Program: RFEM 5, RSTAB 8, RF-DYNAM Pro, DYNAM Pro

Category: Geometrically Linear Analysis, Isotropic Linear Elasticity, Member, Dynamics

Verification Example: 0113 – Suddenly Applied Load to Simply Supported Beam

0113 – Suddenly Applied Load to Simply Supported Beam

Description

A given force F_z is suddenly applied at the mid-span of a simply supported beam at time t_0. Considering only small deformation theory, determine the maximum deflection u_{max} of the beam.

Material	Elastic	Specific Weight	γ	5000.000	kN/m^3
		Modulus of Elasticity	E	50.000	GPa
		Poisson's Ratio	ν	0.500	–
Geometry	Beam	Width	w	0.100	m
		Height	h	0.100	m
		Length	L	1.000	m
Load	Force	Value	F_z	10.000	kN
		Applied at Time	t_0	0.100	s

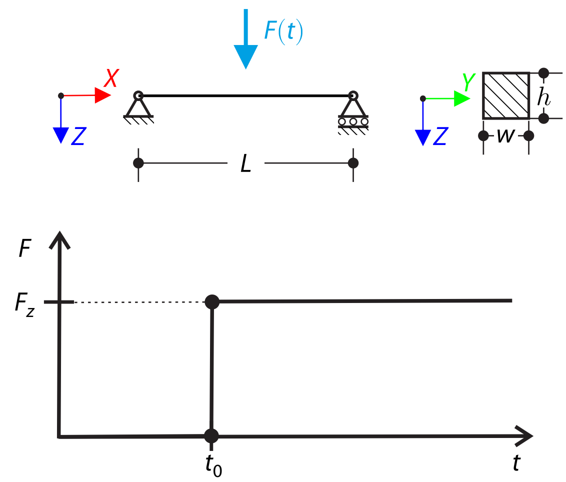

Figure 1: Problem Sketch

Analytical Solution

The problem can be simplified as a single-degree-of-freedom system, the behavior of which can be described, for $t > t_0$, by the second-order differential equation of undamped motion

$$m\frac{d^2u}{dt^2}(t) + ku(t) = F_z \quad (113-1)$$

under the initial conditions

$$u(t_0) = 0, \qquad (113-2)$$

$$\frac{\mathrm{d}u}{\mathrm{d}t}(t_0) = 0 \qquad (113-3)$$

where the mass m has no influence on the maximum deflection and does not have to be defined, k is the bending stiffness of the beam defined as a multiplicative inverse of maximum deflection u_1 of the beam under a unit mid-span force

$$k = \frac{1}{u_1} = \frac{48EI}{L^3} = \frac{4Ewh^3}{L^3} \qquad (113-4)$$

where $I = \frac{wh^3}{12}$ is the second moment of beam cross-section area. Then the equation **(113 – 1)** admits a general solution

$$u(t) = A\cos(\omega t) + B\sin(\omega t) + u_p \qquad (113-5)$$

where u_p is the particular solution of the equation and is equal to the deflection of the system under the static force F_z, namely

$$u_p = \frac{F_z}{k} \qquad (113-6)$$

and ω is angular frequency of the structure:

$$\omega = \sqrt{\frac{k}{m}} \qquad (113-7)$$

Shifting **(113 – 1)** and **(113 – 2)** into the origin, i.e., $t \mapsto t - t_0$, the coefficients A and B can be determined

$$u(0) = 0 \quad \Rightarrow \quad A = -\frac{F_z}{k} \qquad (113-8)$$

$$\frac{\mathrm{d}u}{\mathrm{d}t}(0) = 0 \quad \Rightarrow \quad B = 0 \qquad (113-9)$$

Substituting A and B back into **(113 – 5)**, the resultant deformation reads as

$$u(t) = (1 - \cos(\omega t))\frac{F_z}{k} \qquad (113-10)$$

Hence, it can be stated that for a sudden force the maximum deflection of the beam equals twice the deflection of the statically loaded beam

$$u_{\max} = 2\frac{F_z}{k} = 1.000 \text{ m} \qquad (113-11)$$

RFEM 5 and RSTAB 8 Settings

- Modelled in version RFEM 5.07.07 and RSTAB 8.07.05
- Geometrically linear analysis is considered
- Shear stiffness of members is deactivated
- Mass generated from self-weight of structure is considered in the *Z*-direction
- Root of characteristic polynomial is used as solving method

Results

Structure File	Program
0113.01	RF-DYNAM Pro
0113.02	DYNAM Pro

As can be seen from the table below, good agreement of the analytical result with the numerical output was achieved

Analytical Solution	RF-DYNAM Pro		DYNAM Pro	
u_{max} [mm]	u_{max} [mm]	Ratio [-]	u_{max} [mm]	Ratio [-]
1.000	1.000	1.000	1.000	1.000