Program: RFEM 5, RSTAB 8

Category: Large Deformation Analysis, Isotropic Linear Elasticity, Member, Plate

Verification Example: 0039 – Cable and Membrane

0039 – Cable and Membrane

Description

A steel cable or membrane with pins on both ends is loaded by the distributed loading q. Neglecting its self-weight, determine the maximum deflection of the structure by means of the large deformation theories.

Material	Steel	Modulus of Elasticity	E	210.000	GPa
		Poisson's Ratio	ν	0.300	–
Geometry	Cable	Length	L	5.000	m
		Diameter	d	0.160	m
Load		Distributed	q	10.000	kN/m

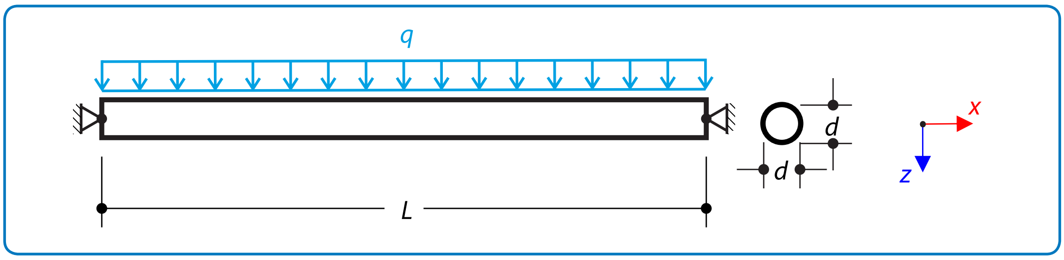

Figure 1: Problem sketch

Analytical Solution

The problem can be analytically approximately solved by the Ritz's method. Considering that the deflection in the z-direction is a multiple of the parabolic function (the complicated solution follows from solution of catenary, which has a same maximum deflection), approximation function can be expressed by the following formula:

$$u_z(x) = \frac{\left(x - \frac{L}{2}\right)^2}{2a} - \frac{L^2}{8a} \tag{39 – 1}$$

where a is an unknown multiplier. Function of rotation can be obtained as its derivative:

$$\varphi = \frac{\mathrm{d}u_z(x)}{\mathrm{d}x} = \frac{2x - L}{2a} \tag{39 – 2}$$

The axial strain of element is caused only by the normal forces and can be expressed as:

$$\varepsilon_x = \frac{\Delta L}{L} = \frac{\mathrm{d}e}{\mathrm{d}x} = \frac{\mathrm{d}s - \mathrm{d}x}{\mathrm{d}x} \tag{39 – 3}$$

where ds is a length of a deformed element (**Figure 2**), which can be expressed using Taylor's polynomial series (higher order terms of the series could be neglected, because they have only small influence to the maximum deflection):

$$\mathrm{d}s = \sqrt{(\mathrm{d}x)^2 + (\mathrm{d}u_z)^2} = \sqrt{1 + \left(\frac{\mathrm{d}u_z}{\mathrm{d}x}\right)^2}\,\mathrm{d}x \approx \left(1 + \frac{1}{2}\left(\frac{\mathrm{d}u_z}{\mathrm{d}x}\right)^2\right)\mathrm{d}x \qquad (39-4)$$

and de is an element's change of length:

$$\mathrm{d}e = \mathrm{d}s - \mathrm{d}x = \left(\frac{\mathrm{d}s}{\mathrm{d}x} - 1\right)\mathrm{d}x \qquad (39-5)$$

Change of the cable length can be than evaluated as follows:

$$\Delta L = e = \int_L \left(\frac{\mathrm{d}s}{\mathrm{d}x} - 1\right)\mathrm{d}x \qquad (39-6)$$

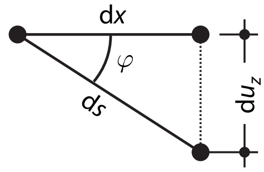

Figure 2: Deformed element

The unknown coefficient a from the equation **(39 – 1)** can be obtained by the principle of the minimum energy of the system:

$$\frac{\mathrm{d}\Pi}{\mathrm{d}a} = 0 \qquad (39-7)$$

where Π is the sum of the internal and the external energy:

$$\Pi = \Pi_{\mathrm{int}} + \Pi_{\mathrm{ext}} \qquad (39-8)$$

The energy of the internal forces can be expressed as:

$$\Pi_{\mathrm{int}} = -\int_V \sigma\varepsilon(x)\mathrm{d}V = -\frac{EA}{2}\int_L \left(\frac{\Delta L}{L}\right)^2 \mathrm{d}s = -\frac{EA}{2L^2}\int_L \left(\int_L \left(\frac{\mathrm{d}s}{\mathrm{d}x} - 1\right)\mathrm{d}x\right)^2 \mathrm{d}s =$$
$$-\frac{EA}{2L^2}\int_L \left(\int_L \left(\sqrt{1 + \left(\frac{\mathrm{d}u_z}{\mathrm{d}x}\right)^2} - 1\right)\mathrm{d}x\right)^2 \sqrt{1 + \left(\frac{\mathrm{d}u_z}{\mathrm{d}x}\right)^2}\,\mathrm{d}x = -\frac{EA}{2L^2} \times$$
$$\int_L \left(\int_L \frac{(2x-L)^2}{8a^2}\mathrm{d}x\right)^2 \left(1 + \frac{(2x-L)^2}{8a^2}\right)\mathrm{d}x = -\frac{EAL^5}{1152a^4}\left(1 + \frac{L^2}{24a^2}\right) \qquad (39-9)$$

where A is the cross-section area. Similarly the energy of the external forces can be obtained as:

$$\Pi_{ext} = q\int_L u_z \mathrm{d}s = q\int_L u_z\sqrt{1+\left(\frac{\mathrm{d}u_z}{\mathrm{d}x}\right)^2}\mathrm{d}x = q\int_L \left(\frac{(2x-L)^2}{8a} - \frac{L^2}{8a}\right)\times \left(1+\frac{(2x-L)^2}{8a^2}\right)\mathrm{d}x = -\frac{qL^3}{12a}\left(1+\frac{L^2}{40a^2}\right) \quad (39-10)$$

Substituting formulae **(39 – 9)** and **(39 – 10)** into formulae **(39 – 7)** and **(39 – 8)** following equation and unknown value a can be obtained:

$$\frac{\mathrm{d}\Pi}{\mathrm{d}a} = 0 = \frac{qL^3}{12}a^{-2} + \frac{qL^5}{160}a^{-4} + \frac{EAL^5}{288}a^{-5} + \frac{EAL^7}{4608}a^{-7} \quad (39-11)$$

$$a = -76047.478 \quad (39-12)$$

Substituting into the formula **(39 – 1)** the maximum deflection at the middle of the cable can be obtained:

$$u_{z,max} = u_z\left(\frac{L}{2}\right) = -\frac{L^2}{8a} = \frac{5000^2}{608379.824} = 41.093 \text{ mm} \quad (39-13)$$

RFEM 5 and RSTAB 8 Settings

- Modeled in version RFEM 5.03.0050 and RSTAB 8.03.0050
- The element size is $l_{FE} = 0.250$ m
- Large deformation analysis is considered
- The number of increments is 5
- The Mindlin plate theory is used
- Shear stiffness of members is activated
- Isotropic linear elastic material model is used
- Member division for large deformation or post-critical analysis is activated

Results

Structure File	Program	Entity	Entity Type	Cross-Section
0039.01	RFEM 5	Member	Cable	Circular
0039.02	RFEM 5	Plate	Membrane	Rectangular
0039.03	RSTAB 8	Member	Cable	Circular

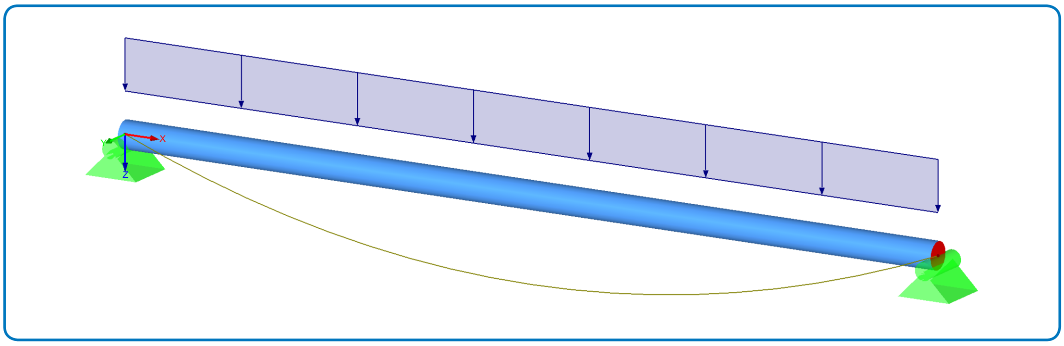

Figure 3: Modeled as a cable in RFEM 5

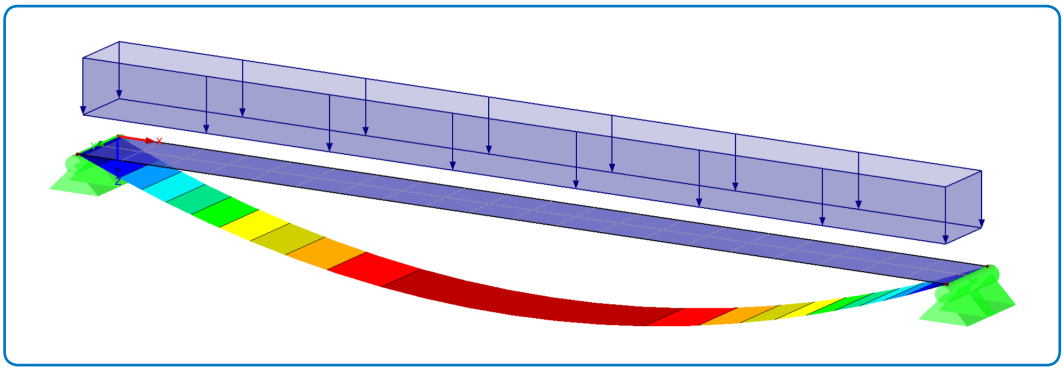

Figure 4: Modeled as a membrane in RFEM 5

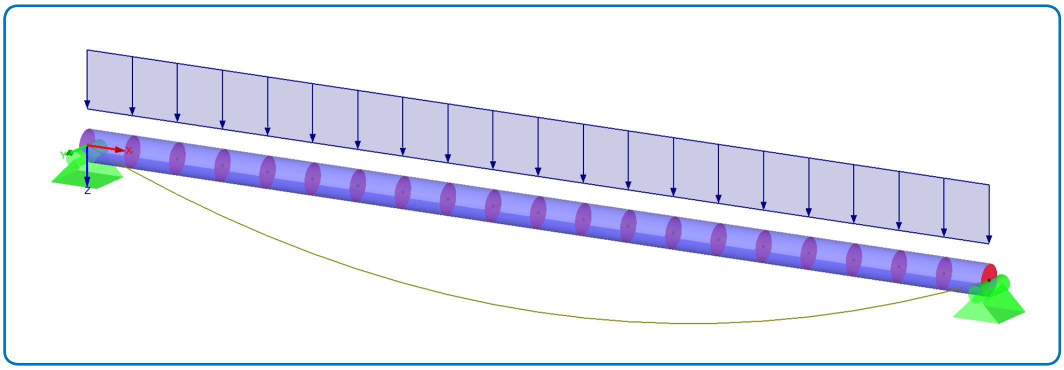

Figure 5: Modeled as a cable in RSTAB 8

As can be seen from the following comparisons, an excellent agreement of analysis solution and outputs of all the three simulations was achieved.

Analytical Solution	RFEM 5 (Cable)		RFEM 5 (Membrane)		RSTAB 8 (Cable)	
$u_{z,\max}$ [mm]	$u_{z,\max}$ [mm]	Ratio [-]	$u_{z,\max}$ [mm]	Ratio [-]	$u_{z,\max}$ [mm]	Ratio [-]
41.093	41.133	1.001	41.203	1.003	41.129	1.001