0033 – Plate with a Hole

Description

The wide plate with a hole is loaded in one direction by means of the tensile stress σ according to the **Figure 1**. The plate width is large with respect to the hole radius and it is very thin considering the state of the plane stress.

Material	Steel	Modulus of Elasticity	E	210000.000	MPa
		Poisson's Ratio	ν	0.270	–
Geometry		Plate Width	$2L$	800.000	mm
		Plate Thickness	t	1.000	mm
		Hole Radius	R	20.000	mm
Load		Tension	σ	100.000	MPa

Determine the radial stress σ_r, tangential stress σ_θ and shear stress $\tau_{r\theta}$ in test points A and B according to the **Figure 1**. The plate is considering infinite wide. Thus the plate modeled in RFEM 5 and RFEM 6 is relatively large.

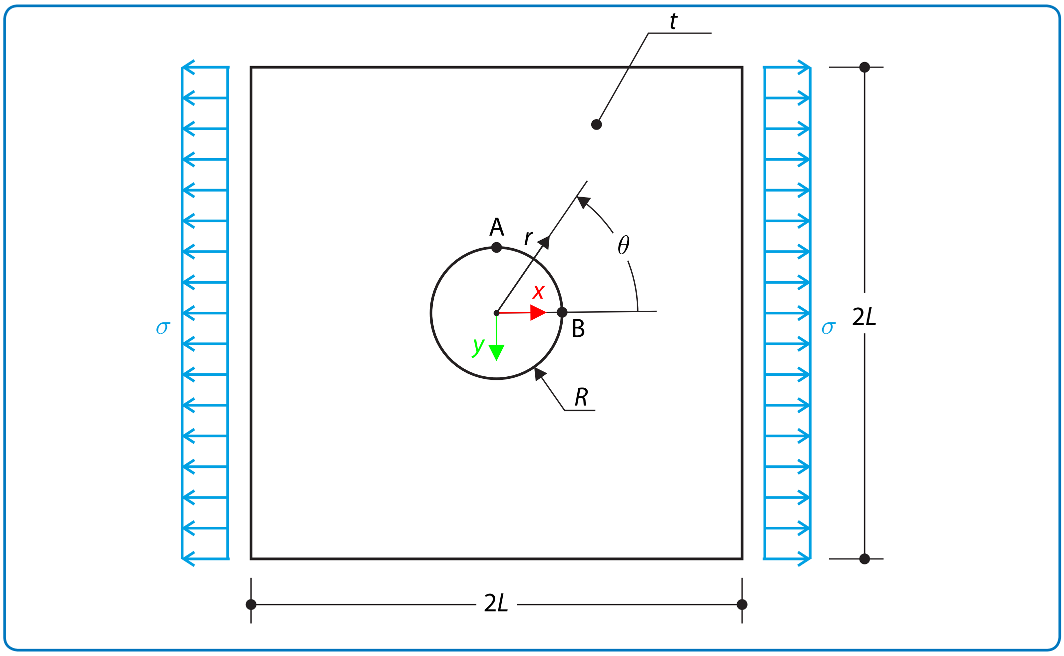

Figure 1: Problem sketch

Analytical Solution

Analytical solution of the stress state can be determined by means of the Airy stress function Φ which is defined in the state of plane stress. Airy stress function Φ is defined as

$$\sigma_x = \frac{\partial^2 \Phi}{\partial y^2} \tag{33 – 1}$$

$$\sigma_y = \frac{\partial^2 \Phi}{\partial x^2} \tag{33 – 2}$$

$$\tau_{xy} = \frac{\partial^2 \Phi}{\partial x \partial y} \tag{33 – 3}$$

It is appropriate to use the polar coordinates. Airy stress function in polar coordinates is defined as follows.

$$\sigma_r = \frac{1}{r}\frac{\partial \Phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \theta^2} \tag{33 – 4}$$

$$\sigma_\theta = \frac{\partial^2 \Phi}{\partial \theta^2} \tag{33 – 5}$$

$$\tau_{r\theta} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \Phi}{\partial \theta}\right) \tag{33 – 6}$$

Where r is the parameter of the radial position and θ is the parameter of the angular position. The Airy stress function has to fullfil the conditions of compatibility, which are defined in the plane stress state by means of Beltrami equation.

$$\Delta(\sigma_r + \sigma_\theta) = \Delta\Delta\Phi = 0 \tag{33 – 7}$$

The radial stress σ_{r} and tangential stress σ_θ in the hole proximity results as follows.

$$\sigma_{\mathrm{r}} = \frac{\sigma}{2}\left(1 - \frac{r^2}{R^2}\right) + \frac{\sigma}{2}\left(1 + \frac{3r^4}{R^4} - \frac{4r^2}{R^2}\right)\cos(2\theta) \tag{33 – 8}$$

$$\sigma_\theta = \frac{\sigma}{2}\left(1 + \frac{r^2}{R^2}\right) - \frac{\sigma}{2}\left(1 + \frac{3r^4}{R^4}\right)\cos(2\theta) \tag{33 – 9}$$

$$\tau_{\mathrm{r}\theta} = -\frac{\sigma}{2}\left(1 + \frac{3r^4}{R^4} + \frac{2r^2}{R^2}\right)\sin(2\theta) \tag{33 – 10}$$

For the test point A ($r = R, \theta = \frac{\pi}{2}$) the stresses then results

$$\sigma_{\mathrm{r}} = 0.000 \text{ MPa} \tag{33 – 11}$$

$$\sigma_\theta = 3\sigma = 300.000 \text{ MPa} \tag{33 – 12}$$

$$\tau_{\mathrm{r}\theta} = 0.000 \text{ MPa} \tag{33 – 13}$$

For the test point B ($r = R, \theta = 0$) the stresses then results

$$\sigma_{\mathrm{r}} = 0.000 \text{ MPa} \tag{33 – 14}$$

$$\sigma_\theta = -\sigma = -100.000 \text{ MPa} \tag{33 – 15}$$

$$\tau_{\mathrm{r}\theta} = 0.000 \text{ MPa} \tag{33 – 16}$$

RFEM Settings

- Modeled in RFEM 5.05.0030 and RFEM 6.01
- The global element size is $l_{FE} = 0.005$ m
- The mesh refinement (circular) is used, $l_{FE} = 0.001$ m
- Geometrically linear analysis is considered
- The number of increments is 5
- Plate entity is used

Results

Structure Files	Program
0033.01	RFEM 5, RFEM 6

Resultant stress fields around the hole are demonstrated in **Figure 2**, **Figure 3** and **Figure 4**.

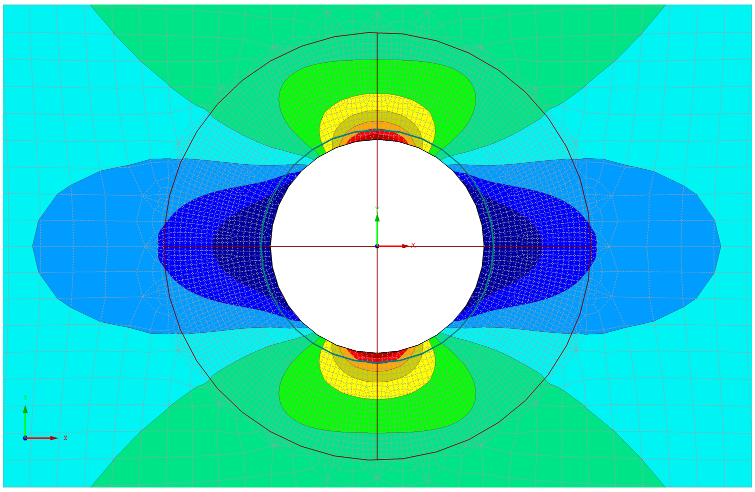

Figure 2: σ_x stress field around the hole.

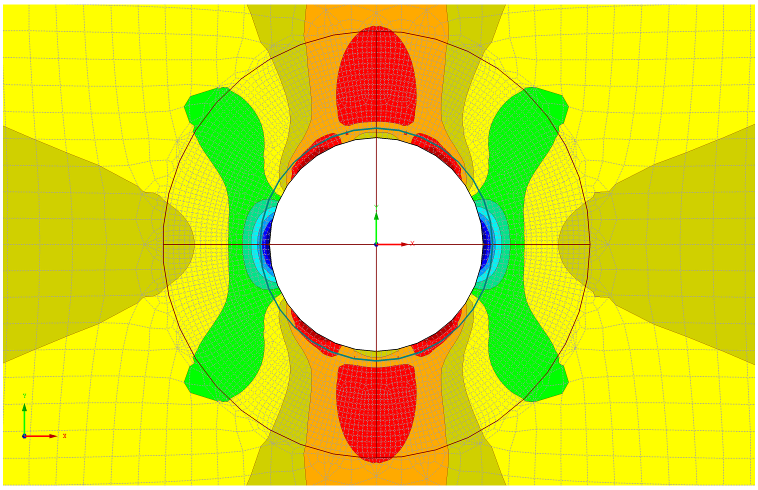

Figure 3: σ_y stress field around the hole.

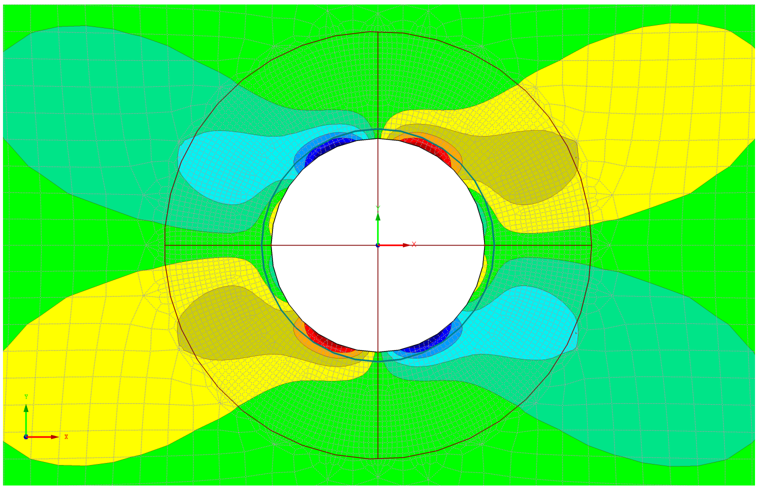

Figure 4: τ_{xy} stress field around the hole.

Test Point A	Analytical Solution	RFEM 5	Ratio	RFEM 6	Ratio
σ_r [MPa]	0.000	2.449	-	2.632	-
σ_θ [MPa]	300.000	300.529	1.002	300.753	1.003
$\tau_{r\theta}$ [MPa]	0.000	-0.002	-	-0.001	-

Test Point B	Analytical Solution	RFEM 5	Ratio	RFEM 6	Ratio
σ_r [MPa]	0.000	-1.753	-	-1.828	-
σ_θ [MPa]	-100.000	-100.216	1.002	-100.398	1.004
$\tau_{r\theta}$ [MPa]	0.000	0.000	-	0.000	-

Remark: In RFEM 5 and RFEM 6, the values of stresses σ_r, σ_θ and $\tau_{r\theta}$ are read from the values of σ_x, σ_y and τ_{xy} in appropriate point and directions.