Program: RFEM 5, RSTAB 8

Category: Large Deformation Analysis, Post-Critical Analysis, Isotropic Linear Elasticity, Member

Verification Example: 0046 – Asymmetric Snap-Through

0046 – Asymmetric Snap-Through

Description

This verification example is a more complex variant of verification example 0045. A structure is made of two trusses of unequal length, which are embedded into the hinge supports according to the **Figure 1**. The structure is loaded by the concentrated force F_z. The problem is described by the following set of parameters.

Material	Steel	Modulus of Elasticity	E	210000.000	MPa
		Poisson's Ratio	ν	0.300	–
Geometry	Structure	Truss 1 Length	L_0	3.000	m
		Truss 2 Length	$2L_0$	6.000	m
		Height	h	1.500	m
	Cross-Section	Width	a	100.000	mm
Load		Force	F_z	122000.000	kN

The self-weight is neglected in this example. Determine the relationship between the loading force F_z and the deflections of the structure u_z and u_x considering large deformations generally. Determine the deflection under the loading force $F_z = 122000$ kN of the connection point of the trusses.

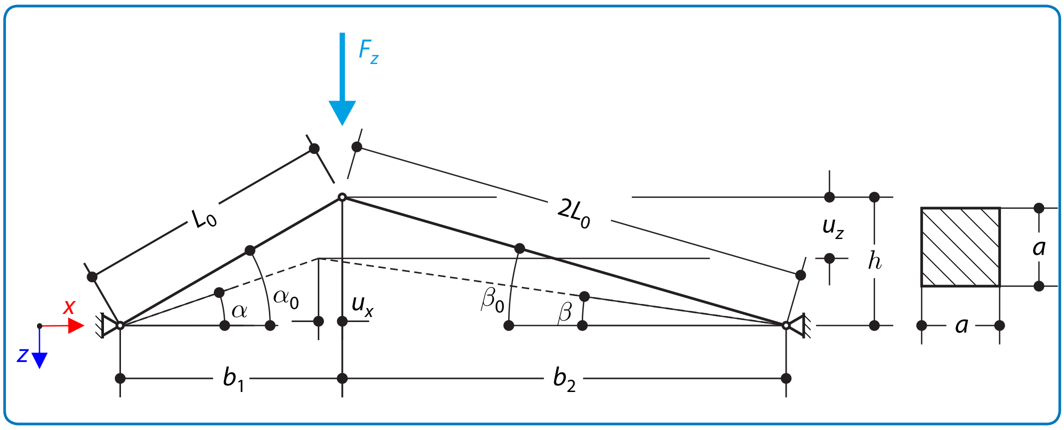

Figure 1: Problem sketch

Analytical Solution

Force equilibrium equations of the structure can be determined according to the **Figure 2**.

$$F_z = N_1 \sin\alpha + N_2 \sin\beta \tag{46 – 1}$$

$$0 = N_1 \cos\alpha - N_2 \cos\beta \tag{46 – 2}$$

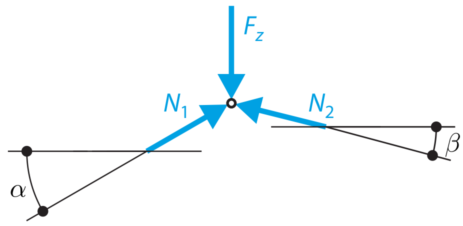

Figure 2: Force equilibrium

Considering the large deformation analysis, the angles α and β are not remaining constant during the loading. The aim of this verification example is to determine the relation between the loading force F_z and the deflections u_z and u_x. Thus the forces in the trusses and angles has to be expressed using the above mentioned deflections. The axial deformations of the trusses can be then determined as follows.

$$\Delta L_1 = L_1 - L_0 = \sqrt{(b_1 - u_x)^2 + (h - u_z)^2} - L_0 \tag{46 – 3}$$

$$\Delta L_2 = L_2 - 2L_0 = \sqrt{(b_2 + u_x)^2 + (h - u_z)^2} - 2L_0 \tag{46 – 4}$$

Where L_1 and L_2 are the lengths of the trusses after the deformation, b_1 and b_2 are the widths of the structure, which can be calculated as follows.

$$b_1 = \sqrt{L_0^2 - h^2} \tag{46 – 5}$$

$$b_2 = \sqrt{(2L_0)^2 - h^2} \tag{46 – 6}$$

The sine and cosine of angles α and β in formulae **(46 – 1)** and **(46 – 2)** can be expressed using following substitutions.

$$\sin\alpha = \frac{h - u_z}{L_1}$$

$$\sin\beta = \frac{h - u_z}{L_2}$$

$$\cos\alpha = \frac{b_1 - u_x}{L_1}$$

$$\cos\beta = \frac{b_2 + u_x}{L_2}$$

The axial force in the truss N can be generally determined from the Hooke's law[1] as

[1] Hooke's law $\sigma = E\varepsilon$. The axial stress is defined as $\sigma = \frac{N}{A}$, where A is the cross-section area.

$$N = \varepsilon EA \tag{46 – 7}$$

Considering the large deformation analysis the logarithmic form of the axial strain ε should be used.

$$\varepsilon = \ln\left(1 - \frac{\Delta L}{L_0}\right) \tag{46 – 8}$$

Using above mentioned formulae the general relationship between loading force F_z and the deflections u_x and u_z can be determined according to the formulae **(46 – 1)** and **(46 – 2)**.

$$F_z = \frac{EA\,(h-u_z)}{\sqrt{(b_1-u_x)^2+(h-u_z)^2}}\ln\left(1-\frac{\sqrt{(b_1-u_x)^2+(h-u_z)^2}-L_0}{L_0}\right)+ \tag{46 – 9}$$

$$\frac{EA\,(h-u_z)}{\sqrt{(b_2+u_x)^2+(h-u_z)^2}}\ln\left(1-\frac{\sqrt{(b_2+u_x)^2+(h-u_z)^2}-2L_0}{2L_0}\right)$$

$$0 = \frac{EA\,(b_1-u_x)}{\sqrt{(b_1-u_x)^2+(h-u_z)^2}}\ln\left(1-\frac{\sqrt{(b_1-u_x)^2+(h-u_z)^2}-L_0}{L_0}\right)- \tag{46 – 10}$$

$$\frac{EA\,(b_2+u_x)}{\sqrt{(b_2+u_x)^2+(h-u_z)^2}}\ln\left(1-\frac{\sqrt{(b_2+u_x)^2+(h-u_z)^2}-2L_0}{2L_0}\right)$$

The system of formulae **(46 – 9)** and **(46 – 10)** is obviously nonlinear and has to be solved numerically to obtain the solution for given loading force $F_z = 122000$ kN. Newton iteration method is used in this case and resultant deflections are following.

$$u_z = 3.545 \text{ m} \tag{46 – 11}$$

$$u_x = 0.154 \text{ m} \tag{46 – 12}$$

RSTAB 8 and RFEM 5 Settings

- Modeled in RSTAB 8.16.01 / RFEM 5.16.01
- The element size is $l_{FE} = 0.025$ m
- The number of increments is 10
- The structure is modeled using members (Truss - only N)
- Shear stiffness of the members is neglected
- Isotropic linear elastic material model is used
- In global calculation parameters there is disabled: *Activate member divisions for large deformation or post-critical analysis*

Results

Structure Files	Program	Solving Method
0046.01	RFEM 5	Post-Critical Analysis – Modified Newton-Raphson
0046.02	RFEM 5	Large Deformation Analysis – Dynamic Relaxation
0046.03	RSTAB 8	Post-Critical Analysis – Modified Newton-Raphson

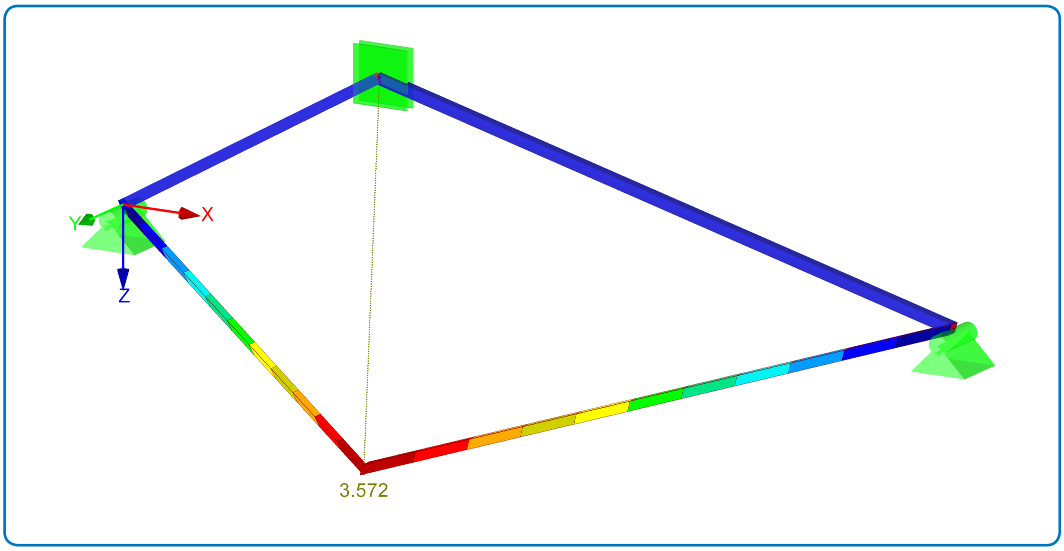

Figure 3: RFEM 5 / RSTAB 8 Results

Model	Analytical Solution	RSTAB 8 and RFEM 5 Solution	
	u_z [m]	u_z [m]	Ratio [-]
RFEM 5 (Modified Newton-Raphson)	3.545	3.568	1.006
RFEM 5 (Dynamic Relaxation)		3.568	1.006
RSTAB 8 (Modified Newton-Raphson)		3.556	1.003

Model	Analytical Solution	RSTAB 8 and RFEM 5 Solution	
	u_x [m]	u_x [m]	Ratio [-]
RFEM 5 (Modified Newton-Raphson)	0.154	0.159	1.032
RFEM 5 (Dynamic Relaxation)		0.159	1.032
RSTAB 8 (Modified Newton-Raphson)		0.157	1.019