Verification Example

Program: RFEM 5, RSTAB 8

Category: Geometrically Linear Analysis, Isotropic Linear Elasticity, Member, Plate

Verification Example: 0023 – Shear Effects

0023 – Shear Effects

Description

The square block of isotropic material is fully fixed at one end and loaded with the uniform vertical pressure. Assuming only small deformation theory and neglecting its self-weight, determine the maximum deflections of the structure while considering or neglecting shear effect.

Material		Modulus of Elasticity	E	0.200	MPa
		Poisson's Ratio	ν	0.000	–
Geometry	Block	Length	L	1.000	m
		Thickness	t	0.500	m
Load		Pressure	p	0.001	MPa

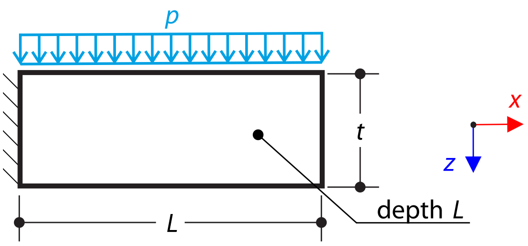

Figure 1: Problem sketch

Analytical Solution

While the shear effect is neglected, deflection of the cantilever is caused only by bending and can be expressed as:

$$u_{z,\text{bending}} = \frac{1}{8}\frac{pL^4}{EI} = 0.060 \text{ m} \qquad (23-1)$$

where $I = \frac{Lt^3}{12}$ is the second moment of inertia. The maximum shear deflection can be evaluated by the following formula:

$$u_{z,\text{shear}} = \frac{pL^2}{2\kappa GA} = 0.012 \text{ m} \qquad (23-2)$$

where $A = Lt$ is the cantilever's cross-section area, $G = \frac{E}{2(1+\nu)} = \frac{E}{2}$ is a shear modulus and κ is a parameter dependent on the shape of the cross-section, in the case of the rectangular cross-section it is equal to $\frac{5}{6}$. When the shear effect is considered, maximum deflection of the construction can be obtained as a sum of the bending and shear deflections:

$$u_{z,\max} = u_{z,\text{bending}} + u_{z,\text{shear}} = 0.072 \text{ m} \tag{23 – 3}$$

RFEM 5 Settings

- Modeled in version RFEM 5.03.0050
- The element size is $l_{\text{FE}} = 0.050$ m
- Geometrically linear analysis is considered
- The number of increments is 1
- Isotropic linear elastic material model is used

Results

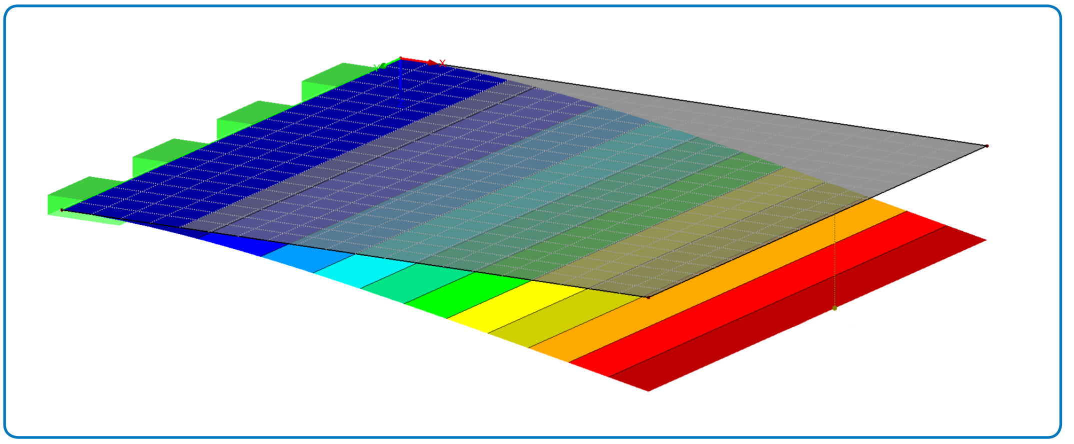

Figure 2: Results for 2D plate in RFEM 5 assuming the Kirchhoff theory (no shear effect)

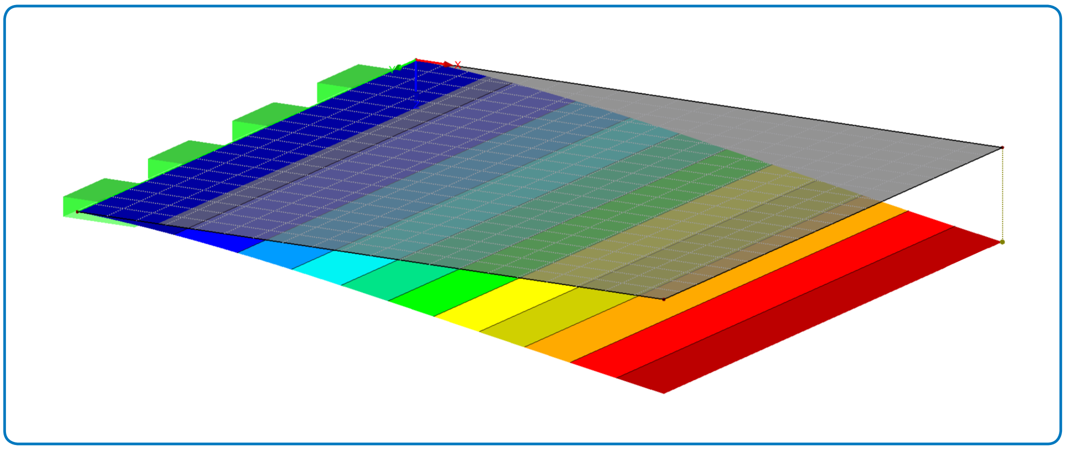

Figure 3: Results for 2D plate in RFEM 5 assuming the Mindlin theory (including shear effect)

Structure File	Program	Entity	Theory
0023.01	RSTAB 8	Member	Bernoulli
0023.02	RSTAB 8	Member	Timoshenko
0023.03	RFEM 5	Member	Bernoulli
0023.04	RFEM 5	Member	Timoshenko
0023.05	RFEM 5	Plate	Kirchhoff
0023.06	RFEM 5	Plate	Mindlin

As can be seen from the tables below, excellent agreements of analytical solutions with numerical simulations were achieved.

Member Theory	Analytical Solution	RSTAB 8		RFEM 5 (Member)	
	u_z [m]	u_z [m]	Ratio [-]	u_z [m]	Ratio [-]
Bernoulli	0.060	0.060	1.000	0.060	1.000
Timoshenko	0.072	0.072	1.000	0.072	1.000

Plate Theory	Analytical Solution	RFEM 5 (Plate)	
	u_z [m]	u_z [m]	Ratio [-]
Kirchhoff	0.060	0.060	1.000
Mindlin	0.072	0.072	1.000