Program: RFEM 5, RSTAB 8, RF-DYNAM Pro, DYNAM Pro

Category: Geometrically Linear Analysis, Isotropic Linear Elasticity, Member, Dynamics

Verification Example: 0114 – Impulse Applied to Simply Supported Beam

0114 – Impulse Applied to Simply Supported Beam

Description

A given force F_z is applied for a short period of time Δt at the mid-span of a simply supported beam. Considering only small deformation theory and assuming that the mass m of the beam is concentrated at its mid-span, determine its maximum deflection u_{max}.

Material	Elastic	Modulus of Elasticity	E	50.000	GPa
		Poisson's Ratio	ν	0.500	–
Geometry	Beam	Width	w	0.100	m
		Height	h	0.100	m
		Length	L	1.000	m
Load	Force	Value	F_z	100.000	kN
		Period of Time	Δt	0.010	s
Mass	Concentrated	Mid-span	m	25000.000	kg

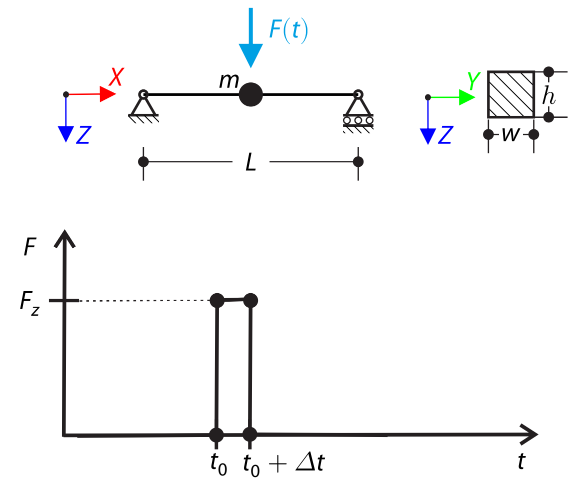

Figure 1: Problem Sketch

Analytical Solution

For the loading to be classified as an impulse, it has to be verified that it is applied for a time interval Δt shorter than one quarter of the natural period $T = 2\pi/\omega$ of the structure, where ω is the angular frequency of the beam

$$\omega = \sqrt{\frac{k}{m}} \qquad (114-1)$$

and k is its bending stiffness defined as a multiplicative inverse of the maximum deflection u_1 of the beam subjected to the unit mid-span force

$$k = \frac{1}{u_1} = \frac{48EI}{L^3} = \frac{4Ewh^3}{L^3} \tag{114 – 2}$$

where $I = \frac{wh^3}{12}$ is the second moment of beam cross-section area. Therefore, it indeed holds that

$$0.010\ \text{s} = \Delta t < \frac{T}{4} = \frac{\pi}{2\omega} = \frac{\pi}{4}\sqrt{\frac{L^3 m}{Ewh^3}} \approx 0.055\ \text{s} \tag{114 – 3}$$

and the load can be classified as an impulse J

$$J = \int_{t_0}^{t_0+\Delta t} F_z(t)\text{d}t \tag{114 – 4}$$

The beam can be simplified as a single-degree-of-freedom system, the behavior of which is described by the second-order differential equation of undamped motion

$$m\frac{\text{d}^2 u}{\text{d}t^2}(t) + ku(t) = 0 \tag{114 – 5}$$

which admits the general solution

$$u(t) = A\cos(\omega t) + B\sin(\omega t) \tag{114 – 6}$$

with unknown real parameters A and B. Setting the end of impulse as the beginning of beam motion, i.e., $t_0 + \Delta t = 0$, the following initial conditions apply

$$u(t_0 + \Delta t) = 0 \tag{114 – 7}$$

$$\frac{\text{d}u}{\text{d}t}(t_0) = 0 \tag{114 – 8}$$

Knowing that the impulse J can be defined as the change in momentum

$$J = m\frac{\text{d}u}{\text{d}t}(t_0 + \Delta t) - m\frac{\text{d}u}{\text{d}t}(t_0) \tag{114 – 9}$$

the coefficients A and B can be determined from **(114 – 5)** and **(114 – 9)**

$$u(0) = 0 \qquad \Rightarrow A = 0 \tag{114 – 10}$$

$$\frac{\text{d}u}{\text{d}t}(0) = \frac{J}{m} \qquad \Rightarrow B = \frac{J}{m\omega} \tag{114 – 11}$$

Hence, **(114 – 6)** reads as

$$u(t) = \frac{J}{m\omega} \sin \omega t \tag{114 – 12}$$

and the formula for the maximal displacement u_{max} can be easily derived

$$u_{\text{max}} = \frac{J}{m\omega} \tag{114 – 13}$$

RFEM 5 and RSTAB 8 Settings

- Modelled in version RFEM 5.07.07 and RSTAB 8.07.05
- Geometrically linear analysis is considered
- Shear stiffness of members is deactivated
- Mass is considered to act in the Z-direction
- Root of characteristic polynomial is used as solving method

Results

Structure File	Program
0114.01	RF-DYNAM Pro
0114.02	DYNAM Pro

As can be seen from the table below, good agreement of the analytical result with the numerical output was achieved

Analytical Solution	RF-DYNAM Pro		DYNAM Pro	
u_{max} [mm]	u_{max} [mm]	Ratio [-]	u_{max} [mm]	Ratio [-]
1.414	1.410	0.997	1.410	0.997