0024 – Three-Layer Sandwich Cantilever

Description

A sandwich cantilever consists of three layers (core and two faces). It is fixed on the left end and loaded by a concentrated force on the right end, see **Figure 1**. The problem is described by the following set of parameters.

Material	Faces	Modulus of Elasticity	$E_1 = E_3$	10.000	MPa
		Poisson's Ratio	$\nu_1 = \nu_3$	0.000	–
	Core	Modulus of Elasticity	E_2	0.020	MPa
		Poisson's Ratio	ν_2	0.000	–
Geometry	Common Parameters	Length	L	10.000	m
		Width	w	1.000	m
		Total Thickness	$t = \sum_{i=1}^{3} t_i$	0.580	m
	Faces	Thickness	$t_1 = t_3$	0.040	m
	Core	Thickness	t_2	0.500	m
Load		Force	F	0.750	kN

Small deformations are considered and the self-weight is neglected in this example. The goal is to determine the maximum deflection of the structure $u_{z,\max}$.

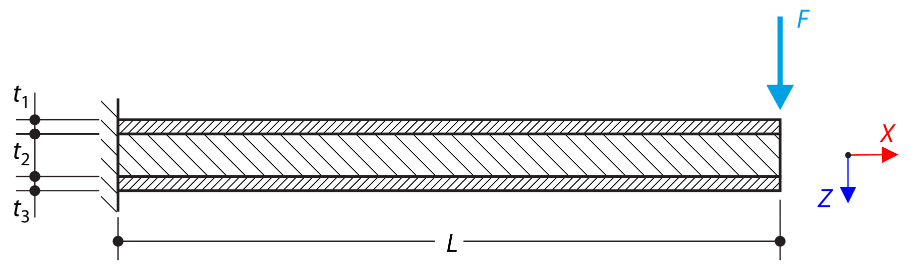

Figure 1: Problem sketch

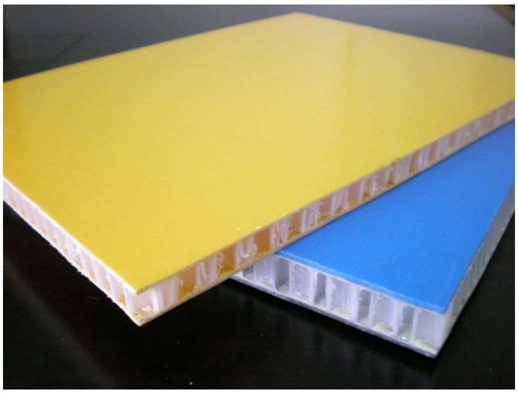

Figure 2: Real sandwich plate

Analytical Solution

The deflection of a single-layer cantilever loaded by a concentrated force, considering only bending, is described by the Bernoulli-Euler formula

$$u_{z,\text{bend}} = \frac{FL^3}{3EI_y} \tag{24 – 1}$$

where I_y is the quadratic moment of the cross-section to the y-axis. Multi-layer beams are analogously described by the formula

$$u_{z,\text{bend}} = \frac{FL^3}{3\sum_k E_k I_{yk}} \tag{24 – 2}$$

where index k sums over all layers. Note that the quadratic moment of the cross-section of the outer layers has to be transformed by means of Steiner's theorem to the central axis of the cantilever[1]. Quadratic moments of the cross-sections I_{yk} are following:

$$I_{y1} = I_{y3} = \frac{1}{12}wt_1^3 + wt_1\left(\frac{t_1+t_2}{2}\right)^2 \tag{24 – 3}$$

$$I_{y2} = \frac{1}{12}wt_2^3 \tag{24 – 4}$$

$$\sum_{k=1}^{3} E_k I_{yk} = 2E_1 I_{y1} + E_2 I_{y2} = 2E_1\left[\frac{1}{12}wt_1^3 + wt_1\left(\frac{t_1+t_2}{2}\right)^2\right] + E_2\frac{1}{12}wt_2^3 \tag{24 – 5}$$

Using **(24 – 2)**, the deflection caused by bending only is equal to

$$u_{z,\text{bend}} = 4.264 \text{ m} \tag{24 – 6}$$

It is suitable to take into account the shear effect also due to the remarkable cantilever height. The total deflection of the structure $u_{z,\max}$ is composed of the partial deflections due to the bending

[1] Steiner's theorem $I_{y2} = I_{y1} + Ad^2$, where A is the cross-section area and $d = y_2 - y_1$ is the perpendicular distance between axis y_1, y_2 to which moments I_{y1}, I_{y2} are related.

$u_{z,\mathrm{bend}}$ and the shear $u_{z,\mathrm{shear}}$, which is described in **Figure 3** where the dash denotes the differentiation with respect to x. The deflection caused by the shear $u_{z,\mathrm{shear}}$ can be calculated according to [1] as follows.

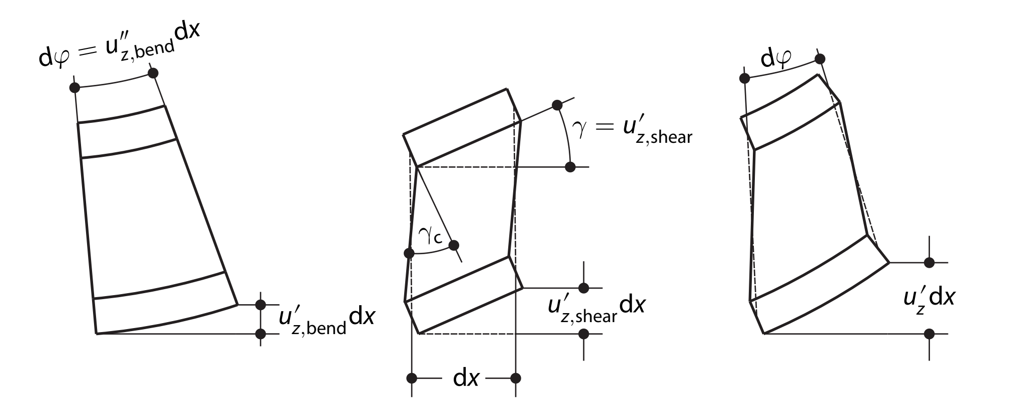

Figure 3: Deformation of an element

The cantilever shear strain γ is related to the shear strain of the sandwich cantilever core γ_c through

$$\gamma_c = \frac{t_2 + t_1}{t_2}\gamma \tag{24 – 7}$$

Thus, the shear stress in the core can be calculated

$$\tau_c = G_2\gamma_c = \frac{t_2 + t_1}{t_2}G_2\gamma \tag{24 – 8}$$

where $G_2 = E_2/(2(1 + \nu_2))$ is the shear modulus of the core. The shear-strain energy stored in the element dx is defined as follows

$$dU = \frac{1}{2}\frac{\tau_c^2 t_2 w}{G_2} = \frac{1}{2}S\gamma^2 \tag{24 – 9}$$

where the quantity S defines the shear stiffness

$$S = \frac{(t_2 + t_1)^2 w}{t_2}G_2 \tag{24 – 10}$$

The shear strain of the cantilever loaded by the force F is then calculated according to the formula

$$\gamma = \frac{F}{S} \tag{24 – 11}$$

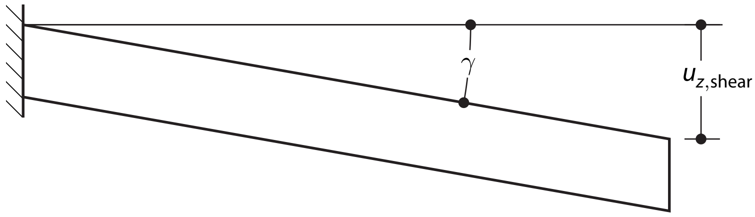

Figure 4: Deflection due to pure shear

The maximum deflection $u_{z,\text{shear}}$ of the cantilever due to the shear can be calculated according to **Figure 4**

$$u_{z,\text{shear}} = \gamma L = \frac{F}{S} L = \frac{FLt_2}{(t_2 + t_1)^2 w G_2} = 1.286 \text{ m} \qquad (24 - 12)$$

The total deflection of the structure is finally calculated

$$u_{z,\max} = u_{z,\text{bend}} + u_{z,\text{shear}} = 4.264 + 1.268 = 5.550 \text{ m} \qquad (24 - 13)$$

RFEM Settings

- Modeled in RFEM 5.26 and RFEM 6.01
- The element size is $l_{\text{FE}} = 0.200$ m
- Geometrically linear analysis is considered
- The number of increments is 5
- Isotropic linear elastic material model is used
- Multilayer Surfaces add-on is used in RFEM 6 for plate models

Results

Structure File	Program	Entity	Theory
0024.01	RFEM 5, RFEM 6	Solid	-
0024.02	RF-LAMINATE, RFEM 6	Plate	Kirchhoff
0024.03	RF-GLASS	Plate	Kirchhoff
0024.04	RF-LAMINATE, RFEM 6	Plate	Mindlin
0024.05	RF-GLASS	Plate	Mindlin

Model	Analytical Solution	RFEM 5	
	$u_{z,max}$ [m]	$u_{z,max}$ [m]	Ratio [-]
RFEM 5, RF-LAMINATE (Kirchhoff Theory)	4.264	4.264	1.000
RFEM 5, RF-GLASS (Kirchhoff Theory)		4.264	1.000
RFEM 5, Solid	5.550	5.579	1.005
RFEM 5, RF-LAMINATE (Mindlin Theory)		5.546	0.999
RFEM 5, RF-GLASS (Mindlin Theory)		5.546	0.999

Model	Analytical Solution	RFEM 6	
	$u_{z,max}$ [m]	$u_{z,max}$ [m]	Ratio [-]
RFEM 6, Multilayer Surfaces (Kirchhoff Theory)	4.264	4.264	1.000
RFEM 6, Solid	5.550	5.579	1.005
RFEM 6, Multilayer Surfaces (Mindlin Theory)		5.545	0.999

References

[1] PLANTEMA, F. J. *Sandwich construction: the bending and buckling of sandwich beams, plates, and shells*. Wiley, 1966.