0090 – Eccentricity Test

Description

Pinned beam of length $2L$ with rectangular cross-section of side length b and height h is subjected to distributed loading q and shifted vertically by eccentricity e. Considering small deformation theory, neglecting self-weight, and assuming that the beam is made of isotropic elastic material with modulus of elasticity E, determine the maximum deflection u_{max}.

Material	Steel	Modulus of Elasticity	E	210000.000	MPa
		Poisson's Ratio	ν	0.296	–
Geometry	Beam	Cross-section Width	b	0.050	m
		Cross-section Height	h	0.200	m
		Beam Length	$2L$	10.000	m
		Eccentricity	e	0.100	m
Loading		Member Load	q	5.000	kN/m

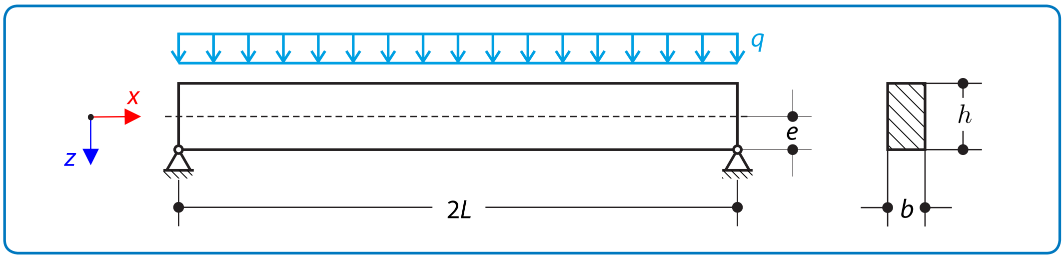

Figure 1: Problem sketch

Analytical Solution

The deflection of the beam is composed of the bending deflection u_b and shear deflection u_s

$$u_{\text{max}} = u_b + u_s. \tag{90 – 1}$$

As a sub-step in the determination process of beam maximum deflection, fictitious force $2F_d$ is applied at the midspan of the beam. Considering $2F_d$ and applied distributed loading q, shear force acting in the beam is

$$V(x) = -qx + qL + F_d. \tag{90 – 2}$$

Integrating **(90 – 2)**, expression for bending moment of the beam can be obtained

$$M(x) = \int V(x)\mathrm{d}x = -\frac{qx^2}{2} + qLx + F_d x + M_a, \qquad (90-3)$$

where $M_a = Ne$ is bending moment produced by axial force N acting with eccentricity e. Knowing the expression for bending moment **(90 – 3)**, energy in the beam produced by loading (with exception of the shear force) can be obtained with following formula

$$U = \int_0^L \frac{M^2(x)}{2EI_y}\mathrm{d}x + U_t, \qquad (90-4)$$

where $I_y = \frac{1}{12}bh^3$ is the second moment of inertia and U_t is the energy in the beam produced by axial force N

$$U_t = \frac{LN^2}{2EA}, \qquad (90-5)$$

where cross-section area $A = bh$. According to the principle of minimum energy $\frac{\mathrm{d}U}{\mathrm{d}N} = 0$, axial force can be calculated as

$$N = -\frac{eLA(2qL + 3F_d)}{6(e^2A + I_y)}. \qquad (90-6)$$

Differentiating **(90 – 4)** by fictitious force F_d and setting $F_d = 0$, maximum deflection in the mid-span of the beam (excluding effect of shear force) can be expressed as

$$u_b = \frac{qL^4(e^2A + 5I_y)}{24EI_y(e^2A + I_y)}. \qquad (90-7)$$

Similarly the deflection of the beam caused by shear force, can be obtained by differentiating shear strain energy U_s

$$U_s = \int_0^L \frac{V^2(x)}{2GA_s}\mathrm{d}x = \frac{5}{12GA}\left(\frac{q^2L^3}{2} + F_d^2L + qF_dL^2\right) \qquad (90-8)$$

by fictitious force F_d and setting $F_d = 0$.

$$u_s = \frac{5qL^2}{12GA} \qquad (90-9)$$

Maximum deflection of the beam can be obtained by summarizing **(90 – 7)** and **(90 – 9)**:

$$u_{\max} = u_b + u_s = \frac{qL^4(e^2A + 5I_y)}{24EI_y(e^2A + I_y)} + \frac{5qL^2}{12GA} \approx 37.267 \text{ mm}. \qquad (90-10)$$

RFEM 5 and RSTAB 8 Settings

- Modelled in version RFEM 5.09.01 and RSTAB 8.09.01
- Geometrically linear analysis is considered
- The element size is $l_{FE} = 0.025$ m
- Shear stiffness of members is activated
- The Mindlin plate theory is used

Results

Structure File	Program	Entity
0090.01	RSTAB 8	Member
0090.02	RFEM 5	Member
0090.03	RFEM 5	Plate

The detail of the eccentricity can be seen in **Figure 2**.

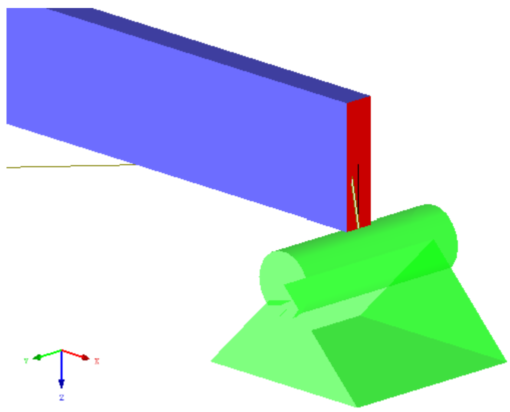

Figure 2: RFEM 5 / RSTAB 8 detail of the eccentricity

Analytical Solution	RSTAB 8 Member		RFEM 5 Member		RFEM 5 Plate	
u_{max} [mm]	u_{max} [mm]	Ratio [-]	u_{max} [mm]	Ratio [-]	u_{max} [mm]	Ratio [-]
37.267	37.295	1.001	37.295	1.001	37.275	1.000