Program: RFEM 5

Category: Isotropic Linear Elasticity, Geometrically Linear Analysis, Shell

Verification Example: 0084 – Thin-walled Spherical Vessel

0084 – Thin-walled Spherical Vessel

Description

A thin-walled spherical vessel is loaded by inner pressure p according to **Figure 1**. While neglecting self-weight, determine the von Mises stress σ_{Mises} and the radial deflection u_R of the vessel.

Material	Modulus of Elasticity	E	210000.0	MPa
	Poisson's Ratio	ν	0.296	–
Geometry	Radius	R	0.500	m
	Shell Thickness	t	5.000	mm
Load	Internal Pressure	p	5.000	MPa

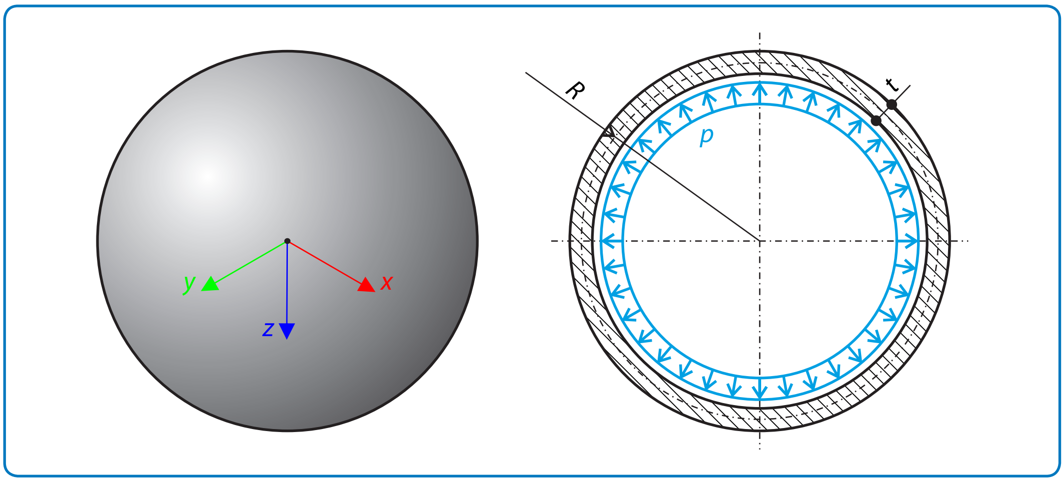

Figure 1: Problem Sketch

Analytical Solution

The analytical solution is based on the theory of thin-walled vessels. This theory assumes the membrane stress state of the shell, thus the following conditions have to be satisfied:

- The thickness of the shell can not change discontinuously.
- The distributed loading can not change discontinuously.
- The curvature radii and positions of centers can not change discontinuously.
- The outer forces including the reaction forces have to be tangential to the shell surface.

The stress state of the thin-walled vessel is described by the Laplace equation

$$\frac{\sigma_1}{R_1} + \frac{\sigma_2}{R_2} = \frac{p}{t}, \tag{84 – 1}$$

where σ_1, σ_2 are stresses in meridian and parallel direction respectively and R_1, R_2 are the radii in the appropriate directions. For the spherical vessel **(84 – 1)** can be simplified due to symmetry ($\sigma_1 = \sigma_2 = \sigma, R_1 = R_2 = R$) into the form

$$\sigma = \frac{pR}{2t}. \tag{84 – 2}$$

In case of thin-walled spherical vessels the principal stresses are equal to $\sigma_1 = \sigma, \sigma_2 = \sigma, \sigma_3 = 0$. The von Mises stress can be determined from the equation for the principal stresses

$$\sigma_{\text{Mises}} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} = \sigma \approx 250.000 \text{ Nmm}^{-2}. \tag{84 – 3}$$

The radial deflection of the vessel follows from Hooke's law

$$u_R = \frac{R}{E}(\sigma - \mu\sigma) \approx 0.419 \text{ mm}. \tag{84 – 4}$$

RFEM 5 Settings

- Modeled in RFEM 5.11.01
- Element size is $l_{\text{FE}} = 0.010$ m
- The number of increments is 10
- Isotropic linear elastic material is used

Results

Structure File	Program	Model
0084.01	RFEM 5	Full Model
0084.02	RFEM 5	Eighth Model

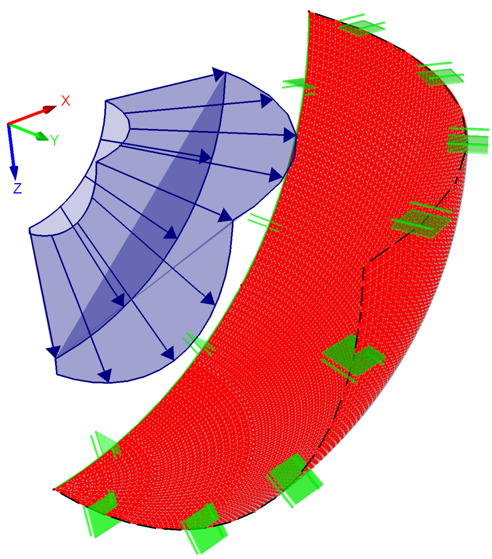

Figure 2: RFEM 5 results - eighth model, von Mises stress σ_{Mises}

Analytical Solution	RFEM 5 Full Model		RFEM 5 Eighth Model	
σ_{Mises} [Nmm^{-2}]	σ_{Mises} [Nmm^{-2}]	Ratio [-]	σ_{Mises} [Nmm^{-2}]	Ratio [-]
250.000	250.117	1.000	249.984	1.000

Analytical Solution	RFEM 5 Full Model		RFEM 5 Eighth Model	
u_R [mm]	u_R [mm]	Ratio [-]	u_R [mm]	Ratio [-]
0.419	0.419	1.000	0.419	1.000