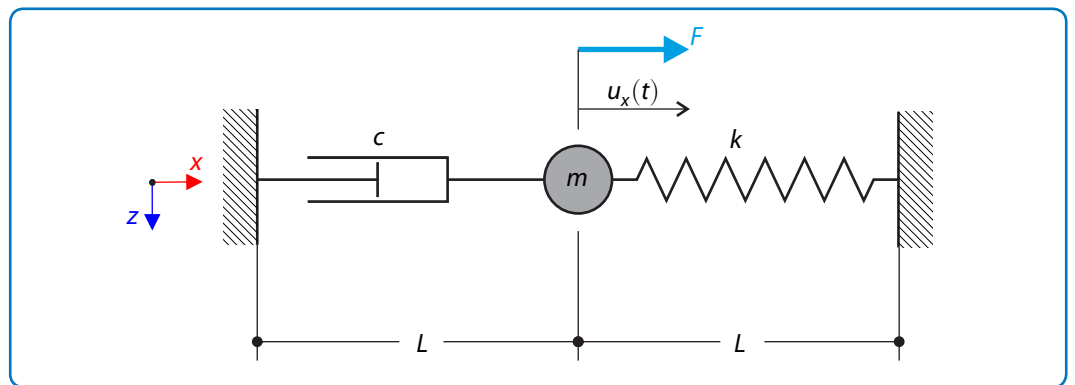


## 0121 – Dynamic Force Distribution

### Description

A single-mass system with dashpot is subjected to a constant loading force  $F$ . Determine the spring force  $S$ , the damping force  $B$  and the inertial force  $D$  at given test time. In this verification example, the Kelvin–Voigt dashpot, namely, a spring and a damper element in serial connection, is decomposed into its purely viscous and purely elastic parts in accord with **Figure 1**, in order to better evaluate the reaction forces. The problem is described by the following parameters.

System Properties	Dashpot	Stiffness	$k$	2000.000	N/m
		Length	$L$	0.200	m
		Damping Parameter	$c$	100.000	Ns/m
	Mass	Weight	$m$	100.000	kg
Load		Force	$F$	200.000	N



**Figure 1:** Problem sketch

### Analytical Solution

The analytical solution is based on the theory introduced in [1]. A damped single-mass system is described by the second-order differential equation<sup>1</sup>

$$m\ddot{u}_x + c\dot{u}_x + ku_x = F, \quad (121 - 1)$$

where each term can be replaced by the corresponding force - inertial force  $D$ , damping force  $B$  and spring force  $S$

$$D + B + S = F. \quad (121 - 2)$$

The deflection of the endpoint is defined by the following formula

<sup>1</sup>  $\dot{u}_x$  and  $\ddot{u}_x$  denote the first and second time derivative of  $u_x$ , respectively.

## Verification Example: 0121 – Dynamic Force Distribution

$$u_x(t) = \frac{F}{2k} \left( 1 - \frac{c_r}{\sqrt{c_r^2 - 1}} \right) e^{(\sqrt{c_r^2 - 1} - c_r)\Omega t} - \frac{F}{2k} \left( 3 - \frac{c_r}{\sqrt{c_r^2 - 1}} \right) e^{(-\sqrt{c_r^2 - 1} - c_r)\Omega t} + \frac{F}{k}. \quad (121 - 3)$$

This deflection and its derivatives are further used for the calculation of the desired forces

$$S = ku_x, \quad (121 - 4)$$

$$B = c\dot{u}_x, \quad (121 - 5)$$

$$D = m\ddot{u}_x. \quad (121 - 6)$$

These forces at test time  $t = 1$  s are equal to

$$S(1) \approx 245.317 \text{ N}, \quad (121 - 7)$$

$$B(1) \approx -26.320 \text{ N}, \quad (121 - 8)$$

$$D(1) \approx -18.997 \text{ N}. \quad (121 - 9)$$

## RFEM 5 and RSTAB 8 Settings

- Modeled in RFEM 5.14.01 and RSTAB 8.14.01
- The global element size is  $l_{FE} = 0.2$  m

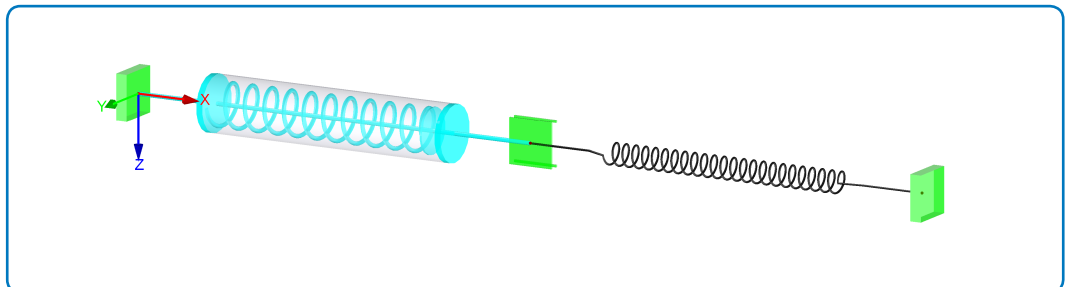


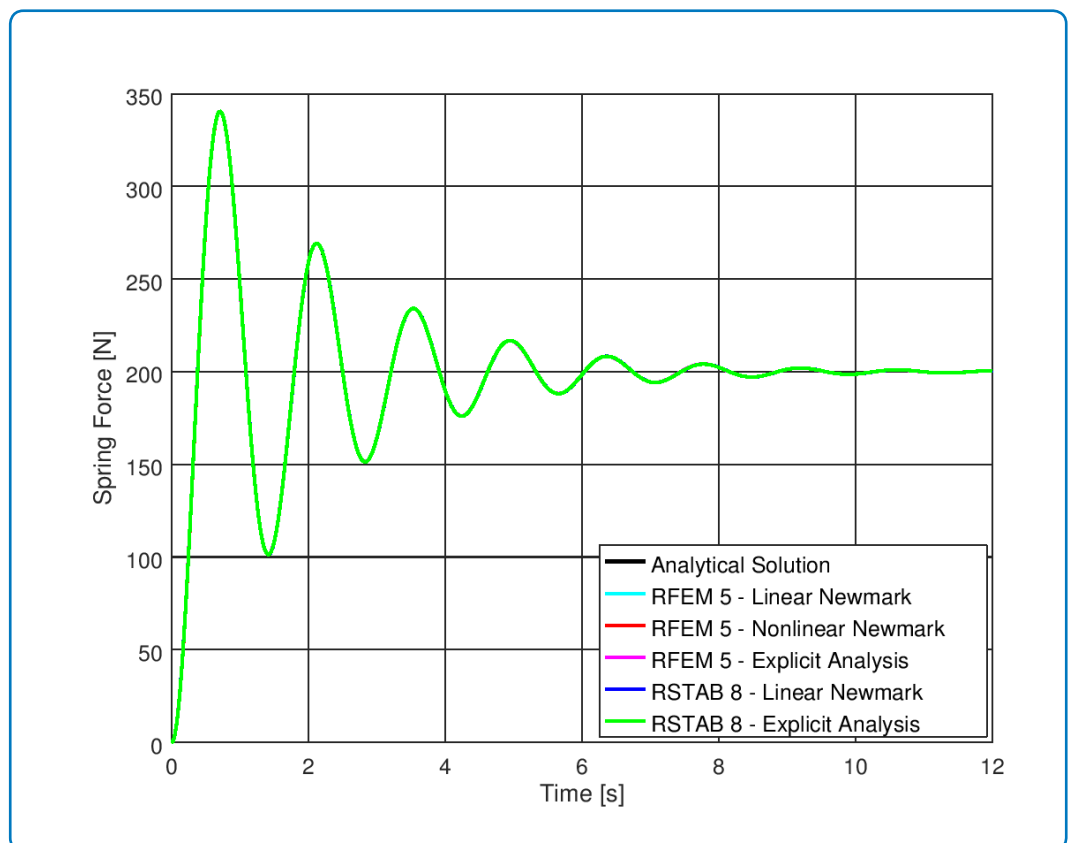
Figure 2: Model in RFEM 5 / RSTAB 8

## Results

Structure Files	Program	Method
0121.01	RFEM 5 – RF-DYNAM Pro	Linear Implicit Newmark Analysis
0121.02	RFEM 5 – RF-DYNAM Pro	Nonlinear Implicit Newmark Analysis
0121.03	RFEM 5 – RF-DYNAM Pro	Explicit Analysis
0121.04	RSTAB 8 – DYNAM Pro	Linear Implicit Newmark Analysis
0121.05	RSTAB 8 – DYNAM Pro	Explicit Analysis

### Verification Example: 0121 – Dynamic Force Distribution

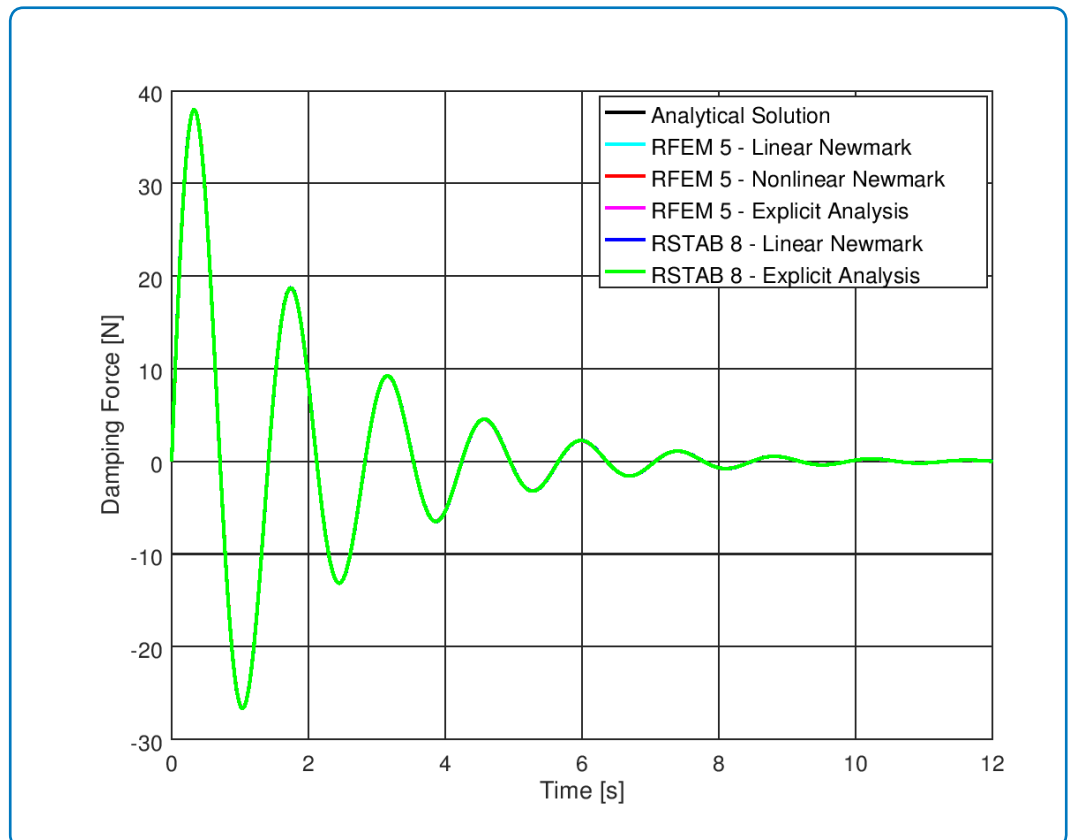
Model	Analytical Solution	RFEM 5 / RSTAB 8	
	S(1) [N]	S(1) [N]	Ratio [-]
RFEM 5, Linear Implicit Newmark Analysis	245.317	245.449	1.001
RFEM 5, Nonlinear Implicit Newmark Analysis		245.448	1.001
RFEM 5, Explicit Analysis		244.393	0.996
RSTAB 8, Linear Implicit Newmark Analysis		245.407	1.000
RSTAB 8, Explicit Analysis		245.368	1.000



**Figure 3:** Analytical and RFEM 5 / RSTAB 8 solution - spring force S

### Verification Example: 0121 – Dynamic Force Distribution

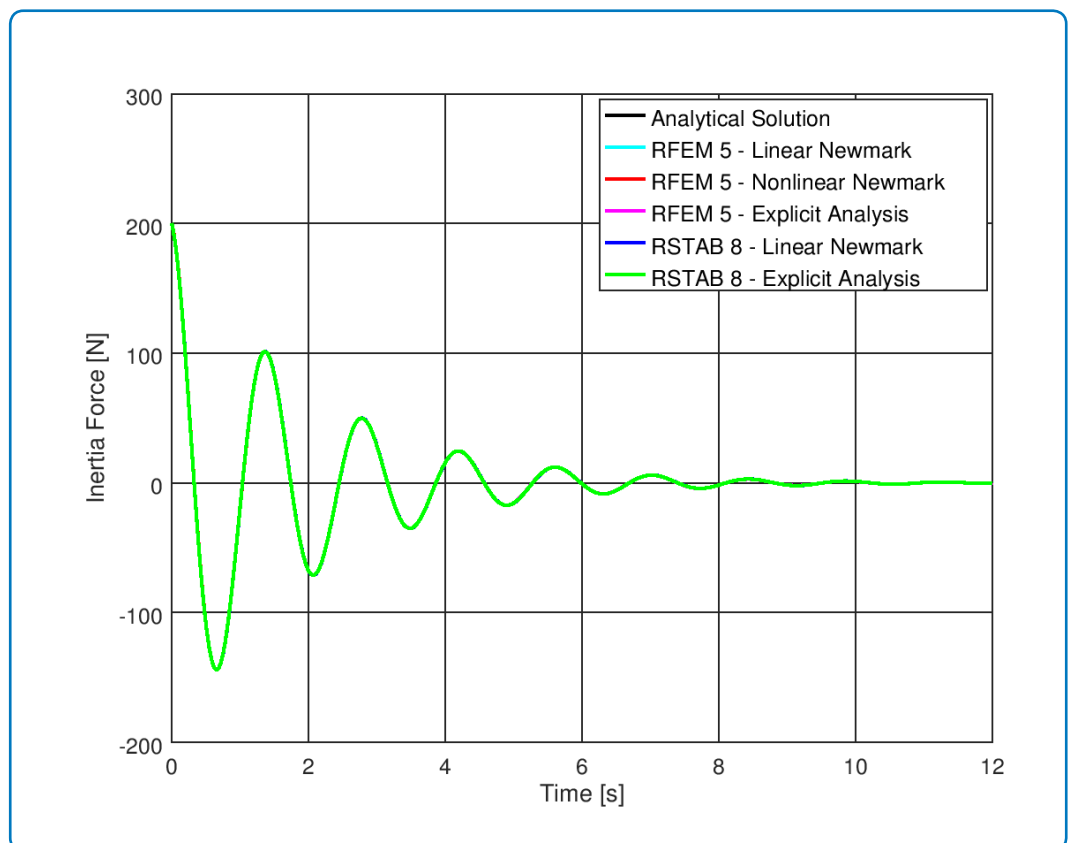
Model	Analytical Solution	RFEM 5 / RSTAB 8	
	$B(1)$ [N]	$B(1)$ [N]	Ratio [-]
RFEM 5, Linear Implicit Newmark Analysis	-26.320	-26.316	1.000
RFEM 5, Nonlinear Implicit Newmark Analysis		-26.316	1.000
RFEM 5, Explicit Analysis		-26.320	1.000
RSTAB 8, Linear Implicit Newmark Analysis		-26.321	1.000
RSTAB 8, Explicit Analysis		-26.318	1.000



**Figure 4:** Analytical and RFEM 5 / RSTAB 8 solution - damping force  $B$

### Verification Example: 0121 – Dynamic Force Distribution

Model	Analytical Solution	RFEM 5 / RSTAB 8	
	$D(1)$ [N]	$D(1)$ [N]	Ratio [-]
RFEM 5, Linear Implicit Newmark Analysis	-18.997	-19.133	1.007
RFEM 5, Nonlinear Implicit Newmark Analysis		-19.133	1.007
RFEM 5, Explicit Analysis		-19.002	1.000
RSTAB 8, Linear Implicit Newmark Analysis		-19.086	1.005
RSTAB 8, Explicit Analysis		-18.997	1.000



**Figure 5:** Analytical and RFEM 5 / RSTAB 8 solution - inertia force  $D$

### References

- [1] DLUBAL SOFTWARE GMBH, *Verification Example 0120 – Single-Mass Oscillation with Dashpot*. 2018a.