Verification Example

Program: RFEM 5

Category: Geometrically Linear Analysis, Isotropic Linear Elasticity, Plate

Verification Example: 0016 – Elastic Bending - Continuous Load

0016 – Elastic Bending - Continuous Load

Description

A thin plate is fully fixed on the left end ($x = 0$) and is subjected to a uniform pressure p according to the **Figure 1**. The problem is described by the following set of parameters.

Material	Elastic-Plastic	Modulus of Elasticity	E	210000.000	MPa
		Poisson's Ratio	ν	0.000	
		Shear Modulus	G	105000.000	MPa
Geometry	Plate	Length	L	1.000	m
		Width	w	0.050	m
		Thickness	t	0.005	m
Load	Plate	Pressure	p	2750.000	Pa

Small deformations are considered and the self-weight is neglected in this example. Determine the maximum deflection $u_{z,\max}$. The aim of this example is to show that a surface of the surface stiffness type *Without Tension* behaves linearly under bending.

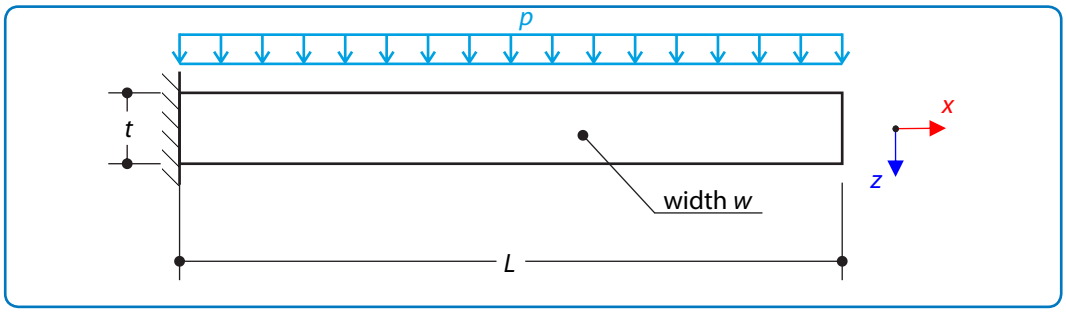

Figure 1: Problem sketch

Analytical Solution

The bending moment M for the plate under continuous loading $q = pw$ is defined by

$$M = -\frac{q(L-x)^2}{2} \qquad (16-1)$$

Recalling the elementary equation for the beam deflection using the Euler-Bernoulli theory

$$EI_y \frac{\mathrm{d}^2 u_z}{\mathrm{d}x^2} = M \tag{16 – 2}$$

where I_y is the quadratic moment of a cross-section with respect to the y-axis and it is given by

$$I_y = \frac{wt^3}{12} \tag{16 – 3}$$

We integrate the equation **(16 – 2)** from 0 to L twice. Upon applying the boundary conditions for cantilever beam $u_z(0) = u_z'(0) = 0$, the solution takes the following form

$$u_z(x) = \frac{qx^2(6L^2 - 4Lx + x^2)}{24EI_y} \tag{16 – 4}$$

The maximum displacement occurs at the point $x = L$:

$$u_{z,\max} = \frac{qL^4}{8EI_y} = \frac{3pL^4}{2Ew^2t} = 157.143 \text{ mm} \tag{16 – 5}$$

RFEM 5 Settings

- Modeled in RFEM 5.04.0024
- The element size is $l_{FE} = 0.020$ m
- Geometrically linear analysis is considered
- The Mindlin plate theory is used
- Isotropic linear elastic material model is used

Results

Structure File	Surface Stiffness Type
0016.01	Standard
0016.02	Without Tension

Surface Stiffness Type	Analytical Solution	RFEM 5	
	$u_{z,\max}$ [mm]	$u_{z,\max}$ [mm]	Ratio [–]
Standard	157.143	157.130	1.000
Without Tension		157.150	1.000