0030 – Glass-Foil-Glass Cantilever Plate

Description

A composite plate, consisting of two glass layers and one foil layer in between, is fully fixed at one end and subjected to the uniform pressure p. Neglecting plate's self weight and assuming only small deformation theory, determine the maximum deflection of the cantilever at the free end.

Material	Glass	Modulus of Elasticity	$E_1 = E_3$	70000.000	MPa
		Poisson's Ratio	$\nu_1 = \nu_3$	0.230	–
	Foil	Modulus of Elasticity	E_2	3.000	MPa
		Poisson's Ratio	ν_2	0.499	–
Geometry	Plan	Length	L	1000.000	mm
		Depth	d	100.000	mm
	Layer 1	Thickness	t_1	10.000	mm
		Minimum z-coordinate	$z_{1,\text{min}}$	−10.150	mm
		Maximum z-coordinate	$z_{1,\text{max}}$	−0.150	mm
	Layer 2	Thickness	t_2	0.300	mm
		Minimum z-coordinate	$z_{2,\text{min}}$	−0.150	mm
		Maximum z-coordinate	$z_{2,\text{max}}$	0.150	mm
	Layer 3	Thickness	t_3	10.000	mm
		Minimum z-coordinate	$z_{3,\text{min}}$	0.150	mm
		Maximum z-coordinate	$z_{3,\text{max}}$	10.150	mm
Load		Pressure	p	5.000	kPa

Analytical Solution

Maximum deflection of the cantilever consists of the deflection due to the bending and shear:

$$u_{z,\text{max}} = u_{z,\text{bending}} + u_{z,\text{shear}} \tag{30 – 1}$$

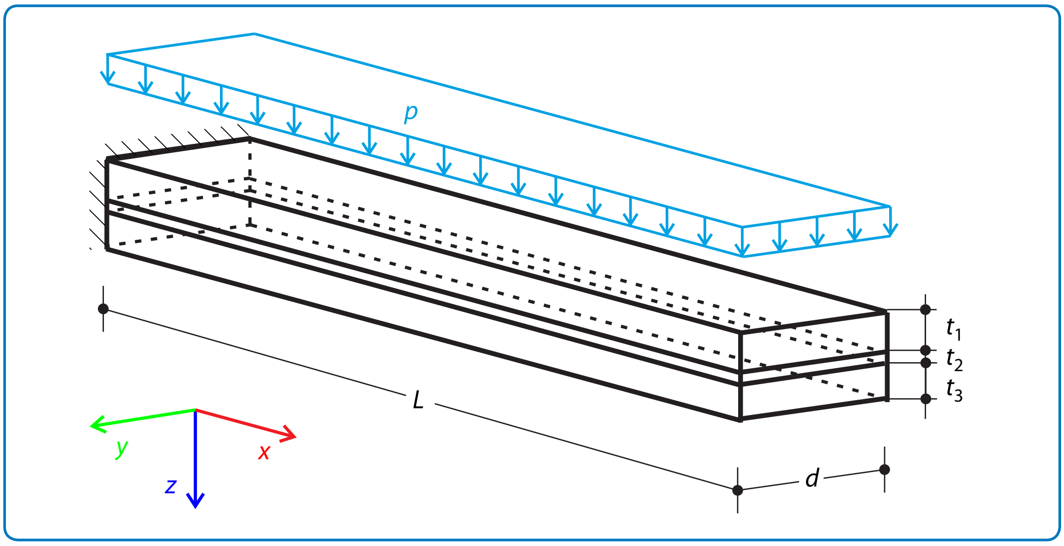

Figure 1: Problem sketch

A maximum bending deflection of cantilever subjected to the constant pressure loading can be expressed by the following formula:

$$u_{z,\text{bending}} = \frac{1}{8}\frac{pdL^4}{EI} \tag{30 – 2}$$

where EI is the bending stiffness and in the case of composite beam consisting of three layers can be expressed as:

$$EI = \sum_{i=1}^{3} E_i I_i \tag{30 – 3}$$

where E_i is each layer's Young's modulus and I_i its moment of inertia given by:

$$I_i = \frac{d(z_{i,\max}^3 - z_{i,\min}^3)}{3} \tag{30 – 4}$$

A maximum shear deflection can be expressed as:

$$u_{z,\text{shear}} = \gamma_{xz} L \tag{30 – 5}$$

where γ_{xz} is a shear strain:

$$\gamma_{xz} = \frac{\tau_{xz}}{G_2} \tag{30 – 6}$$

where τ_{xz} is an equivalent stress acting on the cantilever's free end and can be approximately expressed by the following formula:

$$\tau_{xz} \approx \frac{p}{2} \tag{30 – 7}$$

and G_2 is a foil shear modulus given by:

$$G_2 = \frac{E_2}{2(1+\nu_2)} \tag{30 – 8}$$

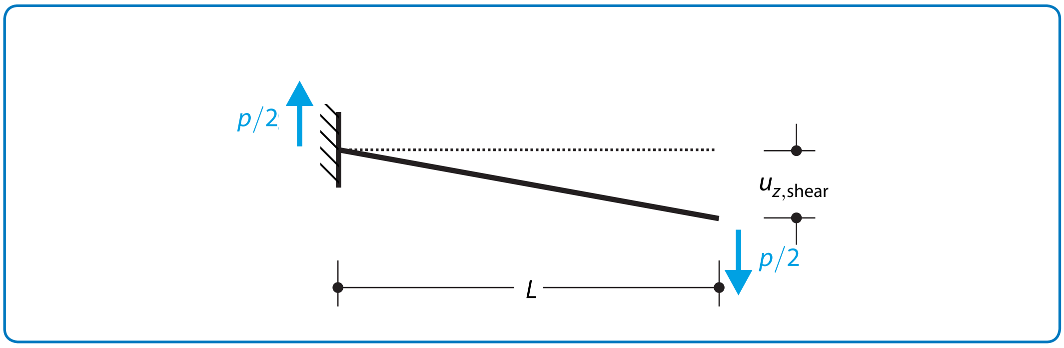

Figure 2: Shear deformation of the core

Using those formulae and input values tabulated above, outputs for deflections can be obtained:

$$u_{z,\text{bending}} = \frac{1}{8}\frac{pdL^4}{\sum_{i=1}^{3} E_i \frac{d(z_{i,\max}^3 - z_{i,\min}^3)}{3}} = 12.808 \text{ mm}$$

$$u_{z,\text{shear}} = \frac{p(1+\nu_2)}{E_2} L = 2.498 \text{ mm}$$

$$u_{z,\max} = u_{z,\text{bending}} + u_{z,\text{shear}} = 15.306 \text{ mm}$$

RFEM 5 Settings

- Modeled in version RFEM 5.04.0108
- The element size is $l_{\text{FE}} = 0.005$ m
- Geometrically linear analysis is considered
- The number of increments is 1
- The Mindlin plate theory is used
- Isotropic linear elastic material model is used
- Coupling of layers is considered

Results

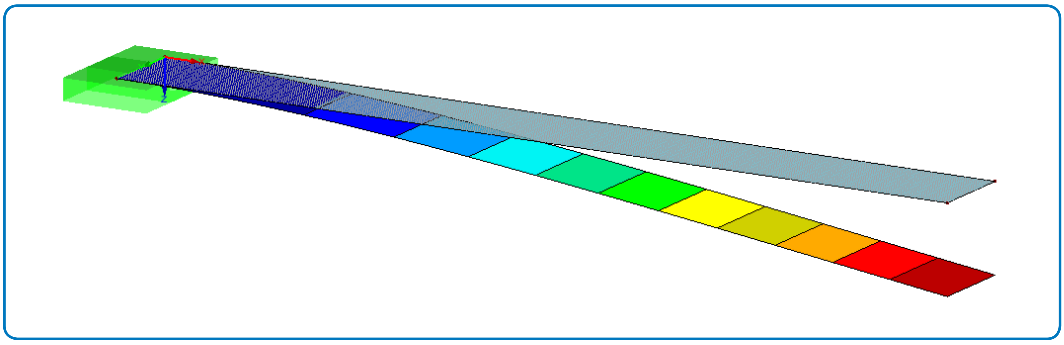

Figure 3: RF-GLASS output

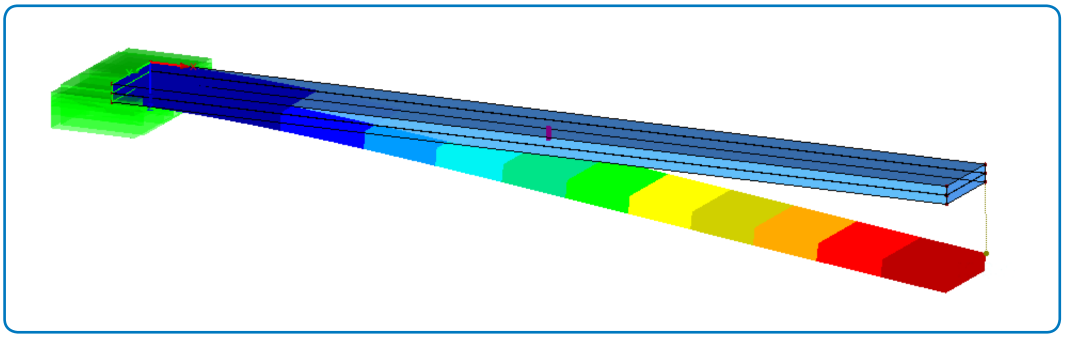

Figure 4: RFEM 5 output for solid

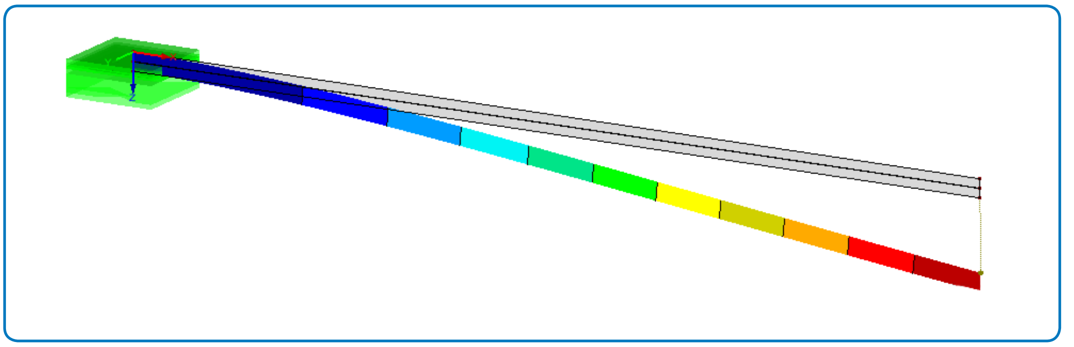

Figure 5: RFEM 5 output for plate

Structure File	Program	Entity
0030.01	RFEM 5	Solid
0030.02	RF-GLASS	Plate
0030.03	RFEM 5	Plate

Comparison of analytical solution with numerical result based on the laminate theory (RF-GLASS output) is tabulated below.

Analytical Solution	RF-GLASS	
$u_{z,\text{bending}}$ [mm]	$u_{z,\text{bending}}$ [mm]	Ratio [-]
12.808	12.936	1.010

Also good agreement of RFEM 5 output with analytical result was achieved.

Analytical Solution	RFEM 5 Solids		RFEM 5 Plates	
$u_{z,\text{max}}$ [mm]	$u_{z,\text{max}}$ [mm]	Ratio [-]	$u_{z,\text{max}}$ [mm]	Ratio [-]
15.306	15.638	1.022	15.740	1.028