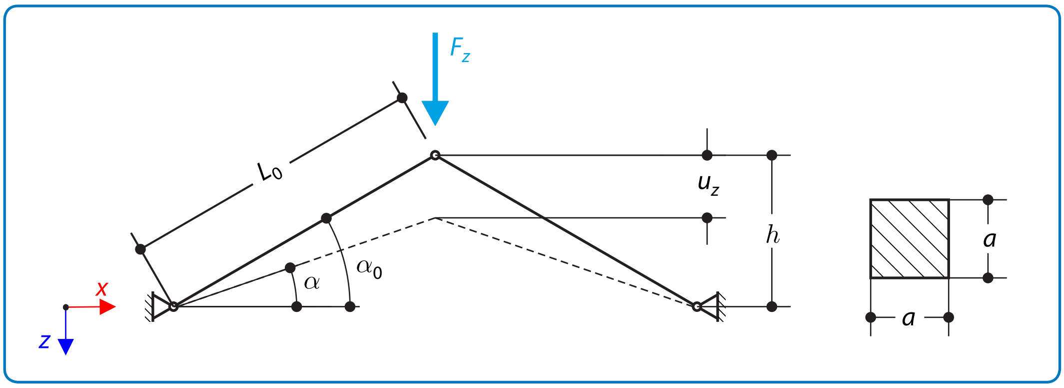

Figure 1: Problem sketch

Analytical Solution

The axial force N in the truss can be determined from the force equilibrium according to the **Figure 2**.

$$N = \frac{F_z}{2 \sin \alpha} \qquad (45-1)$$

Considering Geometrically linear analysis the angle α is remaining constant $\alpha = \alpha_0$ and the formula **(45 – 1)** can be rewritten into the following form.

$$N = F_z \frac{L_0}{2h} \qquad (45 – 2)$$

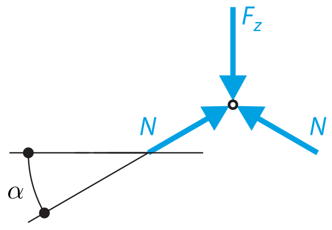

Figure 2: Force equilibrium

Using the Geometrically linear analysis the axial force N is not changing during the deformation of the structure. This is not corresponding with the expectations. The large deformation analysis should be used instead. The axial deformation of the truss can be then determined as follows.

$$\Delta L = L - L_0 = \sqrt{(h - u_z)^2 + b^2} - L_0 \qquad (45 – 3)$$

Where b is the half width of the structure and L is the length of the truss after the deformation. It can be calculated as follows.

$$b = \sqrt{L_0^2 - h^2} \qquad (45 – 4)$$

The formula **(45 – 1)** can be improved into the following form.

$$N = F_z \frac{L}{2(h - u_z)} = F_z \frac{\sqrt{(h - u_z)^2 + b^2}}{2(h - u_z)} \qquad (45 – 5)$$

The axial force N can be also determined from the Hooke's law[1] as

$$N = \varepsilon EA \qquad (45 – 6)$$

Considering the large deformation analysis the logarithmic form of the axial strain ε should be used.

$$\varepsilon = \ln\left(1 - \frac{\Delta L}{L_0}\right) \qquad (45 – 7)$$

Using above mentioned formulae the general relationship between loading force F_z and the deflection u_z can be determined.

[1] Hooke's law $\sigma = E\varepsilon$. The axial stress is defined as $\sigma = \frac{N}{A}$, where A is the cross-section area.

$$F_z = \frac{2EA(h-u_z)\ln\left(1-\frac{\sqrt{(h-u_z)^2+b^2}-L_0}{L_0}\right)}{\sqrt{(h-u_z)^2+b^2}} \qquad (45-8)$$

The formula **(45 – 8)** is obviously nonlinear. The relationship between loading force F_z and the deflection u_z is shown in **Figure 3**. There is also shown the behaviour of the geometrically linear solution. It is obvious, that the instability of the real solution is neglected. The geometrically linear solution is relevant only for small deformation of the structure.

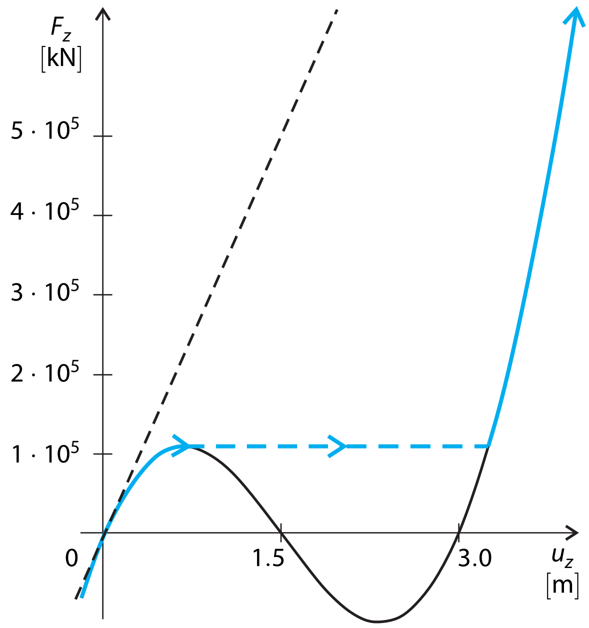

Figure 3: Dependence of loading force on deformation (the blue curve characterizes the advance of the computation process, the dashed line characterizes the geometrically linear solution)

RSTAB 8 and RFEM 5 Settings

- Modeled in RSTAB 8.16.01 / RFEM 5.16.01
- The element size is $l_{FE} = 0.025$ m
- The number of increments is 10
- The structure is modeled using members (Truss - only N)
- Shear stiffness of the members is neglected
- Isotropic linear elastic material model is used
- In global calculation parameters there is disabled: *Activate member divisions for large deformation or post-critical analysis*

Results

Structure Files	Program	Solving Method
0045.01	RFEM 5	Post-Critical Analysis – Modified Newton-Raphson
0045.02	RFEM 5	Large Deformation Analysis – Dynamic Relaxation
0045.03	RSTAB 8	Post-Critical Analysis – Modified Newton-Raphson

Model	Analytical Solution	RSTAB 8 and RFEM 5 Solution	
	u_z [m]	u_z [m]	Ratio [-]
RFEM 5 (Modified Newton-Raphson)	3.282	3.294	1.004
RFEM 5 (Dynamic Relaxation)		3.294	1.004
RSTAB 8 (Modified Newton-Raphson)		3.288	1.002

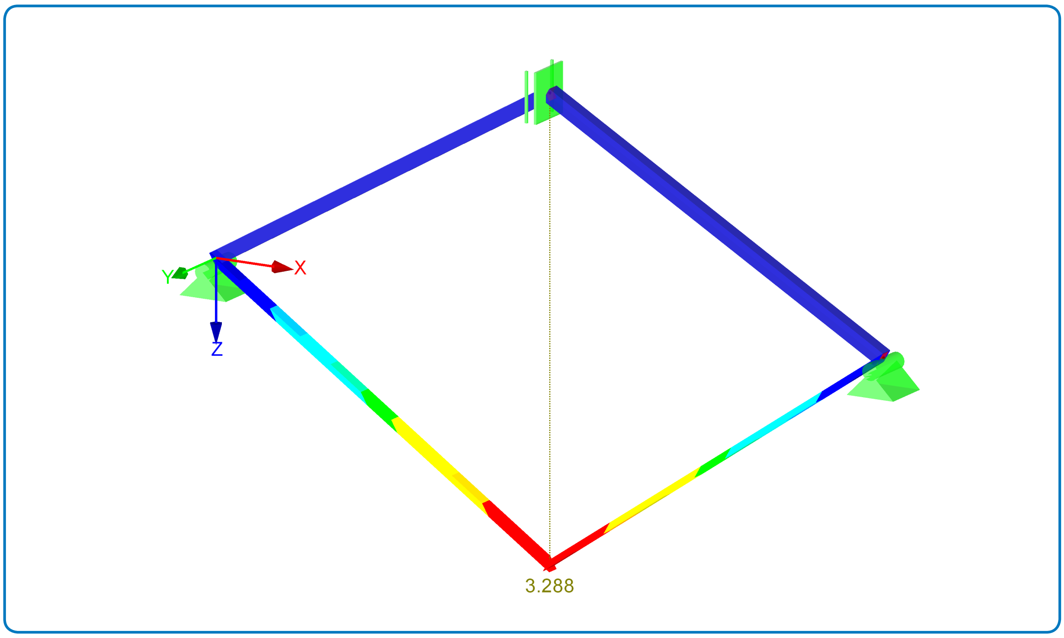

Figure 4: RFEM 5 / RSTAB 8 Results