How to

Evaluate the Capacity Curve for Pushover Analysis

RFEM with RF-DYNAM Pro

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1 Introduction

The Pushover Analysis (POA) is a nonlinear static method for the seismic analysis of structures. A pre-determined lateral load pattern is applied onto the structure and steadily increased to identify yielding and plastic hinge formations and the load at which failure of the various structural components occurs. The Capacity Curve or Pushover Curve represents the nonlinear behaviour of the structure and is a load-deformation curve of the base shear force versus the horizontal roof displacement of the building. Pushover analysis transforms a dynamic problem to a static problem.

In accordance to FEMA 356 [1], at least two load patterns should be used in order to envelope the response. The most common load patterns are:

- Load pattern based on the fundamental mode shape or any other modes of interest. Therefore, the use of RF-DYNAM Pro is recommended and shown in the tutorial
- Uniform load distribution (according to mass distribution)
- FEMA load distribution in accordance to FEMA 356 [1]

The Pushover Curve is connected to the Inelastic Response Spectrum to evaluate a Performance Point which is the maximum displacement the building can cope with. There are different Pushover Analysis methods:

- Capacity Spectrum Method (CSM) applied in ATC 40 [2]
- N2-Method introduced by P.Fajfar [3]
- Displacement Coefficients Method (DCM)

Pushover analysis is based on the assumption that the structure has one dominant eigenvalue and mode shape. It is also assumed that this eigenvalue remains the same during the elastic and inelastic response. Detailed theoretical background of these Pushover Methods can be found in various literature (i.e. [4], [5], [3]). Multi-modal pushover analysis methods are described in [6] and [7].

The following tutorial shows how the Capacity Curve of a two-story steel frame can be evaluated with RFEM. The Capacity Curve is the basis for the pushover analysis employing one of the methods listed above. The following steps are performed in this tutorial:

- Definition of Plastic Hinges in accordance to FEMA 356 [1]
- Definition of Lateral Load Pattern based on the fundamental eigenvalue
- Non-linear static calculation with incrementally increasing load
- Evaluation of the Capacity Curve and plastic hinge deformation

The structural system considered in this tutorial is a two-story steel frame. The structure and cross-sections are shown in Figure 1.1. The material used is steel A992. When applying lateral loads to this system, the largest moments are expected at the joints. Therefore, plastic hinges are defined at the start and end of each beam. The structural system and the position of the plastic hinges represent a strong-column / weak-beam frame building where a beam-sway plastic mechanism is expected.
Figure 1.1: Two-story steel frame used to evaluate the *Pushover Curve*. The cross-sections and member lengths are labeled accordingly. The plastic hinges are illustrated as member end pins.

Plastic hinges are assigned to several members. Within RFEM, plastic hinges in accordance to the *FEMA 356, Chapter 5* [1] are available. The hinge diagram values and acceptance criteria are pre-defined for steel members. The yielding moments and yielding rotations are dependent on the cross-section and the material of the members. *RFEM* will set the yielding values automatically.
2 Plastic Hinges

It is recommended to create separate plastic hinge types for each member type and members of different lengths. All pre-defined hinge values are defined in the Plastic Hinge dialogue box.

2.1 W 8x15 Hinges

1. Edit the first level W 8x15 members, and define new hinges at the start and end of the member.

![Figure 2.1: Edit the members with cross-section W 8x15 and add new member hinges at the start and end of the members.](image)

2. Define a moment hinge and choose the nonlinearity Plastic Hinge.

![Figure 2.2: Selection of the nonlinearity Plastic Hinge for the moment hinge.](image)

3. When you enter the dialogue Nonlinearity - Plastic Hinge - My, the hinge type FEMA 356 Rigid-Plastic Automatic is pre-set. The hinge diagram and the acceptance criteria in accordance to FEMA 356 Table 5.6 - Beam Flexure cannot be adjusted. For the cross-section W 8x15 an interpolation between Line a and Line b of FEMA 356 Table 5.6 - Beam Flexure is not required.
and the values of Line a are used. A selection can be made between the Primary and Secondary acceptance criteria. The yield limits are calculated in accordance to FEMA 356 Equation 5-1 and 5-6. For the W 8x15 with A992 material, the yield moment \( M_{y,\text{yield}} \) and yield rotation \( \varphi_{y,\text{yield}} \) of the hinge are determined with the following equations.

\[
M_{y,\text{yield}} = Z_y \cdot f_{y,\text{yield}} = 13.6 \text{ in}^3 \cdot 50 \text{ ksi}
\]

\[
M_{y,\text{yield}} = 56.667 \text{ kipft}
\]

\[
\varphi_{y,\text{yield}} = \frac{Z_y \cdot f_{y,\text{yield}} \cdot l_b}{6EI_y} = \frac{13.6 \text{ in}^3 \cdot 50 \text{ ksi} \cdot (17.5 \cdot 12)\text{in}}{6 \cdot 29,000 \text{ ksi} \cdot 48 \text{ in}^4}
\]

\[
\varphi_{y,\text{yield}} = 0.0171 \text{ rad}
\]

The following definitions are used in Equations 2.1 and 2.2.

- \( Z_y \): Plastic section modulus
- \( f_{y,\text{yield}} \): A992 yield strength (ANSI/AISC 360-16)
- \( l_b \): Length of the beam
- \( E \): Modulus of elasticity
- \( I_y \): Moment of inertia

In this tutorial, all default values are used. However, values can be adjusted in the dialogue box to account for various scenarios (i.e. connections, interaction with normal forces, etc.).

![Image](https://via.placeholder.com/150)

Figure 2.3: Plastic Hinge with the default FEMA 356 | Rigid-Plastic | Automatic hinge selected. The hinge diagram parameters are set automatically and can be found in FEMA 356 Table 5.6. The yield limits are pre-set for the W 8x15 with material A992 and a member length of \( l_b = 17.5 \text{ ft} \).
2.2 W 6x12 Hinges

4. Define now the plastic hinges for the upper level beams with W 6x12 cross-sections.

The automatic plastic hinges are used. The diagram parameters and acceptance criteria are identical to the hinges defined for the W 8x15 members (compare with Figure 2.3). A cross-section interpolation is not required for the W 6x15. The difference to the previously defined hinges include the yield limits calculated in Equations 2.1 and 2.2, using the W 6x12 plastic section modulus $Z_y = 8.3 \text{ in}^3$ and the moment of inertia $I_y = 22.1 \text{ in}^4$. The resulting yielding limits are shown in Figure 2.5.
2.3 W 8x24 Hinges

5. We define plastic hinges at the bottom of the columns to ensure final collapse of the steel frame.

![Figure 2.6: Edit the members with cross-section W 8x24 and add new member hinges at the start of the members.](image)

6. Again, we choose the default automatic plastic hinge option for this cross-section. For the cross-section W 8x24 an interpolation between Line a and Line b of FEMA 356 Table 5.6 - Beam Flexure (Columns - Flexure with $P/P_{u}$ << 0.2) [1] is required as the flange slenderness condition is not fulfilled for Line a. The plastic hinge dialogue is shown in Figure 2.7. The diagram parameters are different to the previously defined hinges for the W 8x15 and W 6x12.

![Figure 2.7: Plastic Hinge with the default FEMA 356 | Rigid-Plastic | Automatic hinge selected. The hinge diagram parameters are interpolated automatically to match the criteria for flange and web slenderness in accordance to FEMA 356 Table 5.6 (Columns - Flexure with $P/P_{u}$ << 0.2). The secondary acceptance criteria is selected. The yield limits are set automatically for the W 8x24 with A992 material and a member length of $l_b = 10$ ft.](image)
RFEM automatically calculates the slenderness checks and interpolates the correct values for diagram parameters and acceptance criteria as shown below.

\[
\frac{b_f}{2t_f} \leq \frac{52}{\sqrt{f_{\text{yield}}}}
\]

(2.3)

\[
\frac{b_f}{2t_f} \geq \frac{65}{\sqrt{f_{\text{yield}}}}
\]

(2.4)

with

- \(b_f\) : Width of the cross-section, \(b_f = 6.5\) in
- \(t_f\) : Thickness of the flange, \(t_f = 0.4\) in
- \(f_{\text{yield}}\) : A992 yield strength (ANSI/AISC 360-16), \(f_{\text{yield}} = 50\) ksi

For the \(W\ 8\times 24\) the flange slenderness conditions are not fulfilled, and the hinge diagram values and acceptance criteria are linearly interpolated between Line a and Line b of FEMA 356 Table 5.6. The interpolated values are shown in Figure 2.7.

Web slenderness is not an issue for the \(W\ 8\times 24\). This is checked with the following condition:

\[
\frac{h}{t_w} \leq \frac{300}{\sqrt{f_{\text{yield}}}}
\]

(2.5)

With

- \(h\) : Height of the cross-section in between the flanges, \(h = 7.13\) in
- \(t_w\) : Thickness of the web, \(t_w = 0.25\) in
- \(f_{\text{yield}}\) : A992 yield strength (ANSI/AISC 360-16:2016), \(f_{\text{yield}} = 50\) ksi

This results in \(28.52 \leq 42.43\) and no interpolation due to web slenderness needs to be done.

\[
\frac{h}{t_w} \leq \frac{418}{\sqrt{f_{\text{yield}}}}
\]

(2.6)

\[
\frac{h}{t_w} \leq \frac{640}{\sqrt{f_{\text{yield}}}}
\]

(2.7)
3 Load Pattern

Increasing lateral load patterns need to be applied to the structure up to the inelastic state. The goal is to monitor the progressive yielding of the structure. *FEMA 356* [1] recommends applying at least two load patterns in order to envelope the response. In this tutorial only one type of load pattern is shown.

The most common load distribution is in accordance to the structure’s dominant mode shape. The eigenvalues and dominant mode shape can easily be determined in *RFEM* with the add-on module *RF-DYNAM Pro*. In addition, with *RF-DYNAM Pro - Equivalent Loads*, a response spectrum analysis is performed and equivalent loads are exported into *Load Cases* within *RFEM*.

*RF-DYNAM Pro* evaluates eigenvalues, the mass distribution of the structure and the linear response spectrum in accordance to the *ASCE 7*, *NBCC* and other international building standards. This information is required for the *Pushover Analysis* when utilizing the *N2-Method* or the *Capacity Spectrum Method*.

1. Open the add-on module *RF-DYNAM Pro* and select the *Response spectrum analysis with generation of equivalent loads*.

   ![Figure 3.1: Within the General tab of the add-on module RF-DYNAM Pro, the response spectra analysis with generation of equivalent loads is selected.](image)

2. Masses are defined in the *Mass Case* and *Mass Combination* tab. Defined masses in this example are the self-weight of the structure and imposed loads. The masses are imported from *Load Cases* into the module *RF-DYNAM Pro* and are combined in *Mass Combinations*. Combination factors can be adjusted.
3. Load Pattern

Figure 3.2: The self-weight and imposed loads imported from LC2 are defined as separate mass cases.

Figure 3.3: The masses from self-weight and imposed loads are combined with a factor of 1.0.

3. The eigenvalues and mode shapes are calculated based on the defined masses. In this example, only the deformation in the X-direction is of interest. The masses are lumped at the structure's nodes. The settings of the Natural Vibration Case are illustrated in Figure 3.4.

Figure 3.4: The eigenvalues and mode shapes are calculated with the shown settings. The masses are lumped at nodes and act only in the X-direction.

4. The linear elastic response spectrum is defined in accordance to the ASCE 7-16 [8]. The chosen parameters are shown in Figure 3.5.
3 Load Pattern

Figure 3.5: Linear elastic response spectrum with applicable parameters in accordance to the ASCE 7-16 [8].

The acceleration values of the response spectrum are automatically calculated in accordance to the ASCE 7-16 [8]. These tabulated values can be exported to Excel as shown in Figure 3.6. This is useful to convert the linear spectrum into the inelastic spectrum required for the Pushover Analysis.

Figure 3.6: Tabulated values of the linear elastic response spectrum can be exported to Excel.

5. The multi-modal response spectrum analysis can now be performed. The structure is only excited in the X-direction. The generated equivalent loads are exported into Load Cases. The settings for the response spectrum analysis are shown in Figure 3.7.
In this tutorial, only the fundamental mode shape is evaluated. The selection of eigenvalues is shown in Figure 3.8. Other eigenvalues can easily be analyzed by selecting them in the Mode Shapes tab. The equivalent loads of each selected eigenvalue are exported into separate Load Cases.

6. Calculate the RF-DYNAM Pro with the button [OK & Calculate]. The load case LC3 contains the equivalent loads in accordance to the dominant mode shape. The load distribution is shown in Figure 3.9. Due to the hinge definition, two FE mesh points exist at each node. Consequently, two loads are exported for each node.
3 Load Pattern

Figure 3.9: Load distribution in accordance to the dominant mode shape of the structure. The equivalent loads are exported from RF-DYNAM Pro.

7. The base shear force for the dominant mode is listed in Table 5.8 as shown in Figure 3.10

Figure 3.10: A list of all generated equivalent loads together with the base shear force is provided in Table 5.8 separate for each mode.
4 Non-Linear Static Calculation with Incrementally Increasing Load

The load case LC3 exported from RF-DYNAM Pro includes the equivalent loads and is directly used for the non-linear static analysis for the Pushover Curve.

1. Open the Calculation Parameters for LC3 and adjust the parameters. The final settings used in this tutorial are shown in Figure 4.1.
2. Perform a Large Deformation Analysis to enable the Incrementally Increasing Loading feature.
3. Modify the load with a factor to scale the equivalent loads exported from RF-DYNAM Pro. To ensure the load steps are small enough, a small Initial load factor is recommended.
4. Activate the Incrementally Increasing Loading feature. Set the Initial Load Factor to 0.010. To increase the loads by $\sum F = 0.1$ kip in each calculation step, the load factor increment must be set to 0.001.
   These settings highly depend on the structure and the expected deformation. The smaller the Load Increment value, the longer the calculation time. However, additional data points will be available in the Pushover Curve for a more exact Pushover Analysis. A Load Increment convergence study should be performed to find the ideal balance between data points and calculation time. A stopping condition is not necessarily required.
5. Save the result of all load increments.
6. Calculate the load case LC3.
5 Capacity Curve

Once LC3 is calculated, the Pushover Curve is available. This curve is required to evaluate the Performance Point using the N2-Method or the Capacity Spectrum Method, which is not included in this tutorial.

1. Go to the Global Calculation Parameters and select the Calculation Diagram tab.
2. Define a new Calculation Diagram with the settings shown in Figure 5.1.

![Figure 5.1: The definition of the Calculation Diagram to obtain the Pushover Curve. The sum of all lateral loads (base shear) is displayed on the vertical axis. The roof level deformation is displayed on the horizontal axis.](image)

3. Zoom into the Pushover Curve using the button. Data values can be exported to Excel using the button. This is useful for the Pushover Analysis.
4. Each load increment can be viewed graphically. The displayed plastic hinge color depends on the defined acceptance criteria. An overview for these possibilities are shown in Figure 5.3.
Literature


