0066 – Plastic Thick-Walled Vessel

Description

A thick–walled vessel is loaded by inner pressure, which is chosen so that the vessel reaches the elastic–plastic state. The problem is modeled as a quarter model, see **Figure 1**, and is described by the following set of parameters.

Material	Modulus of Elasticity	E	200000.000	MPa
	Poisson's Ratio	ν	0.250	–
	Yield Strength	f_y	200.000	MPa
Geometry	Inner radius	r_1	200.000	mm
	Outer radius	r_2	300.000	mm
Load	Inner pressure	p_1	80.000	MPa

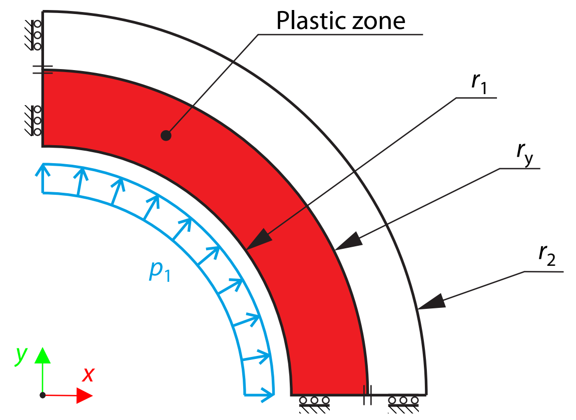

Figure 1: Problem Sketch

While neglecting self-weight, determine and compare the analytical and numerical solution for the radial position of the plastic zone border r_y under the Tresca hypothesis for the yield surface.

Analytical Solution

The stress state of the thick–walled vessel is described by the equation of equilibrium

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_t}{r} = 0 \qquad (66-1)$$

where σ_r and σ_t are the radial and tangent stresses, respectively.

The Tresca criterion implies the tensile yield strength f_y to be equal to

$$f_y = \sigma_t - \sigma_r \qquad (66-2)$$

which then with the following boundary condition

$$\sigma_r(r_1) = -p_1 \tag{66 – 3}$$

renders **(66 – 1)** into the relation

$$\sigma_r(r) = f_y \ln \frac{r}{r_1} - p_1 \tag{66 – 4}$$

Denoting p_y the pressure at the yield radius r_y, see **(66 – 6)**, and substituting into **(66 – 4)** the yield radius,

$$\sigma_r(r_y) = f_y \ln \frac{r_y}{r_1} - p_1 \tag{66 – 5}$$

$$\sigma_r(r_y) = -p_y \tag{66 – 6}$$

p_y reads as

$$p_y = p_1 - f_y \ln \frac{r_y}{r_1} \tag{66 – 7}$$

Further, the elastic part of the vessel has to be described, for the details of which see [1]. From the Tresca at the yield radius r_y it follows that

$$f_y = \sigma_t(r_y) - \sigma_r(r_y) \tag{66 – 8}$$

$$\sigma_r(r_y) = -p_y \tag{66 – 9}$$

$$\sigma_t(r_y) = 2K + p_y \tag{66 – 10}$$

whence

$$p_y = \frac{f_y(r_2^2 - r_y^2)}{2r_2^2} \tag{66 – 11}$$

Lastly, combining **(66 – 7)** and **(66 – 11)** yields the sought relation

$$p_1 = f_y \left(\ln \frac{r_y}{r_1} + \frac{(r_2^2 - r_y^2)}{2r_2^2} \right) \tag{66 – 12}$$

the numerical solution of which yields

$$r_y \approx 278.103 \text{ mm} \tag{66 – 13}$$

RFEM Settings

- Modeled in RFEM 5.06 and RFEM 6.01
- The element size is $l_{FE} = 0.002$ m
- The number of increments is 10
- Isotropic plastic 2D/3D material model is used

Results

Structure Files	Program
0066.01	RFEM 5, RFEM 6

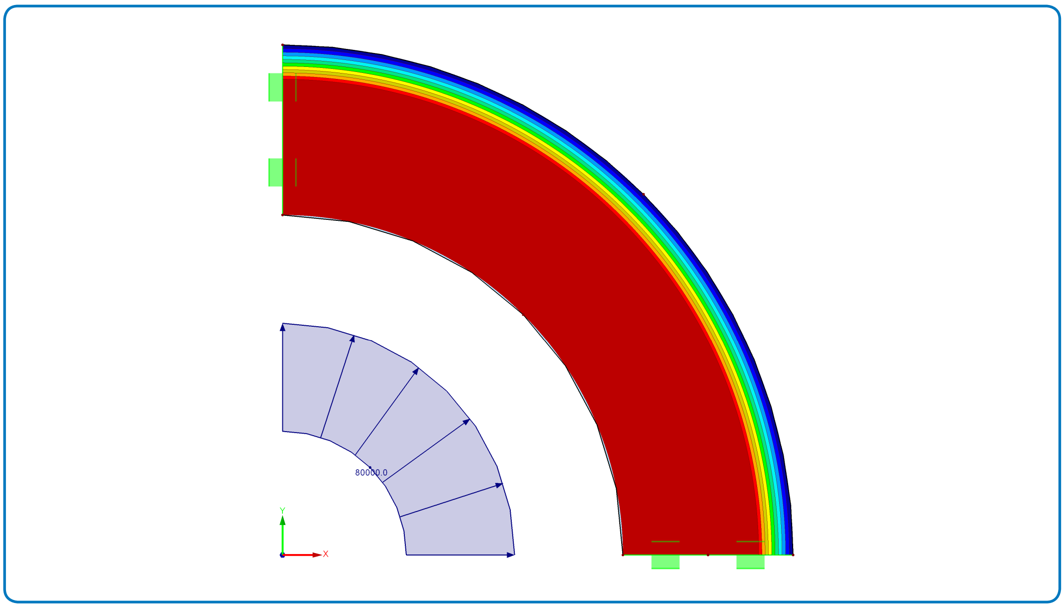

Figure 2: Results in RFEM - Tresca stress

Quantity	Analytical Solution	RFEM 5	Ratio	RFEM 6	Ratio
r_y [mm]	278.103	276.200	0.993	276.000	0.992

References

[1] DLUBAL SOFTWARE GMBH, *Verification Example 0064 – Thick-Walled Vessel*. 2016.