0087 – Curved Beam with Distributed Loading

Description

A curved beam according to **Figure 1** consists of two beams of length L and rectangular cross-section $w \times h$. It is loaded by a distributed loading p. While neglecting self-weight, determine the maximal stress $\sigma_{x,\max}$ on the top surface of the horizontal beam.

Material	Modulus of Elasticity	E	210000.000	MPa
	Poisson's Ratio	ν	0.296	–
Geometry	Length	L	1.000	m
	Cross-section Width	w	25.000	mm
	Cross-section Height	h	50.000	mm
Load	Distributed Loading	p	10.000	N/mm

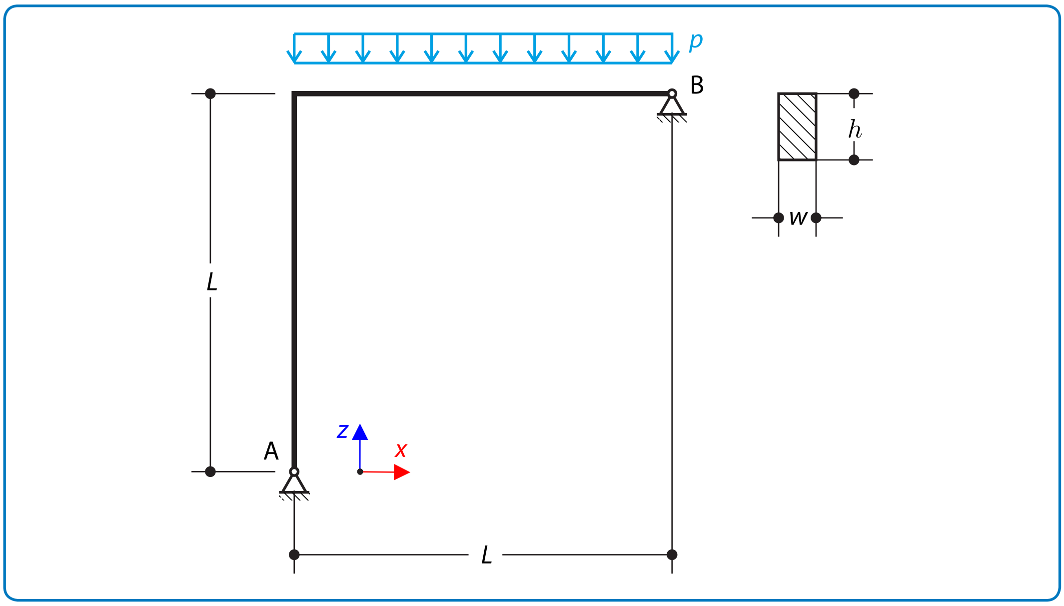

Figure 1: Problem Sketch

Analytical Solution

The equations of equilibrium yields that the given structure is statically indeterminate. To complete the set of equations, further constraint has to be found

$$A_x - B_x = 0, \tag{87 – 1}$$
$$pL - A_z - B_z = 0, \tag{87 – 2}$$
$$\frac{pL^2}{2} - B_zL - B_xL = 0, \tag{87 – 3}$$

where A_x, A_z, B_x, B_z are the corresponding reaction forces, see **Figure 2**. The missing equation is defined by means of the condition of zero deflection at point B in z-direction

$$u_{zB} = 0. \tag{87 – 4}$$

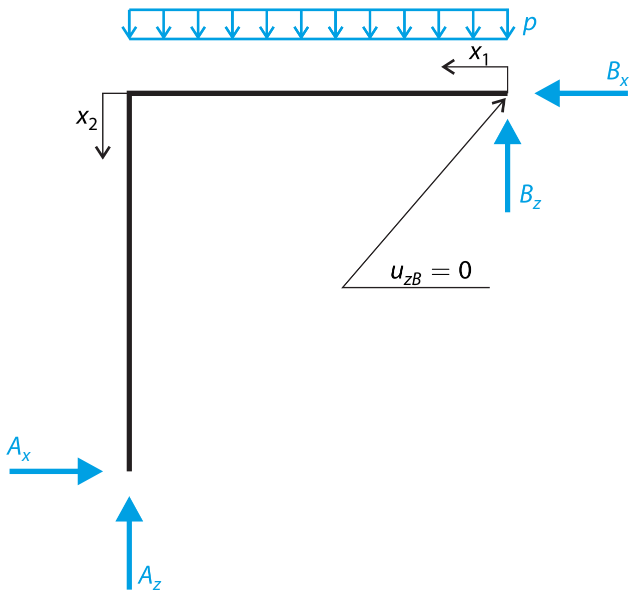

Figure 2: Free body diagram

The general deflection v of beams and curved beams can conveniently be determined by Maxwell-Mohr integral

$$v = \frac{1}{EI_y} \int_L M(x)m(x)\mathrm{d}x, \tag{87 – 5}$$

where I_y is the second moment of area, $M(x)$ is the bending moment caused by the outer forces and $m(x)$ is the bending moment caused by the unitary force, which is added to the investigated point in appropriate direction. The following formulas define these bending moments in two regions with coordinate x_1

$$x_1 \in [0, L], \tag{87 – 6}$$
$$M(x_1) = \frac{px_1^2}{2} - B_zx_1, \tag{87 – 7}$$
$$m(x_1) = x_1, \tag{87 – 8}$$

and coordinate x_2

$$x_2 \in [0, L], \tag{87 – 9}$$

$$M(x_2) = \frac{pL^2}{2} - B_x x_2 - B_z L, \tag{87 – 10}$$

$$m(x_2) = L - x_2. \tag{87 – 11}$$

The deflection of the point B is then equal to

$$u_{zB} = \frac{1}{EI_y} \left(\int_0^L M(x_1) m(x_1) \mathrm{d}x_1 + \int_0^L M(x_2) m(x_2) \mathrm{d}x_2 \right) = 0. \tag{87 – 12}$$

Considering equations **(87 – 1)**, **(87 – 2)**,**(87 – 3)** and **(87 – 12)** the reaction forces are equal to

$$A_x = \frac{1}{16} pL, \tag{87 – 13}$$

$$A_z = \frac{9}{16} pL, \tag{87 – 14}$$

$$B_x = \frac{1}{16} pL, \tag{87 – 15}$$

$$B_z = \frac{7}{16} pL. \tag{87 – 16}$$

The maximum stress occurs at the point with maximum bending moment M_{max}. This point is on the horizontal beam at distance

$$x_1 = \frac{7}{16} L. \tag{87 – 17}$$

The horizontal beam is also loaded by the axial reaction force B_x. The maximum stress $\sigma_{x,\mathrm{max}}$ on the top surface is composed of the maximum bending stress and the pressure stress caused by the axial reaction force B_x, hence

$$\sigma_{x,\mathrm{max}} = \sigma_{b,\mathrm{max}} + \sigma_a = \frac{6M_{\mathrm{max}}}{wh^2} + \frac{-B_x}{wh} = -92.375 \text{ MPa}. \tag{87 – 18}$$

RFEM 5 Settings

- Modeled in RFEM 5.12.02
- Element size is $l_{\mathrm{FE}} = 0.050$ m
- The number of increments is 10
- Isotropic linear elastic material is used
- Shear stiffness of the members is deactivated
- Kirchhoff bending theory for plates is used

Results

Structure File	Entity
0087.01	Member
0087.02	Plate

Entity	Theory	RFEM 5	
	$\sigma_{x,\text{max}}$ [MPa]	$\sigma_{x,\text{max}}$ [MPa]	Ratio [-]
Member	-92.375	−91.774	0.993
Plate		−92.422	1.001

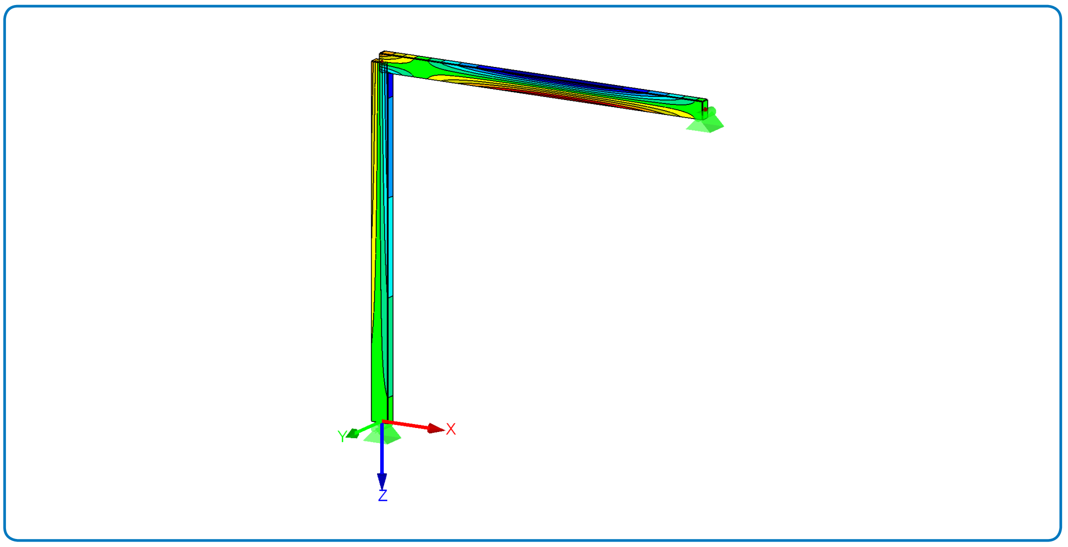

Figure 3: RFEM 5 results – σ_x stress distribution along the curved beam

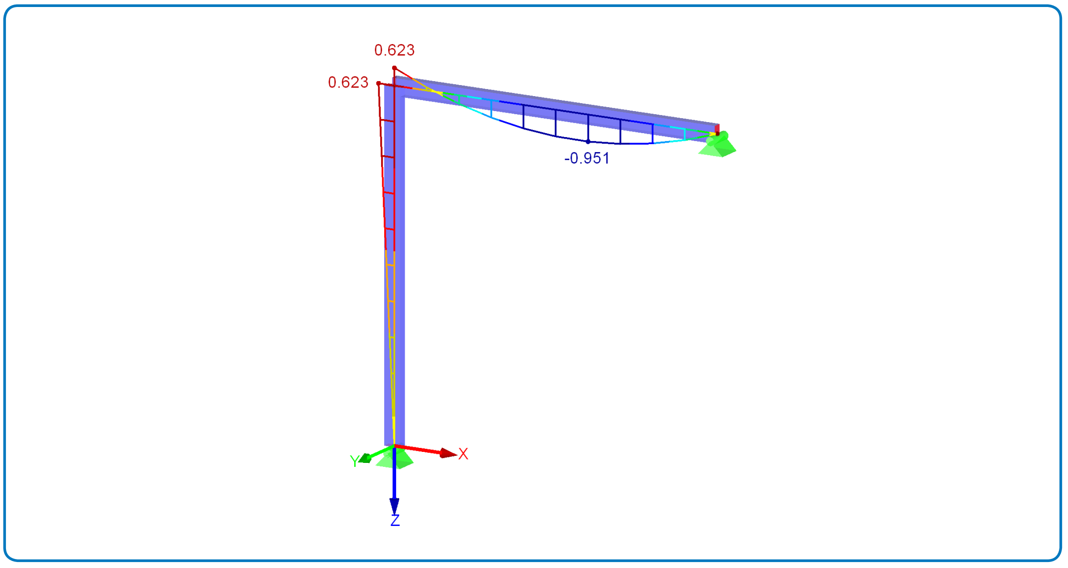

Figure 4: RFEM 5 results – bending moment behaviour along the curved beam