Program: RFEM 5, RSTAB 8, RF-STABILITY, RSBUCK

Category: Isotropic Linear Elasticity, Geometrically Linear Analysis, Post-Critical Analysis, Stability, Member

Verification Example: 0093 – Buckling of Beam with Various Cross-sections

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Description

A column is composed of a concrete part - rectangle 100/200 and of a steel part - profile I 200. It is subjected to pressure force P according to **Figure 1**. Determine the critical load P_{cr} and corresponding load factor f. The problem is described by the following parameters.

Material	Steel	Modulus of Elasticity	E_1	210000.000	MPa
		Poisson's Ratio	ν_1	0.300	–
	Concrete	Modulus of Elasticity	E_2	35000.000	MPa
		Poisson's Ratio	ν_2	0.200	–
Geometry	Beam	Length	L_1	2.000	m
	Column	Length	L_2	2.000	m
Load		Force	P	100.000	kN

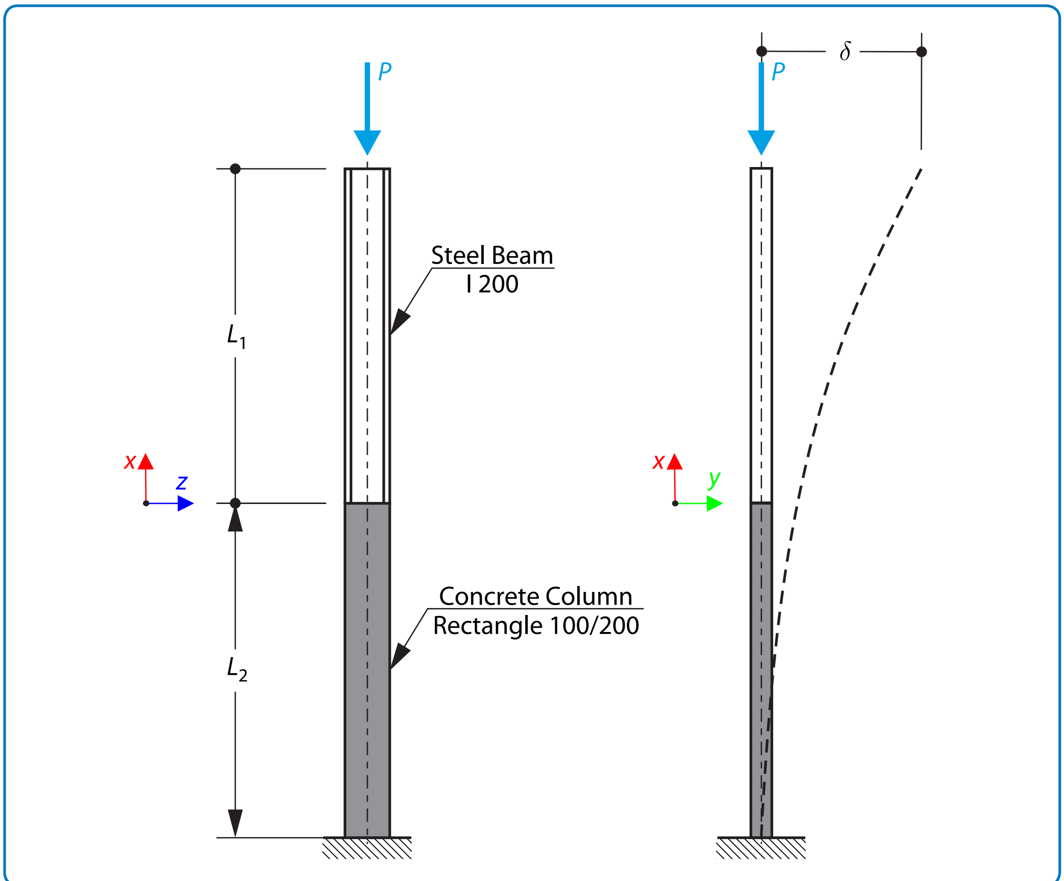

Figure 1: Problem Sketch

Analytical Solution

The theoretical solution is based on the buckling of a simple beam. In this case two regions have to be taken into account due to different moment of inertia and material properties. Considering the deflection of the endpoint δ the problem is described by the set of following differential equations, [1]

$$E_1 I_1 \frac{\mathrm{d}^2}{\mathrm{d}x^2} y_1 = P(\delta - y_1), \tag{93 – 1}$$

$$E_2 I_2 \frac{\mathrm{d}^2}{\mathrm{d}x^2} y_2 = P(\delta - y_2), \tag{93 – 2}$$

where the lower index number denotes the relation to the appropriate part of the column[1]. Using the substitutions

$$\alpha_1 = \sqrt{\frac{P}{E_1 I_1}}, \tag{93 – 3}$$

$$\alpha_2 = \sqrt{\frac{P}{E_2 I_2}}, \tag{93 – 4}$$

the solution of differential equations reads as

$$y_1 = C_1 \cos(\alpha_1 x) + C_2 \sin(\alpha_1 x) + \delta, \tag{93 – 5}$$

$$y_2 = C_3 \cos(\alpha_2 x) + C_4 \sin(\alpha_2 x) + \delta. \tag{93 – 6}$$

Integration constants C_1, C_2, C_3 and C_4 can be obtained from the following boundary conditions

$$y_2(0) = 0, \tag{93 – 7}$$

$$\frac{\mathrm{d}}{\mathrm{d}x} y_2(0) = 0, \tag{93 – 8}$$

$$y_1(L) = \delta, \tag{93 – 9}$$

$$y_1(L_2) = y_2(L_2). \tag{93 – 10}$$

The integration constants are then

$$C_1 = -C_2 \tan(\alpha_1 L), \tag{93 – 11}$$

$$C_2 = \frac{\delta \cos(\alpha_2 L_2) \cos(\alpha_1 L)}{\sin(\alpha_1 L_1)}, \tag{93 – 12}$$

$$C_3 = -\delta, \tag{93 – 13}$$

$$C_4 = 0. \tag{93 – 14}$$

[1] The moments of inertia I_1 and I_2 correspond to the minimum values of the cross-section and are taken from RFEM, $I_1 = 1.667 \cdot 10^7\ \mathrm{mm}^4$, $I_2 = 1.170 \cdot 10^6\ \mathrm{mm}^4$.

Furthermore, two parts of the column have the same tangent at $x = L_2$

$$\frac{\mathrm{d}}{\mathrm{d}x}y_1(L_2) = \frac{\mathrm{d}}{\mathrm{d}x}y_2(L_2). \tag{93 – 15}$$

Using the above determinated integration constants, the transcendental equation is obtained

$$\tan(\alpha_1 L_1)\tan(\alpha_2 L_2) = \frac{\alpha_1}{\alpha_2}. \tag{93 – 16}$$

This can be rewritten using **(93 – 3)** and **(93 – 4)** and solved numericaly to obtain the critical value of the loading force P_{cr} according to the Euler buckling theory. For the given parameters of the problem the critical force P_{cr} is equal to

$$P_{cr} \approx 70.782 \text{ kN}. \tag{93 – 17}$$

The load factor f is then

$$f = \frac{P_{cr}}{P} \approx 0.708. \tag{93 – 18}$$

RFEM 5 and RSTAB 8 Settings

- Modeled in RFEM 5.16.01 and RSTAB 8.16.01
- Element size is $l_{\text{FE}} = 0.1$ m
- The number of increments is 10
- Isotropic linear elastic material is used
- Lanczos method is used for eigenvalue analysis

Results

Structure File	Program	Method
0093.01	RFEM 5 - RF-STABILITY	Eigenvalue Analysis
0093.02	RFEM 5 - RF-STABILITY	Nonlinear Analysis
0093.03	RSTAB 8 - RSBUCK	Eigenvalue Analysis
0093.04	RFEM 5	Postcritical Analysis
0093.05	RSTAB 8	Postcritical Analysis

Model	Analytical Solution	RFEM 5 / RSTAB 8	
	f [-]	*f* [-]	Ratio [-]
RF-STABILITY, Eigenvalue Analysis	0.708	0.708	1.000
RF-STABILITY, Nonlinear Analysis		0.708	1.000
RSBUSCK		0.708	1.000
RFEM 5, Postcritical Analysis*		0.708	1.000
RSTAB 8, Postcritical Analysis*		0.708	1.000

* Remark: The postcritical analysis (modified Newton-Raphson method) is used as a variant to the solution in add-on modules for buckling. The critical force or the load factor can be approximately determined from the column deflection behaviour. It is convenient to use the incrementally increasing loading with refinement of the last load increment.

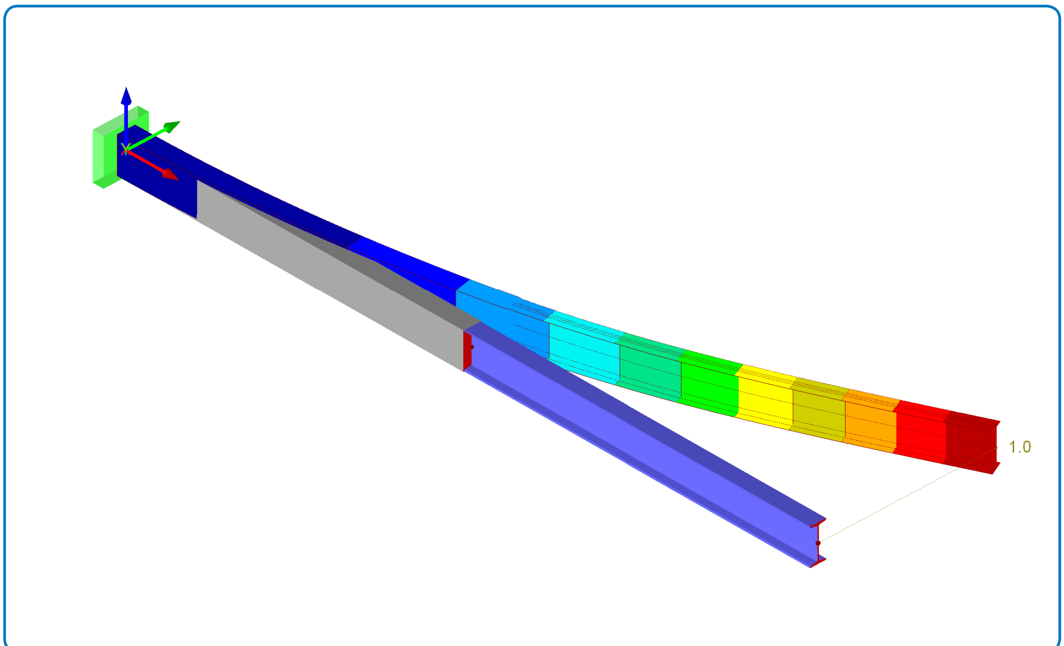

Figure 2: Results in RFEM 5 / RSTAB 8 – First eigenvector, load factor $f = 0.708$.

References

[1] TIMOSHENKO, S. and GERE, J. *Theory of Elastic Stability*. McGraw-Hill Book Company, 1963.