0050 – Cantilever Under Torsion Without Warping

Description

A thin-walled cantilever of QRO-profile is fully fixed on the left end ($x = 0$), and the warping is enabled. The cantilever is subjected to a torque M at $x = L_1$ according to the **Figure 1** [1]. The problem is described by the following set of parameters.

Material	Steel	Modulus of Elasticity	E	210000.000	MPa
		Shear Modulus	G	81000.000	MPa
		Plastic Strength	f_y	355.000	MPa
Geometry	QRO Cantilever	Length	L	4.000	m
		Width and Height	b	200.000	mm
		Face Thickness	t	6.000	mm
Load		Torque	M	80.000	kNm
		Position	L_1	2.800	m

Small deformations are considered and the self-weight is neglected. Determine the maximum rotation $\varphi_{x,\max}$ and control the values of the moments M_{Tpri}, M_{Tsec} and M_ω at the point $x = L_1$.

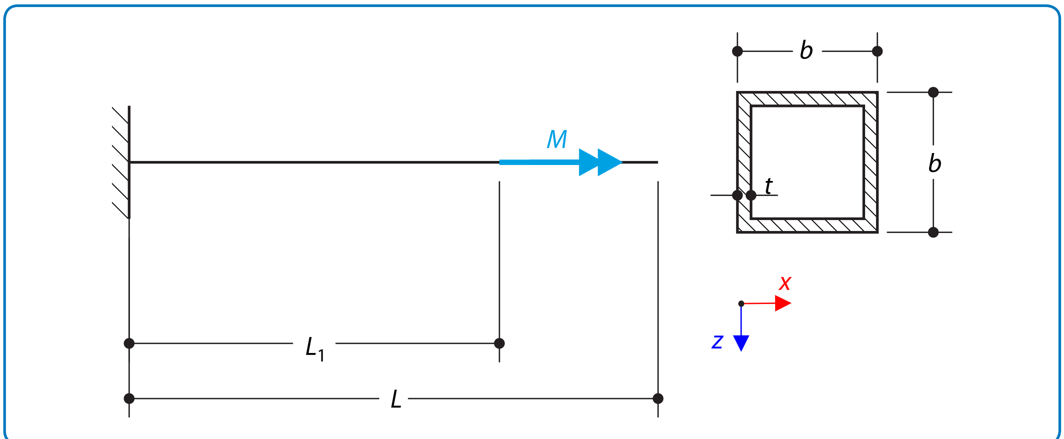

Figure 1: Problem sketch

Analytical Solution

The relative twist ϑ of the cantilever can be calculated according to the following formula

$$\vartheta = \frac{M}{GJ} = 2.252 \cdot 10^{-5}\ \text{rad/mm} \qquad (50-1)$$

where J is the torsional constant[1]. For the given profile it is

$$J = 4.386 \cdot 10^7 \text{ mm}^4 \quad (50-2)$$

The maximum rotation about the x axis $\varphi_{x,\max}$ occurs at the point $x = L_1$ and it has the same value to the free end.

$$\varphi_{x,\max} = \vartheta L_1 = 0.063 \text{ rad} \quad (50-3)$$

For this thin-walled profile (radii in the corners are neglected) is the warping constant $C_\omega = 0$. Thus the moments M_{Tsec} and M_ω, which are defined below, should be zero on the full length of the profile. The primary torsional moment M_{Tpri} should be coincident with the total torsional moment M_{T}.

$$M_\omega(x) = -EC_\omega \varphi'(x) \quad (50-4)$$

$$M_{\text{Tsec}} = \frac{\mathrm{d}M_\omega(x)}{\mathrm{d}x} \quad (50-5)$$

RFEM 5 and RSTAB 8 Settings

- Modeled in RFEM 5.05.0029 and RSTAB 8.05.0029
- The element size is $l_{\text{FE}} = 0.200$ m
- The number of increments is 5
- Isotropic linear elastic material model is used
- The structure is modeled using members

Results

Structure Files	Program
0050.01	RFEM 5
0050.02	RSTAB 8
0050.03	RF-FE-LTB

	Analytical Solution	RFEM 5	Ratio	RSTAB 8	Ratio	RF-FE-LTB	Ratio [-]
$\varphi_{x,\max}$ [rad]	0.063	0.063	1.000	0.063	1.000	0.063	1.000

In the following graphs there is demonstrated the behaviour of all the moments on the given cantilever calculated in RF-FE-LTB module. The primary torsional moment M_{Tpri} should be coincident with the total torsional moment M_{T}. The moments M_{Tsec} and M_ω should be zero on the full length

[1] The torsional constant for the given profile can be approximately calculated as $J_{\text{approx.}} = t(b-t)^3$. The exact value is taken from RFEM 5 / RSTAB 8.

of the profile. It can be seen there is the affected area in the nearby of the loading point ($x = L_1$), when RF-FE-LTB module is used (warping is considered). Note that the above mentioned effect is getting smaller with the smoother mesh.

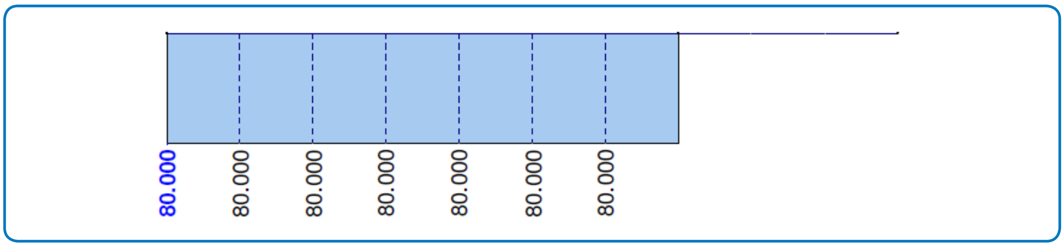

Figure 2: Total torsional moment M_T [kNm] behaviour

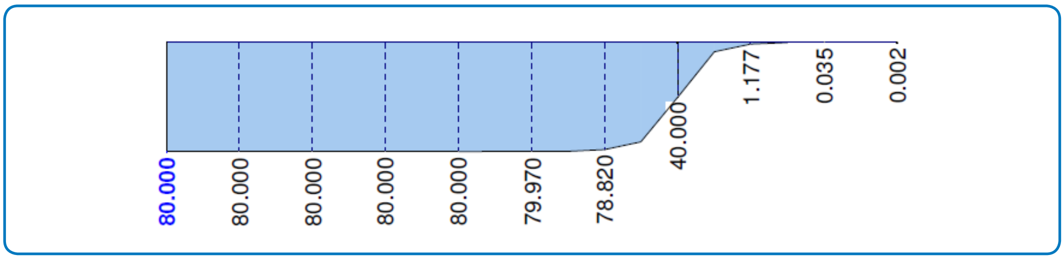

Figure 3: Primary torsional moment M_{Tpri} [kNm] behaviour

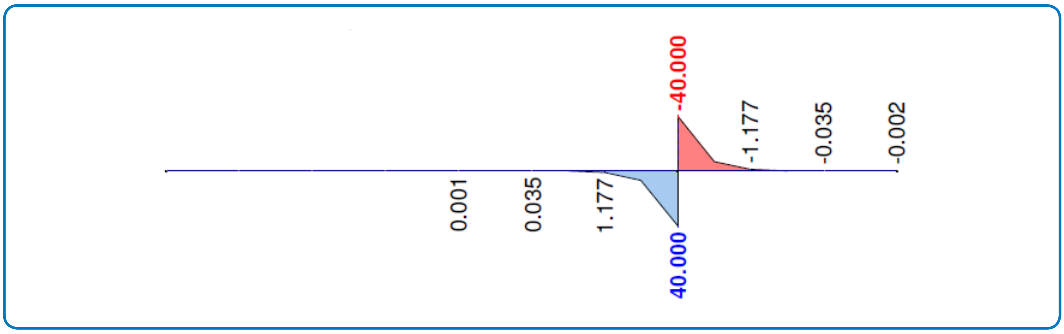

Figure 4: Secondary torsional moment M_{Tsec} [kNm] behaviour

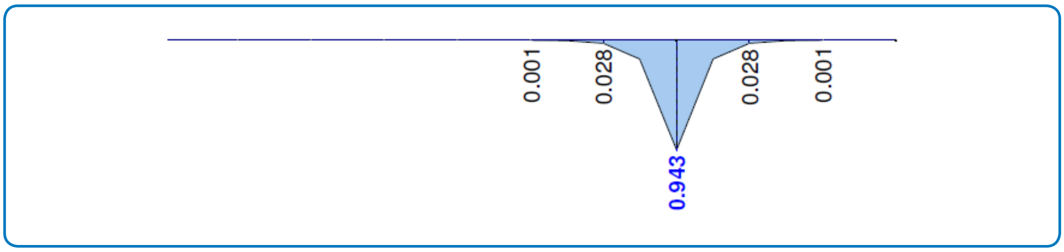

Figure 5: Warping moment M_ω [kNm^2] behaviour

References

[1] LUMPE, G. and GENSICHEN, V. *Evaluierung der linearen und nichtlinearen Stabstatik in Theorie und Software: Prüfbeispiele, Fehlerursachen, genaue Theorie*. Ernst.