Verification Example

Program: RFEM 5, RFEM 6, RSTAB 8, RSTAB 9, RF-FE-LTB, FE-LTB

Category: Second-Order Analysis, Isotropic Linear Elasticity, Warping, Member

Verification Example: 0049 – Twisted Beam Under Complex Loads with Warping

0049 – Twisted Beam Under Complex Loads with Warping

Description

A beam is fully fixed (warping is restraint) on the left end ($x = 0$) and supported by the fork support (warping is enabled) on the right end ($x = 4a$). The beam is subjected to a torque M, longitudinal force F_x and transverse force F_z according to the **Figure 1** [1]. The problem is described by the following set of parameters.

Material	Steel	Modulus of Elasticity	E	210000.000	MPa
		Shear Modulus	G	81000.000	MPa
Geometry	Beam	Section Length	a	2.000	m
		Height	h	400.000	mm
		Width	b	180.000	mm
		Web Thickness	s	10.000	mm
		Flange Thickness	t	14.000	mm
Load		Torque	M	6.000	kNm
		Longitudinal Force	F_x	800.000	kN
		Transverse Force	F_z	100.000	kN

Determine the behaviour of the primary torsional moment M_{Tpri}, secondary torsional moment M_{Tsec} and warping moment M_ω.

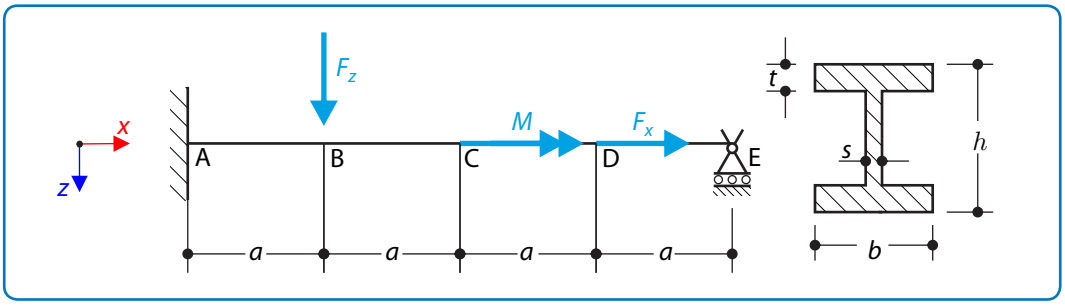

Figure 1: Problem sketch

Analytical Solution

Analytical solution is not available, results given in [1] are taken as a reference.

RFEM and RSTAB Settings

- Modeled in RFEM 5.05.0029, RSTAB 8.05.0029 and RFEM 6.01, RSTAB 9.01
- The element size is $l_{FE} = 0.025$ m
- Isotropic linear elastic material model is used
- Second-Order Analysis is used
- RF-FE-LTB and FE-LTB module is used in RFEM 5 and RSTAB 8
- Torsional Warping and Steel Design add-on is used in RFEM 6 and RSTAB 9
- Member Transverse Stiffener is used to model warping restraint (RFEM 6 and RSTAB 9)

Results

Structure Files	Program
0049.01	RFEM 5 – RF-FE-LTB, RFEM 6
0049.02	RSTAB 8 – FE-LTB, RSTAB 9

In the following graphs there are shown behaviours of the total torsional moment M_T, primary torsional moment M_{Tpri}, secondary torsional moment M_{Tsec}[1] and warping moment M_ω. The quantities are compared with the reference results in [1] at the test points A-E in the following tables.

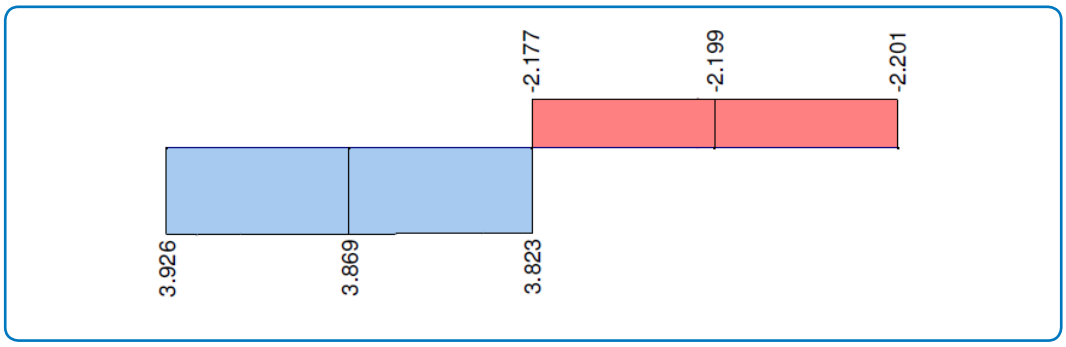

Figure 2: RSTAB 8 – FE-LTB, Total torsional moment M_T [kNm] behaviour

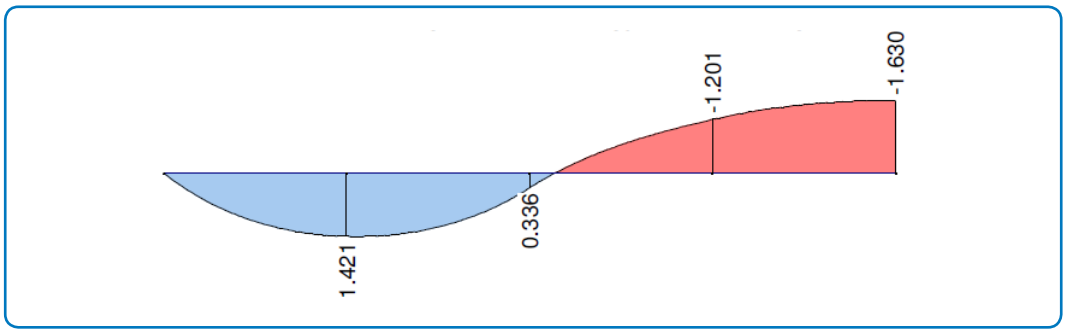

Figure 3: RSTAB 8 – FE-LTB, Primary torsional moment M_{Tpri} [kNm] behaviour

[1] Displayed results of the secondary torsional moment M_{Tsec} are including the influence of the axial force F_x.

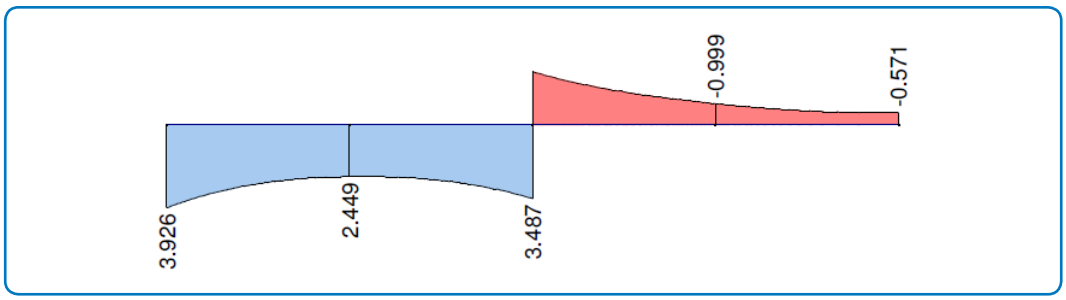

Figure 4: RSTAB 8 – FE-LTB, Secondary torsional moment M_{Tsec} [kNm] behaviour

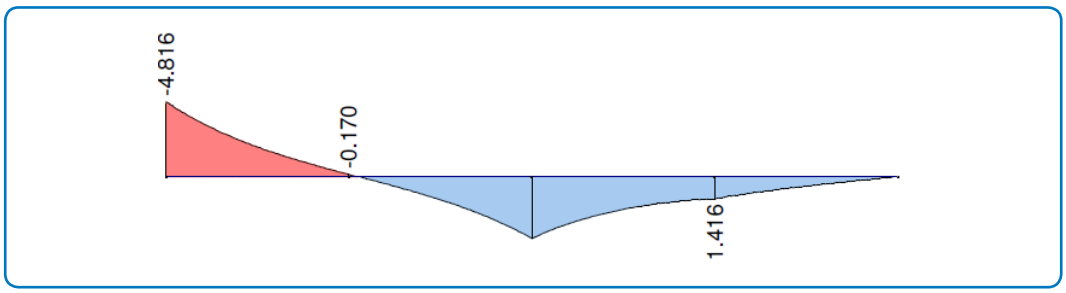

Figure 5: RSTAB 8 – FE-LTB, Warping moment M_{ω} [kNm^2] behaviour

Point A	S3D [2]	ANSYS 15 (BEAM188)*	RFEM 5 RF-FE-LTB	Ratio	RSTAB 8 FE-LTB	Ratio [-]
M_T [kNm]	3.92	–	3.95	1.008	3.92	1.000
M_{Tpri} [kNm]	0.00	–	0.00	–	0.00	-
M_{Tsec} [kNm]	3.92	–	3.95	1.008	3.92	1.000
M_{ω} [kNm^2]	-4.77	-4.74	-4.73	0.992	-4.81	1.008

Point A	S3D [2]	ANSYS 15 (BEAM188)*	RFEM 6	Ratio	RSTAB 9	Ratio [-]
M_T [kNm]	3.92	–	3.90	1.000	3.92	1.000
M_{Tpri} [kNm]	0.00	–	0.00	–	0.01	-
M_{Tsec} [kNm]	3.92	–	3.90	0.995	3.91	0.997
M_{ω} [kNm^2]	-4.77	-4.74	-4.80	1.006	-4.80	1.006

Point B	S3D [2]	ANSYS 15 (BEAM188)*	RFEM 5 RF-FE-LTB	Ratio	RSTAB 8 FE-LTB	Ratio [-]
M_T [kNm]	3.86	–	3.89	1.008	3.87	1.003
M_{Tpri} [kNm]	1.44	–	1.41	0.979	1.42	0.986
M_{Tsec} [kNm]	2.42	–	2.48	1.025	2.45	1.012
M_{ω} [kNm^2]	-0.17	-0.17	-0.16	0.941	-0.17	1.000

Point B	S3D [2]	ANSYS 15 (BEAM188)*	RFEM 6	Ratio	RSTAB 9	Ratio [-]
M_T [kNm]	3.86	–	3.88	1.005	3.86	1.000
M_{Tpri} [kNm]	1.44	–	1.40	0.972	1.42	0.986
M_{Tsec} [kNm]	2.42	–	2.48	1.025	2.44	1.008
M_ω [kNm^2]	-0.17	-0.17	-0.19	1.118	-0.17	1.000

Point C	S3D [2]	ANSYS 15 (BEAM188)*	RFEM 5 RF-FE-LTB	Ratio	RSTAB 8 FE-LTB	Ratio [-]
ΔM_T [kNm]	-6.00	–	-6.00	1.000	-6.00	1.000
M_{Tpri} [kNm]	0.34	–	0.34	1.000	0.34	1.000
ΔM_{Tsec} [kNm]	-6.00	–	-6.00	1.000	-6.00	1.000
M_ω [kNm^2]	3.88	3.89	3.87	0.997	3.95	1.008

Point C	S3D [2]	ANSYS 15 (BEAM188)*	RFEM 6	Ratio	RSTAB 9	Ratio [-]
ΔM_T [kNm]	-6.00	–	-6.00	1.000	-6.00	1.000
M_{Tpri} [kNm]	0.34	–	0.34	1.000	0.33	0.971
ΔM_{Tsec} [kNm]	-6.00	–	-6.000	1.000	-5.99	0.998
M_ω [kNm^2]	3.88	3.89	3.95	1.018	3.94	1.015

Remark: ΔM_T and ΔM_{Tsec} are values of quantity change at point D. This change is caused by the loading moment M.

Point D	S3D [2]	ANSYS 15 (BEAM188)*	RFEM 5 RF-FE-LTB	Ratio	RSTAB 8 FE-LTB	Ratio [-]
M_T [kNm]	-2.21	–	-2.19	0.991	-2.20	0.995
M_{Tpri} [kNm]	-1.22	–	-1.19	0.975	-1.20	0.984
M_{Tsec} [kNm]	-0.99	–	-1.00	1.010	-1.00	1.010
M_ω [kNm^2]	1.39	1.40	1.41	1.014	1.41	1.014

Point D	S3D [2]	ANSYS 15 (BEAM188)*	RFEM 6	Ratio	RSTAB 9	Ratio [-]
M_T [kNm]	-2.21	–	-2.19	0.991	-2.20	0.995
M_{Tpri} [kNm]	-1.22	–	-1.19	0.975	-1.20	0.984
M_{Tsec} [kNm]	-0.99	–	-1.00	1.010	-1.00	1.010
M_ω [kNm^2]	1.39	1.40	1.42	1.022	1.42	1.022

Point E	S3D [2]	ANSYS 15 (BEAM188)*	RFEM 5 RF-FE-LTB	Ratio	RSTAB 8 FE-LTB	Ratio [-]
M_T [kNm]	-2.21	–	-2.19	0.991	-2.20	0.995
M_{Tpri} [kNm]	-1.65	–	-1.63	0.988	-1.63	0.988
M_{Tsec} [kNm]	-0.56	–	-0.57	1.018	-0.57	1.018
M_ω [kNm^2]	0.00	0.00	0.00	–	0.00	–

Point E	S3D [2]	ANSYS 15 (BEAM188)*	RFEM 6	Ratio	RSTAB 9	Ratio [-]
M_T [kNm]	-2.21	–	-2.19	0.991	-2.20	0.995
M_{Tpri} [kNm]	-1.65	–	-1.62	0.982	-1.63	0.988
M_{Tsec} [kNm]	-0.56	–	-0.57	1.018	-0.57	1.018
M_ω [kNm^2]	0.00	0.00	0.00	–	0.00	–

Remark: Values of the warping moment M_ω in all test points are furthermore calculated in ANSYS 15 using BEAM188 element. Note that element solution values are interpolated in desired test points.

* Remark: Numerical solution in ANSYS 15 was carried out by the company Designtec s.r.o.

References

[1] LUMPE, G. and GENSICHEN, V. *Evaluierung der linearen und nichtlinearen Stabstatik in Theorie und Software: Prüfbeispiele, Fehlerursachen, genaue Theorie.* Ernst, 2014.

[2] LUMPE, G. *S3D (Vers. 25.09.2011).* Hochschule Biberach, 2011.