Verification Example

Program:	RFEM 5
Category:	Geometrically Linear Analysis, Isotropic Nonlinear Elasticity, Isotropic Plasticity, Orthotropic Plasticity, Member, Plate, Solid
Verification Example:	0013 – One-Dimensional Plasticity with Hardening

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Description

Consider the example 0008 with hardening. A three-dimensional block, made of elastic-plastic material, of height h, width w and depth d, is fixed on both ends in z direction and partly fixed in x and y direction as it is shown in the **Figure 1**, all the rotations are constrained. The block's middle plane is subjected to the pressure load p. The block is made either from the orthotropic elastic-plastic material or from the isotropic elastic-plastic material, whose material properties are set do that both cases yield the equivalent behaviour. In the isotropic case, the problem is described by the following set of parameters.

Material	Elastic (upper block)	Modulus of Elasticity	E	11000.000	MPa
		Poisson's Ratio	ν	0.000	–
	Elastic-Plastic (lower block)	Modulus of Elasticity	E	11000.000	MPa
		Poisson's Ratio	ν	0.000	–
		Elastic-Plastic Modulus	E_p	3666.000	MPa
		Tensile Plastic Strength	f_y	14.000	MPa

In the orthotropic case, the problem is described by the following set of parameters.

Material	Elastic-Plastic	Modulus of Elasticity	$E_x = E_y = E_z$	11000.000	MPa
		Shear Modulus	$G_{yz} = G_{xz} = G_{xy}$	5500.000	MPa
		Poisson's Ratio	$\nu_{yz} = \nu_{xz} = \nu_{xy}$	0.000	–
		Elastic-Plastic Modulus	$E_{p,x}$	3666.000	MPa
		Tensile Plastic Strength	$f_{t,x} = f_{t,y}$	16.000	MPa
			$f_{t,z}$	14.000	MPa
		Compressive Plastic Strength	$f_{c,x} = f_{c,y}$	21.000	MPa
			$f_{c,z}$	24.000	MPa
		Shear Plastic Strength	$f_{v,yz} = f_{v,xz}$	2.700	MPa
			$f_{v,xy}$	1.000	MPa

In both isotropic and orthotropic case the upper block remains in the elastic state.

Geometry	Block	Width	w	0.050	m
		Depth	d	0.050	m
		Height	h	2.000	m
Load		Pressure	p	32.000	MPa

Small deformations are considered and the self-weight is neglected in this example. Determine the maximum deflection $u_{z,\max}$.

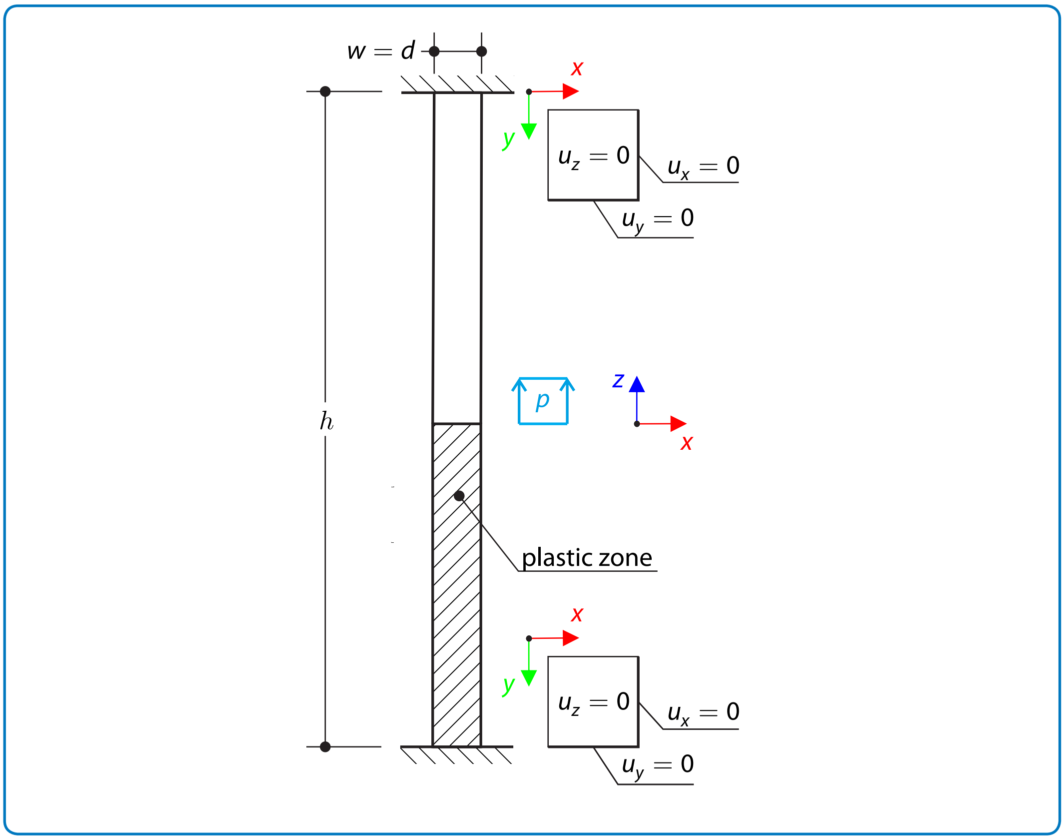

Figure 1: Problem sketch

Analytical Solution

Linear Analysis

Considering linear analysis (only elasticity) the maximum deflection of the structure can be calculated as follows:

$$u_{z,\max,\mathrm{el}} = \left(\frac{p_1}{2E}\right)\frac{h}{2} = 1.45\overline{45}\ \mathrm{mm} \qquad (13-1)$$

Isotropic Case

Let us derive the analytical solution in the isotropic case, the orthotropic case is identical (with different notation only). The stress or pressure in the elastic-plastic block is described by the following formula according to the **Figure 2**.

$$p = \underbrace{E\varepsilon}_{\text{elastic part of the block}} + \underbrace{E\varepsilon_0 + E_\mathrm{p}\left(\varepsilon - \varepsilon_0\right)}_{\text{plastic part of the block}} \qquad (13-2)$$

The total strain ε is after that

$$\varepsilon = \frac{p - \left(E - E_{\mathrm{p}}\right)\varepsilon_0}{E + E_{\mathrm{p}}} \tag{13 – 3}$$

where ε_0 is defined as

$$\varepsilon_0 = \frac{f_{\mathrm{y}}}{E} \tag{13 – 4}$$

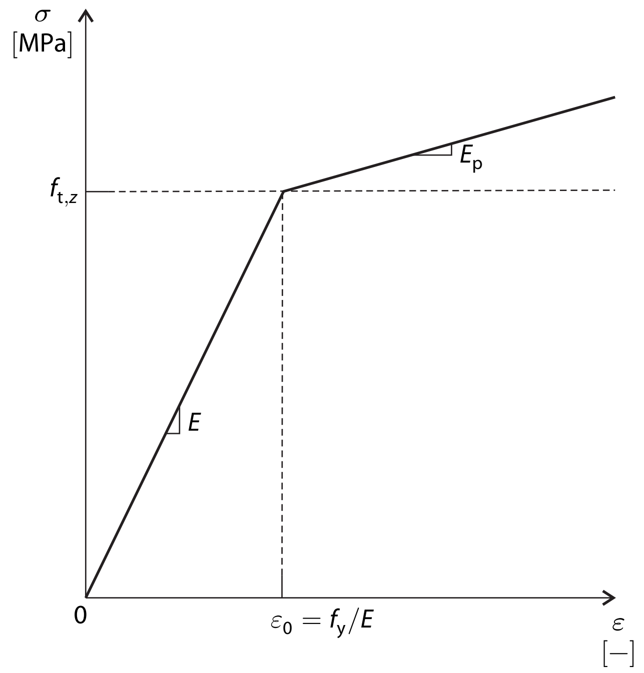

Figure 2: Stress-strain diagram

The maximum displacement $u_{z,\mathrm{max}}$ can be then calculated

$$u_{z,\mathrm{max}} = \varepsilon\frac{h}{2} = 1.54\overline{54}\ \mathrm{mm} \tag{13 – 5}$$

Orthotropic Case

The whole construction remains stable due to the material property $f_{c,z} > f_{t,z}$. The calculation is identical with the isotropic case.

RFEM 5 Settings

- Modeled in RFEM 5.03.0050
- The element size is $l_{\mathrm{FE}} = 0.100$ m
- Geometrically linear analysis is considered
- The number of increments is 5

Results

Structure File	Entity	Material model
0013.01	Member	Isotropic Plastic 1D
0013.02	Plate	Isotropic Plastic 2D/3D
0013.03	Plate	Isotropic Nonlinear Elastic 2D/3D
0013.04	Solid	Isotropic Plastic 2D/3D
0013.05	Plate	Orthotropic Plastic 2D
0013.06	Solid	Orthotropic Plastic 3D

Model	Theory	RFEM 5	
	$u_{z,max}$ [mm]	$u_{z,max}$ [mm]	Ratio [-]
Isotropic Plastic 1D	1.545	1.545	1.000
Isotropic Plastic 2D/3D, Plate		1.545	1.000
Isotropic Nonlinear Elastic 2D/3D , Plate		1.545	1.000
Isotropic Plastic 2D/3D, Solid		1.545	1.000
Orthotropic Plastic 2D		1.547	1.001
Orthotropic Plastic 3D		1.547	1.001