Program: RFEM 5, RF-LAMINATE, RF-GLASS

Category: Geometrically Linear Analysis, Isotropic Linear Elasticity, Glass, Laminate, Plate, Solid

Verification Example: 0024 – Three Layer Sandwich Cantilever

0024 – Three Layer Sandwich Cantilever

Description

A sandwich cantilever consists of three layers (the core and two faces). It is fixed on the left end and loaded by the concentrated force on the right end according to the **Figure 1**. The problem is described by the following set of parameters.

Material	Faces	Modulus of Elasticity	$E_1 = E_3$	10.000	MPa
		Poisson's Ratio	$\nu_1 = \nu_3$	0.000	–
	Core	Modulus of Elasticity	E_2	0.020	MPa
		Poisson's Ratio	ν_2	0.000	–
Geometry	Common Parameters	Length	L	10.000	m
		Width	w	1.000	m
		Total Thickness	$t = \sum_{i=1}^{3} t_i$	0.580	m
	Faces	Thickness	$t_1 = t_3$	0.040	m
	Core	Thickness	t_2	0.500	m
Load		Force	F	0.750	kN

Small deformations are considered and the self-weight is neglected in this example. Determine the maximum deflection of the structure $u_{z,\max}$.

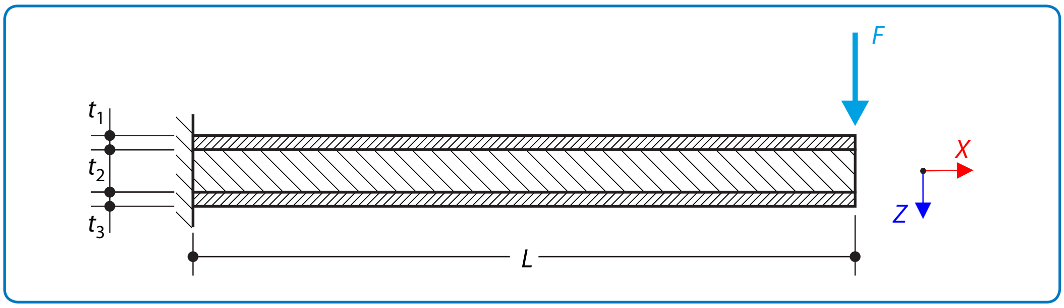

Figure 1: Problem sketch

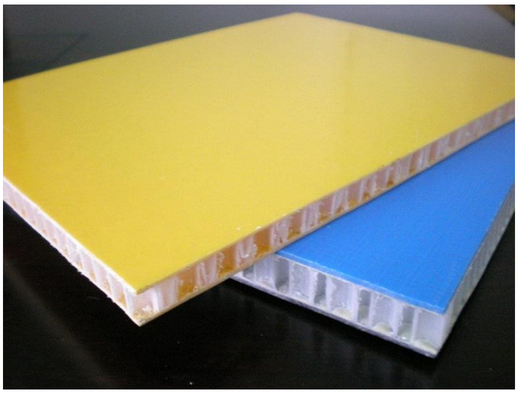

Figure 2: Real sandwich plate

Analytical Solution

The deflection of the single-layer cantilever loaded by a concentrated force, considering only bending, is described by the Bernoulli-Euler formula

$$u_{z,\text{bend}} = \frac{FL^3}{3EI_y} \tag{24 – 1}$$

where I_y is the quadratic moment of the cross-section to the y-axis. Multi-layer beams are analogously described by the formula

$$u_{z,\text{bend}} = \frac{FL^3}{3\sum_k E_k I_{yk}} \tag{24 – 2}$$

where index k sums over all layers. Note, that the quadratic moment of outer layers cross-section has to be transformed by means of Steiner's theorem to the central axis of the cantilever[1]. Quadratic moments of the cross-sections I_{yk} are following:

$$I_{y1} = I_{y3} = \frac{1}{12}wt_1^3 + wt_1\left(\frac{t_1+t_2}{2}\right)^2 \tag{24 – 3}$$

$$I_{y2} = \frac{1}{12}wt_2^3 \tag{24 – 4}$$

$$\sum_{k=1}^{3} E_k I_{yk} = 2E_1 I_{y1} + E_2 I_{y2} = 2E_1\left[\frac{1}{12}wt_1^3 + wt_1\left(\frac{t_1+t_2}{2}\right)^2\right] + E_2\frac{1}{12}wt_2^3 \tag{24 – 5}$$

The deflection caused only by the bend using the formula **(24 – 1)** then yields

$$u_{z,\text{bend}} = 4.264 \text{ m} \tag{24 – 6}$$

It is suitable to take account of the shear effect due to the remarkable cantilever height. The total deflection of the structure $u_{z,\text{max}}$ is composed of the partial deflections due to the bending $u_{z,\text{bend}}$ and the shear $u_{z,\text{shear}}$, which is described in the **Figure 3**, where the dash denotes the differentiation

[1] Steiner's theorem $I_{y2} = I_{y1} + Ad^2$, where A is the cross-section area and $d = y_2 - y_1$ is the perpendicular distance between axis y_1, y_2 to which moments I_{y1}, I_{y2} are related.

with respect to the x. The deflection caused by the shear $u_{z,shear}$ can be calculated according to [1] as follows.

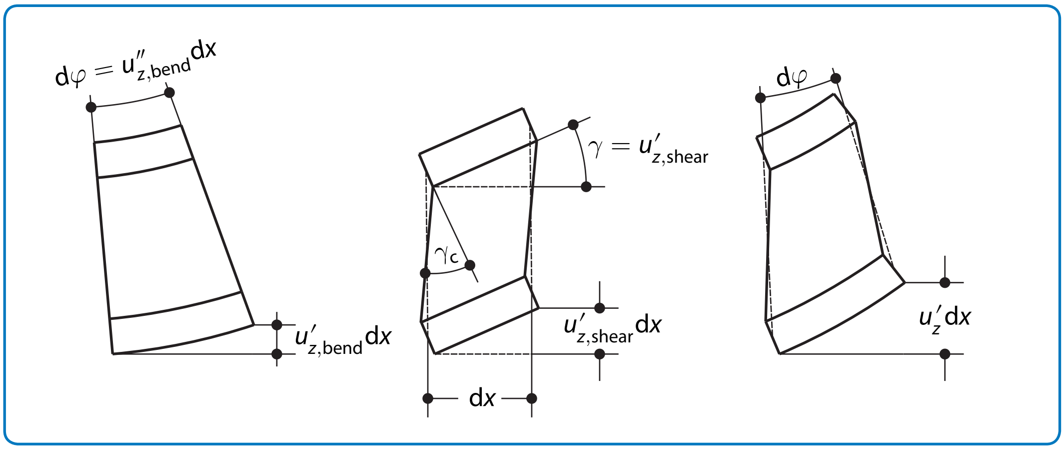

Figure 3: Deformation of an element

The cantilever shear strain γ is realted to the shear strain in the core of the sandwich cantilever γ_c by the formula

$$\gamma_c = \frac{t_2 + t_1}{t_2}\gamma \tag{24 – 7}$$

Thus, the shear stress in the core can be calculated

$$\tau_c = G_2\gamma_c = \frac{t_2 + t_1}{t_2}G_2\gamma \tag{24 – 8}$$

where the shear modulus of the core is $G_2 = E_2/(2(1+\nu_2))$. The shear strain energy stored in the element dx is defined as follows

$$\mathrm{d}U = \frac{1}{2}\frac{\tau_c^2 t_2 w}{G_2} = \frac{1}{2}S\gamma^2 \tag{24 – 9}$$

where the quantity S defines the shear stiffness

$$S = \frac{(t_2 + t_1)^2 w}{t_2}G_2 \tag{24 – 10}$$

The shear strain of the cantilever loaded by the force F is then calculated according to the formula

$$\gamma = \frac{F}{S} \tag{24 – 11}$$

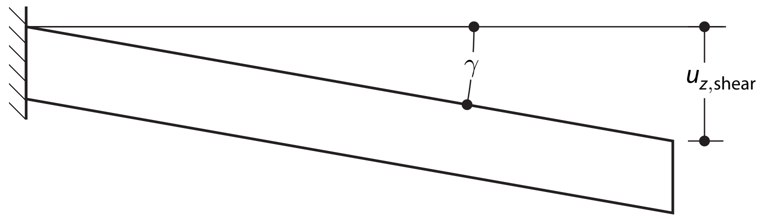

Figure 4: Deflection due to the pure shear

The maximum deflection $u_{z,\text{shear}}$ of the cantilever due to the shear can be calculated according to the **Figure 4**

$$u_{z,\text{shear}} = \gamma L = \frac{F}{S} L = \frac{FLt_2}{(t_2 + t_1)^2 w G_2} = 1.286 \text{ m} \qquad (24-12)$$

The total deflection of the structure is finally calculated

$$u_{z,\max} = u_{z,\text{bend}} + u_{z,\text{shear}} = 4.264 + 1.268 = 5.550 \text{ m} \qquad (24-13)$$

RFEM 5 Settings

- Modeled in RFEM 5.03.0050
- The element size is $l_{\text{FE}} = 0.200$ m
- Geometrically linear analysis is considered
- The number of increments is 5
- Isotropic linear elastic material model is used

Results

Structure File	Program	Entity	Theory
0024.01	RFEM 5	Solid	-
0024.02	RF-LAMINATE	Plate	Kirchhoff
0024.03	RF-GLASS	Plate	Kirchhoff
0024.04	RF-LAMINATE	Plate	Mindlin
0024.05	RF-GLASS	Plate	Mindlin

Model	Analytical Solution	RFEM 5	
	$u_{z,max}$ [m]	$u_{z,max}$ [m]	Ratio [-]
RF-LAMINATE (Kirchhoff Theory)	4.264	4.264	1.000
RF-GLASS (Kirchhoff Theory)		4.264	1.000
RFEM 5, Solid	5.550	5.578	1.005
RF-LAMINATE (Mindlin Theory)		5.546	0.999
RF-GLASS (Mindlin Theory)		5.546	0.999

References

[1] PLANTEMA, F. J. *Sandwich construction: the bending and buckling of sandwich beams, plates, and shells*. Wiley.