0070 – Rectangular Plate Under Lateral and Transversal Load

Description

A simply supported rectangular Kirchhoff plate is subjected to uniform lateral pressure p_z and stretched by a distributed load f_x, see **Figure 1**.

Assuming only the small deformation theory and neglecting self-weight of the plate, determine its maximum out-of-plane deflection u_{max}.

Material	Linear Elastic	Modulus of Elasticity	E	50.000	GPa
		Poisson's Ratio	ν	0.200	–
Geometry	Rectangle	Thickness	t	0.200	m
		Larger edge length	l_x	2.000	m
		Shorter edge length	l_y	1.000	m
Load		Lateral pressure	p_z	100.000	kN/m
		Edge tension	f_x	10.000	MPa

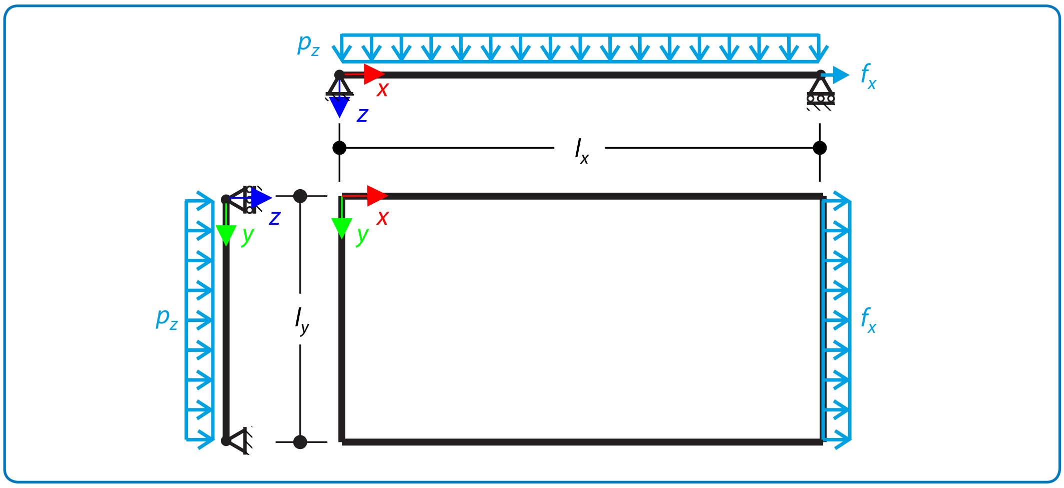

Figure 1: Problem sketch

Analytical Solution

A Kirchhoff-plate deformation field $\mathbf{u}(x,y) = [u_x(x,y),\ u_y(x,y),\ u_z(x,y)]^\top$ under the transversal load p_z fulfills

$$u_x = -z\frac{\partial u}{\partial x}, \quad u_y = -z\frac{\partial u}{\partial y}, \quad u_z = u \tag{70 – 1}$$

for some out-of-plane displacement $u(x, y)$, and the behavior of the plate is governed by the non-homogeneous biharmonic equation

$$D\nabla^2\nabla^2 u(x, y) = p_z \qquad (70-2)$$

where $D = Et^3/[12(1-\nu^2)]$ is the flexural rigidity of the plate and $\nabla^2 w = \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}$ the Laplace operator.

The membrane forces f_x, f_y are then added as an equivalent transversal force

$$p_z^\star = f_x \frac{\partial^2 u_z}{\partial x^2} + f_y \frac{\partial^2 u_z}{\partial y^2} \qquad (70-3)$$

see Section 3.3 in [1] for the details of the derivation through Tailor expansion and neglecting higher order terms.

Hence, equation **(70 – 2)** reads as

$$D\nabla^2\nabla^2 u(x, y) = p_z + f_x \frac{\partial^2 u}{\partial x^2} + f_y \frac{\partial^2 u_z}{\partial y^2} \qquad (70-4)$$

The transversal load p_z and the deflection u can be expressed in a double Fouries series

$$p_z = \frac{16 p_z}{\pi^2} \sum_m^\infty \sum_n^\infty \frac{1}{mn} \sin \frac{m\pi x}{l_x} \sin \frac{n\pi y}{l_y} \quad \text{for } m, n = 1, 3, 5, \ldots \qquad (70-5)$$

$$u(x, y) = \sum_m^\infty \sum_n^\infty U_{mn} \sin \frac{m\pi x}{l_x} \sin \frac{n\pi y}{l_y} \quad \text{for } m, n = 1, 3, 5, \ldots \qquad (70-6)$$

Substituting **(70 – 5)** and **(70 – 6)** into **(70 – 4)**, the unknown coefficients U_{mn} can be obtained

$$U_{mn} = \frac{16 p_z}{D\pi^6 mn \left[\left(\frac{m^2}{l_x^2} + \frac{n^2}{l_y^2} \right)^2 + \frac{f_x m^2}{\pi^2 D l_x^2} \right]} \qquad (70-7)$$

The combination of **(70 – 7)** and **(70 – 6)** leads to the formula for the plate deflection $u(x, y)$

$$u(x, y) = \frac{16 p_z}{D\pi^6} \sum_m^\infty \sum_n^\infty \frac{\sin \frac{m\pi x}{l_x} \sin \frac{n\pi y}{l_y}}{mn \left[\left(\frac{m^2}{l_x^2} + \frac{n^2}{l_y^2} \right)^2 + \frac{f_x m^2}{\pi^2 D l_x^2} \right]} \quad \text{for } m, n = 1, 3, 5, \ldots \qquad (70-8)$$

Assuming that the plate will be deflected the most at its center where $x = \frac{l_x}{2}$ and $y = \frac{l_y}{2}$, the formula for the maximum deflection $u_{\max}$ can be easily derived as

$$u_{\max} = \frac{16 p_z}{D\pi^6} \sum_m^\infty \sum_n^\infty \frac{\sin \frac{m\pi l_x}{2} \sin \frac{n\pi l_y}{2}}{mn \left[\left(\frac{m^2}{l_x^2} + \frac{n^2}{l_y^2} \right)^2 + \frac{f_x m^2}{\pi^2 D l_x^2} \right]} \quad \text{for } m, n = 1, 3, 5, \ldots \qquad (70-9)$$

$$u_{\max} \approx 2.920 \text{ mm}$$

RFEM 5 Settings

- Modeled in version RFEM 5.06.3039
- Element size is $l_{FE} = 0.010$ m
- Geometrically linear analysis is considered
- Number of increments is 1
- Kirchhoff plate theory is used

Results

Structure File	Program
0070.01	RFEM 5

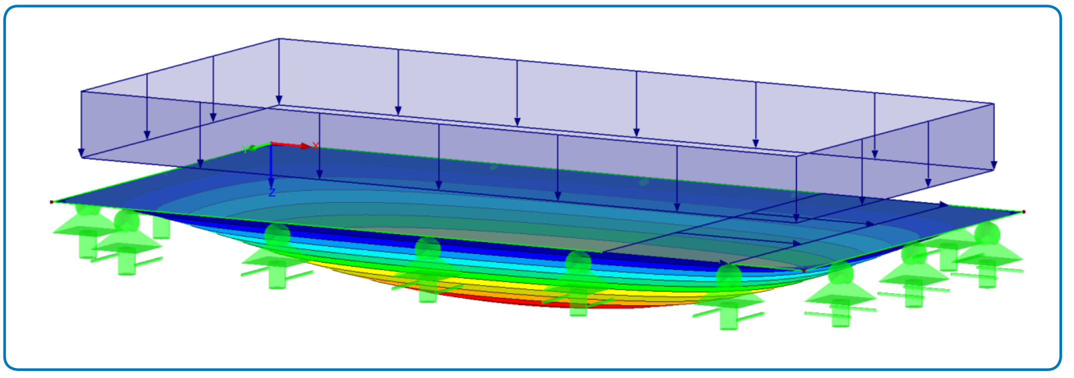

Figure 2: RFEM 5 Solution

As can be seen from the table below, excellent agreement of numerical output with the analytical result was achieved.

Analytical Solution	RFEM 5	
$u_{z,\text{max}}$ [mm]	$u_{z,\text{max}}$ [mm]	Ratio [-]
2.920	2.917	1.001

References

[1] SZILARD, R. *Theories and Application of Plate Analysis: Classical Numerical and Engineering Method*. Hoboken, New Jersey.