Program: RFEM 5

Category: Geometrically Linear Analysis, Isotropic Linear Elasticity, Temperature Dependency, Plate, Solid

Verification Example: 0078 – Bimetallic Strip

0078 – Bimetallic Strip

Description

A bimetallic strip is composed of the invar[1] and copper. The left end of the bimetallic strip is fixed, and the right end is free, loaded by temperature difference T_c according to **Figure 1**. While neglecting self-weight, determine the deflection $u_{z,\max}$ of the bimetallic strip (free end). The problem is described by the following set of parameters.

Material	Invar	Modulus of Elasticity	E_i	137000.000	MPa
		Poisson's Ratio	ν_i	0.280	–
		Coefficient of Thermal Expansion	α_i	1.200×10^{-6}	°C^{-1}
	Copper	Modulus of Elasticity	E_c	130000.000	MPa
		Poisson's Ratio	ν_c	0.354	–
		Coefficient of Thermal Expansion	α_c	2.000×10^{-5}	°C^{-1}
Geometry		Cross-section Width	w	5.000	mm
		Layer Thickness	$t_i = t_c = t$	1.000	mm
		Length	L	100.000	mm
Load		Thermal Loading	T_c	100.000	°C

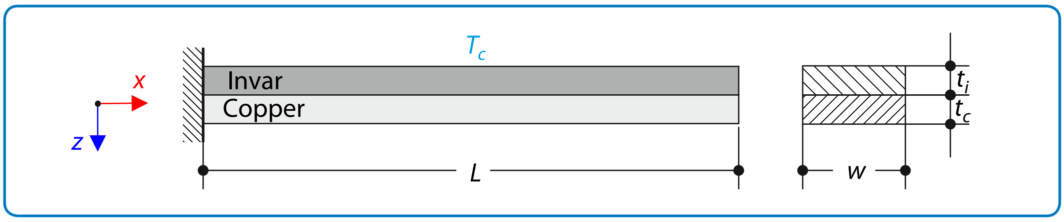

Figure 1: Problem Sketch

[1] Invar is an alloy of iron and nickel with very low coefficient of thermal expansion.

Analytical Solution

The bimetallic strip is made of two metals with different coefficients of thermal expansion. They are frequently used in instruments to sense or control temperatures. When the ambient temperature is changing, the bimetallic strip is bending. That is caused by the different elongations of the used metals.

The analytical solution is based on the approach introduced in [1]. The following formula gives the dimensionless coefficient K_1 which can be used for the calculation of the equivalent stiffness

$$K_1 = 4 + 6\frac{t_c}{t_i} + 4\left(\frac{t_c}{t_i}\right)^2 + \frac{E_c}{E_i}\left(\frac{t_c}{t_i}\right)^3 + \frac{E_i t_i}{E_c t_c}. \qquad (78-1)$$

The maximum deflection of the bimetallic strip is then defined by the following formula

$$u_{z,\max} = \frac{6(\alpha_i - \alpha_c)(t_i + t_c)T_c}{t_s^2 K_1}\frac{L^2}{2} \approx -7.049 \text{ mm}. \qquad (78-2)$$

RFEM 5 Settings

- Modeled in RFEM 5.16.01
- The minimum element size is $l_{\mathrm{FE}} = 0.250$ mm
- Isotropic linear elastic model is used

Results

Structure Files	Program	Entity
0078.01	RFEM 5	Plate
0078.02	RFEM 5	Solid

Model	Analytical Solution	RFEM 5	
	$u_{z,\max}$ [mm]	$u_{z,\max}$ [mm]	Ratio [-]
RFEM 5, Plate	-7.049	−7.047	1.000
RFEM 5, Solid		−7.064	1.002

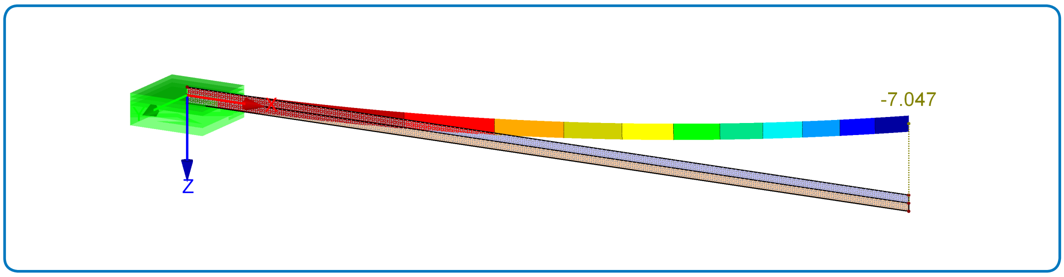

Figure 2: RFEM 5 results – Plate model

References

[1] YOUNG, W., BUDYNAS, R. and SADEGH, A. *Roark's Formulas for Stress and Strain, 8th Edition*. McGraw-Hill Education, 2011.