Program: RFEM 5, RFEM 6

Category: Geometrically Linear Analysis, Isotropic Linear Elasticity, Plate

Verification Example: 0034 – Torsion of a Thin Plate

0034 – Torsion of a Thin Plate

Description

A thin plate is fixed on one side ($\varphi_z = 0$) and loaded by means of the distributed torque on the other side. At first the plate is modeled as a planar plate. Furthermore the plate is modeled as one quarter of the cylinder surface. The planar model has the same width as length of one quarter of the circle of the curved model. The curved model has thus almost equal torsional constant J as the planar model. Both models are shown in **Figure 1**.

Material	Steel	Modulus of Elasticity	E	210000.000	MPa
		Poisson's Ratio	ν	0.300	–
Geometry		Curved Model Radius	r	100.000	mm
		Planar Model Width	s	157.080	mm
		Plate Thickness	t	3.000	mm
		Plate Height	h	200.000	mm
Load		Distributed Torque	m	1268.720	Nm/m

Determine the maximum rotation of the plate $\varphi_{z,\text{max}}$ for both geometrical models and compare the results using both Kichhoff and Mindlin plate theory.

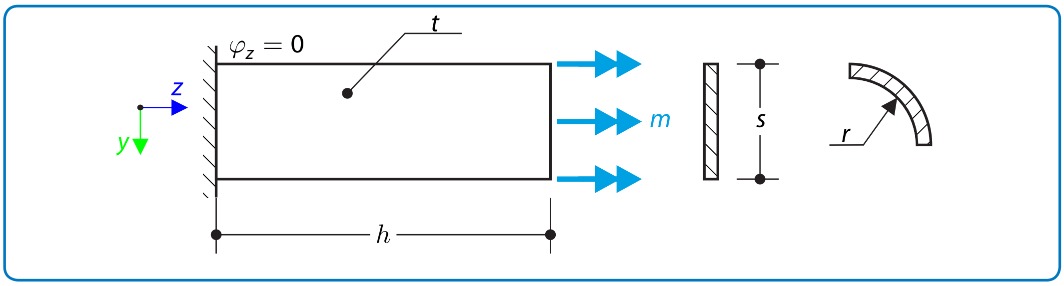

Figure 1: Problem sketch

Analytical Solution

The torsional constant for the planar plate (rectangular cross-section) can be calculated according to the the following formula

$$J = \frac{st^3}{3} = 1413.720 \text{ mm}^4 \qquad (34-1)$$

Considering the same width of the planar and curved plate[1] the identical torsional constant can be used[2]. Using torsional constant, which is calculated in **(34 – 1)**, the maximum rotation on the top of the plate ($z = h$) can be determined as follows.

$$\varphi_{z,\max} = \frac{msh}{GJ} = 0.349 \text{ rad} = 20.000^\circ \tag{34 – 2}$$

Where G is the shear modulus $G = \frac{E}{2(1+\nu)}$.

RFEM Settings

- Modeled in RFEM 5.26 and RFEM 6.01
- The element size is $l_{FE} = 0.002$ m
- Geometrically linear analysis is considered
- The number of increments is 5
- Plate entity is used
- Quadrangular elements are used

Results

Structure Files	Geometrical Model	Plate Theory	Surface Type
0034.01	Planar	Kirchhoff	Plane
0034.02	Curved	Kirchhoff	Quadrangle
0034.03	Planar	Mindlin	Plane
0034.04	Curved	Mindlin	Quadrangle

Model	Analytical Solution	RFEM 5		RFEM 6	
	$\varphi_{z,\max}$ [°]	$\varphi_{z,\max}$ [°]	Ratio [-]	$\varphi_{z,\max}$ [°]	Ratio [-]
Planar, Kirchhoff	20.000	20.163	1.008	20.163	1.008
Curved, Kirchhoff		20.163	1.008	20.163	1.008
Planar, Mindlin		20.733	1.037	20.666	1.033
Curved, Mindlin		20.863	1.044	20.797	1.040

Remark: The results deviation is also caused by the difference between the analytical and numerically calculated torsional constant.

[1] The width of the planar plate is the same as length of one quarter of the circle of the curved model: $s = \frac{\pi r}{2}$.

[2] The torsional constants of the planar plate and curved plate are compared also using program SHAPE-THIN: $J_p = 1396,710 \text{ mm}^4$, $J_c = 1392.670 \text{ mm}^4$.