Program: RFEM 5, RSTAB 8, RF-FE-LTB, FE-LTB

Category: Isotropic Linear Elasticity, Geometrically Linear Analysis, Post-Critical Analysis, Stability, Member

Verification Example: 0095 – Lateral Buckling of a Beam in Pure Bending

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Description

A simply supported beam is loaded by means of pure bending – moment M_y according to **Figure 1**. Determine the critical load $M_{y,cr}$ and corresponding load factor f due to lateral buckling. The problem is described by the following parameters.

Material	Steel	Modulus of Elasticity	E	210000.000	MPa
		Poisson's Ratio	ν	0.300	–
Geometry	Beam	Length	L	10.000	m
	Cross-section	Height	h	500.000	mm
		Width	b	100.000	mm
		Web Thickness	s	5.000	mm
		Flange Thickness	t	5.000	mm
Load		Bending Moment	M_y	8.000	kNm

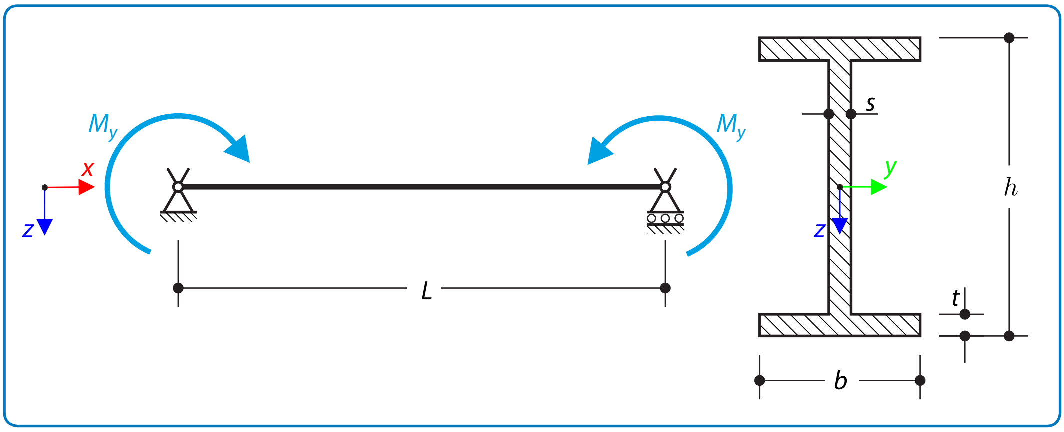

Figure 1: Problem Sketch

Analytical Solution

The analytical solution is based on the theory introduced in [1]. Deformation of a beam is defined by means of the following differential equations, which describe bending in two parpendicular planes and warping

$$EI_y \frac{d^2 u_z}{dx^2} - M_y = 0, \tag{95 – 1}$$

$$EI_z \frac{d^2 u_y}{dx^2} - \varphi_x M_y = 0, \tag{95 – 2}$$

$$GJ \frac{d\varphi_x}{dx} - EC_\omega \frac{d^3 \varphi_x}{dx^3} + \frac{du_y}{dx} M_y = 0, \tag{95 – 3}$$

where I_y, I_z and C_ω are moments of inertia in appropriate directions and warping constant respectively. These constants are taken from cross-section properties in RFEM 5. Differentiating **(95 – 3)** and substituting $\frac{d^2 u_y}{dx^2}$ from **(95 – 2)** the following differential equation is obtained

$$EC_\omega \frac{d^4 \varphi_x}{dx^4} - GJ \frac{d^2 \varphi_x}{dx^2} - \frac{M_y^2}{EI_z} \varphi_x = 0. \tag{95 – 4}$$

Considering substituting constants α and β,

$$\alpha = \frac{GJ}{2EC_\omega}, \tag{95 – 5}$$

$$\beta = \frac{M_y^2}{E^2 I_z C_\omega}, \tag{95 – 6}$$

(95 – 4) can be rewritten into more suitable form as

$$\frac{d^4 \varphi_x}{dx^4} - 2\alpha \frac{d^2 \varphi_x}{dx^2} - \beta \varphi_x = 0. \tag{95 – 7}$$

The general solution of **(95 – 4)** is

$$\varphi_x = C_1 \sin mx + C_2 \cos mx + C_3 e^{nx} + C_4 e^{-nx}, \tag{95 – 8}$$

where C_1, C_2, C_3, C_4 are integration constants and m, n are following substituting constants

$$m = \sqrt{-\alpha + \sqrt{\alpha^2 + \beta}}, \tag{95 – 9}$$

$$n = \sqrt{\alpha + \sqrt{\alpha^2 + \beta}}. \tag{95 – 10}$$

Although beam supports restrain axial rotation φ_x the beam is free to warp at both ends, so that the following boundary conditions can be written

$$\varphi_x(0) = 0, \tag{95 – 11}$$

$$\varphi_x(L) = 0, \tag{95 – 12}$$

$$\frac{d^2 \varphi_x(0)}{dx^2} = 0, \tag{95 – 13}$$

$$\frac{d^2 \varphi_x(L)}{dx^2} = 0. \tag{95 – 14}$$

Using these boundary conditions, $C_2 = C_3 = C_4$ a

$$\sin mL = 0. \tag{95 – 15}$$

The smallest non-negative value of m is

$$m = \frac{\pi}{L}. \tag{95 – 16}$$

The shape of buckling is given by the equation

$$\varphi_x = C_1 \sin mx \tag{95 – 17}$$

Taking above mentioned substitutions into account, the critical value of the bending moment $M_{y,cr}$ can be expressed as follows

$$M_{y,cr} = \frac{\pi}{L}\sqrt{EI_z\left(\frac{\pi^2 EC_\omega}{L^2} + GJ\right)} \approx 7.681 \text{ kNm}, \tag{95 – 18}$$

and the corresponding load factor f is

$$f = \frac{M_{y,cr}}{M_y} \approx 0.960. \tag{95 – 19}$$

RFEM 5 and RSTAB 8 Settings

- Modeled in RFEM 5.18.01 and RSTAB 8.18.01
- Element size is $l_{FE} = 0.100$ m
- The number of increments is 100
- Isotropic linear elastic material is used

Results

Structure File	Program	Description
0095.01	RFEM 5 - RF-FE-LTB	Geometrically Linear Analysis
0095.02	RSTAB 8 - FE-LTB	Geometrically Linear Analysis
0095.03	RFEM 5	Postcritical Analysis
0095.04	RSTAB 8	Postcritical Analysis

Model	Analytical Solution	RFEM 5 / RSTAB 8	
	f [-]	*f* [-]	Ratio [-]
RFEM 5 - RF-FE-LTB	0.960	0.955	0.995
RSTAB 8 - FE-LTB		0.955	0.995
RFEM 5, Postcritical Analysis*		0.940	0.979
RSTAB 8, Postcritical Analysis*		0.980	1.021

* Remark: The postcritical analysis (modified Newton-Raphson method) is used as a variant to the solution in add-on modules for lateral torsional buckling (RF-FE-LTB and FE-LTB). The critical force or the load factor can be approximately determined from the beam deflection behaviour.

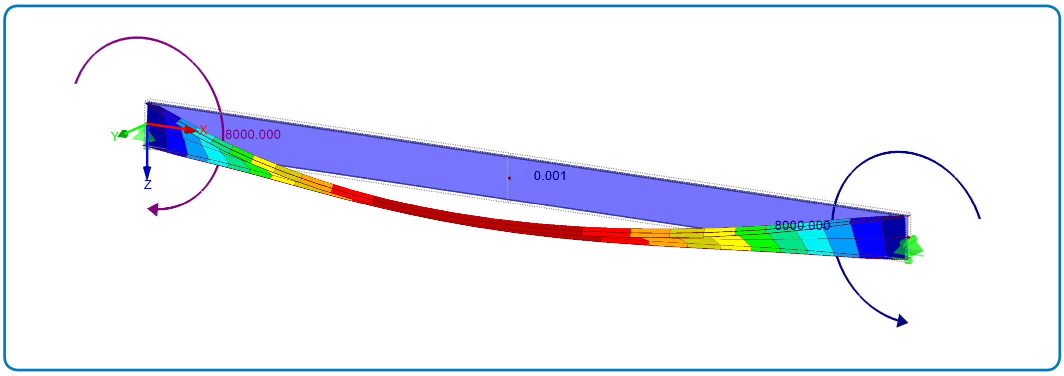

Figure 2: Buckled shape in RFEM 5 / RSTAB 8 – Postcritical Analysis

References

[1] TIMOSHENKO, S. and GERE, J. *Theory of Elastic Stability*. McGraw-Hill Book Company, 1963.