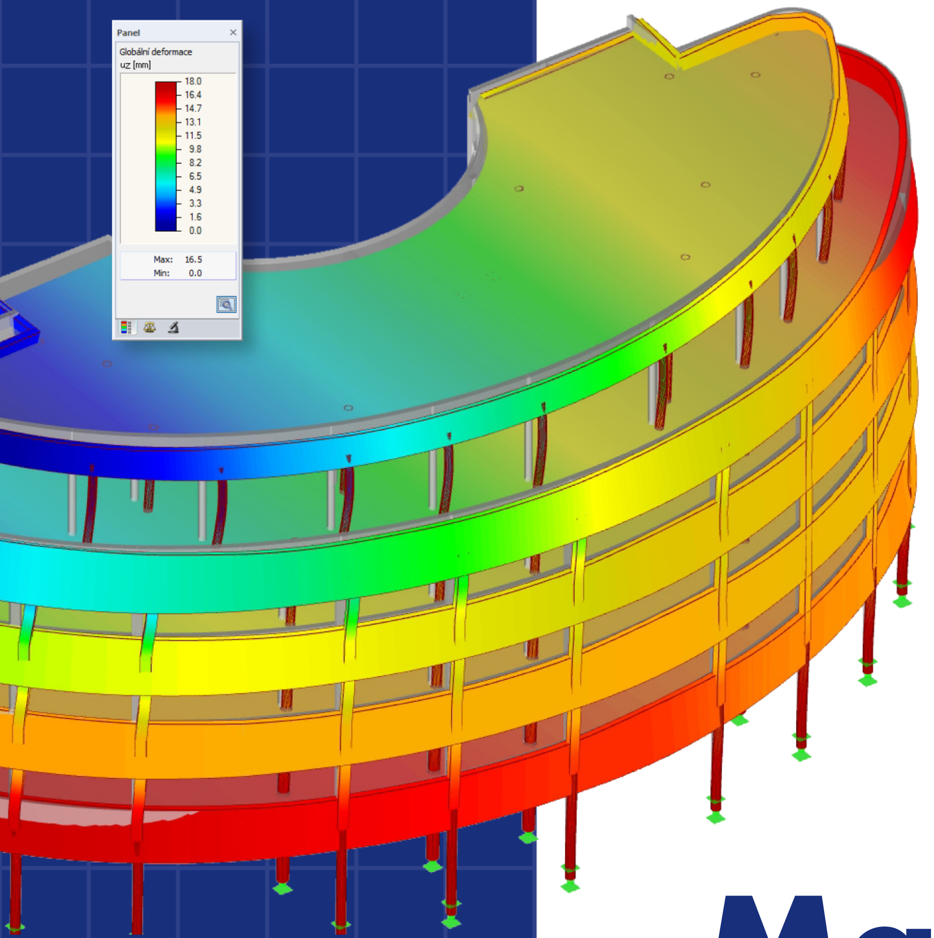
RF-CONCRETE Surfaces

Reinforced concrete design of surfaces (plates, walls, planar structures, shells)



Manual

Version

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Dlubal Software

Short Overview

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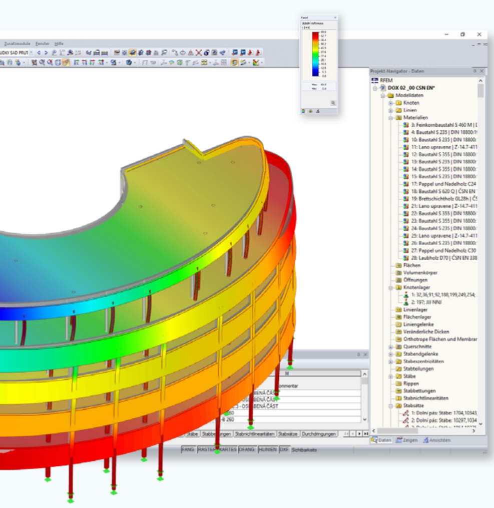
Using the Manual

This program description is divided into chapters, which are based on the order and structure of the input and result windows. In the chapters, the individual windows are presented column by column. They help to understand the functional processes that affect the add-on module. General functions are described in the manual of the main program RFEM.



Hint

The text of the manual shows the described buttons in square brackets, for example [OK]. In addition, they are pictured on the left. Expressions that appear in dialog boxes, tables, and menus are set in *italics* to clarify the explanation. You can also use the search function for the Knowledge Base and FAQs to find a solution in the posts about add-on modules.

Topicality

The high quality standards placed on the software are guaranteed by a continuous development of the program versions. This may result in differences between program description and the current software version you are using. Thank you for your understanding that no claims can be derived from the figures and descriptions. We always try to adapt the documentation to the current state of the software.

Table of Contents

1 Introduction

1.1 Add-on Module RF-CONCRETE Surfaces

Although reinforced concrete is at least as frequently used for plate structures as it is for frameworks, standards and technical literature provide comparatively little information on the design of two-dimensional structural components. In particular, the design of shell structures that are simultaneously subjected to moments and axial forces is rarely described in reference books. Since the finite element method allows for a realistic modeling of plate structures, design assumptions and algorithms must be found to close this "regulatory gap" between member-oriented regulations and computer-generated internal forces of plate structures.

Dlubal Software GmbH meets this challenge with the add-on module RF-CONCRETE Surfaces. A consistent design algorithm for dimensioning reinforcement directions consisting of two and three layers has been developed based on the compatibility conditions defined by Baumann [1] in 1972. However, the module is more than just a tool for determining the statically required reinforcement: It also includes regulations concerning the allowable maximum and minimum reinforcement ratios for the different types of structural components (2D plates, 3D shells, walls, deep beams) like they are given as design specifications in the standards.

When determining reinforcing steel, **RF-CONCRETE Surfaces** checks if the concrete's plate thickness, which stiffens the reinforcement mesh, is sufficient to meet all requirements arising from bending and shear loading.

In addition to the ultimate limit state design, the serviceability limit state design is also possible in the module. These designs include the limitation of the concrete compressive and the reinforcing steel stresses, the minimum reinforcement for crack control, as well as crack control by limiting rebar diameter and rebar spacing. For this purpose, analytical and nonlinear design check methods are available for selection.

If you also have a license for **RF-CONCRETE Deflect**, you can calculate the deformations with the influence of creep, shrinkage, and tension stiffening according to the analytical method.

National Annexes for EN 1992-1-1

With a license of **RF-CONCRETE NL**, you can consider the influence of creep and shrinkage when determining deformations, crack widths, and stresses according to the nonlinear method.

The design is possible according to the following standards:

- EN 1992-1-1:2004/A1:2014
- ACI 318-19
- CSA A23.3-19
- SIA 262:2017
- GB 50010-2010

The figure on the left shows the National Annexes to EN 1992-1-1 that are currently implemented in RF-CONCRETE Surfaces.

All intermediate results for the design are comprehensively documented. In line with the philosophy of Dlubal Software, this provides a notable transparency and traceability of design results.

We hope you will enjoy working with RF-CONCRETE Surfaces.

Your Dlubal team

1.2 Using the Manual

Topics like installation, graphical user interface, results evaluation, and printout are described in detail in the manual of the main program RFEM. This manual focuses on typical features of the add-on module RF-CONCRETE Surfaces.

The descriptions in this manual follow the sequence and structure of the module's input and result windows. In the text, the described **buttons** are given in square brackets, for example [Apply]. At the same time, they are pictured on the left. **Expressions** that appear in dialog boxes, tables, and menus are highlighted in *italics* to clarify explanations.

In the PDF manual, you can perform a full-text search as usual with [Ctrl]+[F]. If you do not find what you are looking for, you can also use the search function of the Knowledge Base on our website to find a solution in the articles about the concrete modules. Our FAQs also provide a number of helpful hints.

1.3 Opening the RF-CONCRETE Surfaces Add-on Module

RFEM provides the following options to start the RF-CONCRETE Surfaces add-on module.

Menu

You can start the add-on module with the RFEM menu item

Add-on Modules → Design - Concrete → RF-CONCRETE Surfaces.

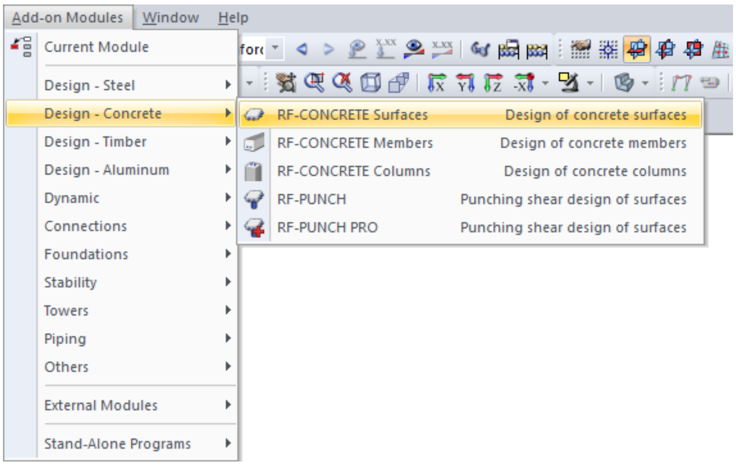


Figure 1.1 *Add-on Modules → Design - Concrete → RF-CONCRETE Surfaces* menu item

Navigator

Alternatively, you can start the add-on module in the *Data* navigator by selecting

Add-on Modules → RF-CONCRETE Surfaces.

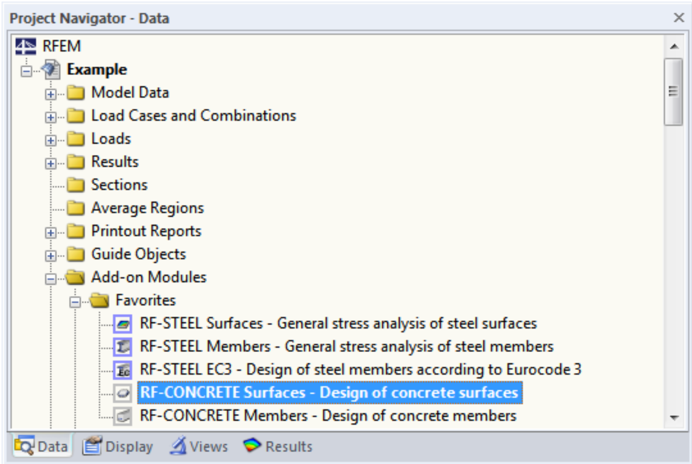
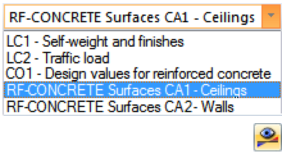


Figure 1.2 *Data navigator: Add-on Modules → RF-CONCRETE Surfaces*

Panel



If results from RF-CONCRETE Surfaces are already available in the RFEM model, you can also start the design module in the panel:

Set the relevant design case of RF-CONCRETE Surfaces in the load case list of the menu bar. Then use the [Show Results] button to display the reinforcements graphically.

Now you can click the [RF-CONCRETE Surfaces] button in the panel to open the add-on module.

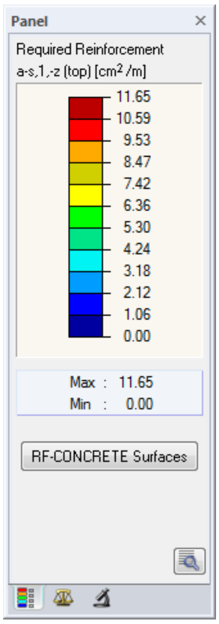


Figure 1.3 [RF-CONCRETE Surfaces] panel button

2 Theoretical Background

2.1 Type of Model

The *Type of Model* that is defined when creating a new model has a crucial influence on how the structural components are stressed.

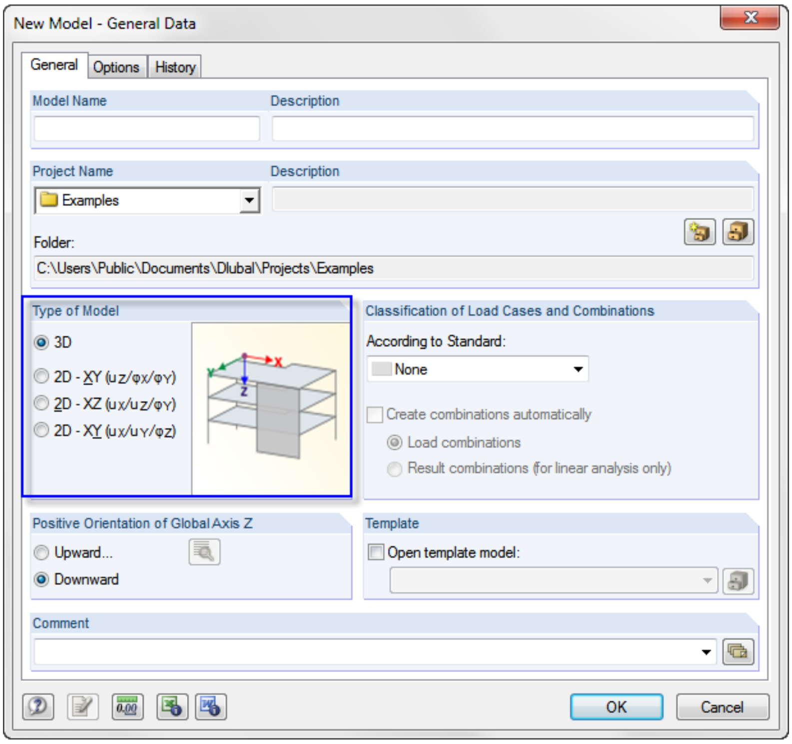


Figure 2.1 *New Model - General Data* dialog box, *Type of Model* dialog section

When the model type *2D - XY* ($u_Z/\varphi_X/\varphi_Y$) is selected, the plate is only subjected to bending. The designed internal forces are then exclusively represented by moments whose vectors lie in the plane of the structural component.

However, when you select *2D - XZ* ($u_X/u_Z/\varphi_Y$) or *2D - XY* ($u_X/u_Y/\varphi_Z$), the wall or diaphragm is only subjected to compression or tension. The internal forces used for the design are solely axial forces whose vectors lie in the plane of the structural component.

In a spatial 3D model type, both loadings (moments and axial forces) are combined. A structural component defined in this way can be subjected to tension/compression and bending simultaneously. Thus, the internal forces to be designed are both axial forces and moments whose vectors lie in the component's plane.

2.2

Design of 1D and 2D Structural Components

To design the ultimate limit state of a one- or two-dimensional structural component consisting of reinforced concrete, it is always necessary to find a state of equilibrium between the acting internal forces and the resisting internal forces of the deformed component. However, in addition to this common feature in the ultimate limit state design of one-dimensional components (members) and two-dimensional components (surfaces), there is a crucial difference:

1D structural component (member)

In a member, the acting internal force is always oriented in such a way that it can be compared to the resisting internal force that is determined from the design strengths of the materials. An example of this is a member subjected to the axial compressive force N.

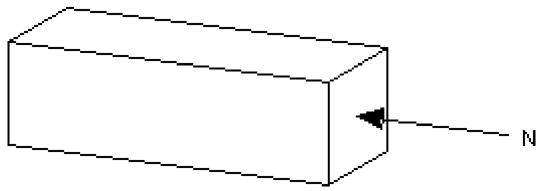


Figure 2.2 Design of a member

With the dimensions of the structural component and the design value of the concrete compressive strength, it is possible to determine the resisting compressive force. If it is smaller than the acting compressive force, the required area of the compressive reinforcement can be determined by means of the existing steel strain with an allowable concrete compressive strain.

2D structural component (surface)

In a surface, the direction of the acting internal force is only in exceptional cases (trajectory reinforcement) oriented in such a way that the directly acting internal force can be set in contrast with the resisting internal force: In an orthogonally reinforced wall, for example, the directions of the two principal axial forces n_1 and n_2 are generally not identical to the reinforcement directions.

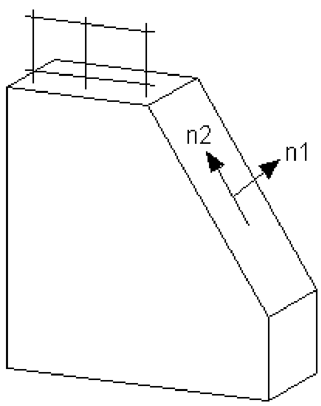


Figure 2.3 Design of a wall

Hence, for the dimensioning of the mesh's reinforcement, it is not possible to use such an approach as is used when determining a member's reinforcement. Internal forces that run in the direction of the reinforcement mesh's layers are required in order to determine the concrete loading. These internal forces are called **design internal forces**.

To better understand design internal forces, we can look at an element of a loaded reinforcement mesh. For simplicity's sake, we assume the second principal axial force n_2 to be zero.

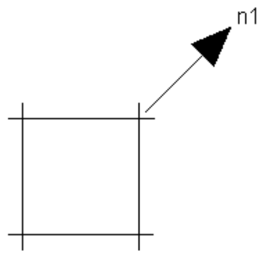


Figure 2.4 Reinforcement mesh element with loading

The reinforcement mesh deforms under the given loading as follows.

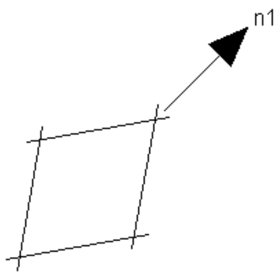


Figure 2.5 Deformation of reinforcement mesh element

The size of the deformation is limited by introducing a concrete compression strut into the reinforcement mesh element.

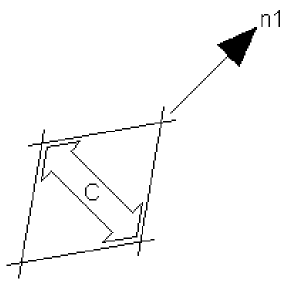


Figure 2.6 Introducing a concrete compression strut

The concrete compression strut induces tensile forces in the reinforcement.

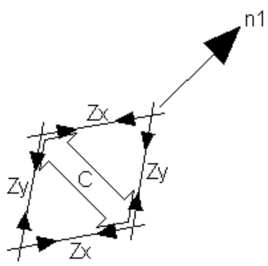


Figure 2.7 Tensile forces in the reinforcement

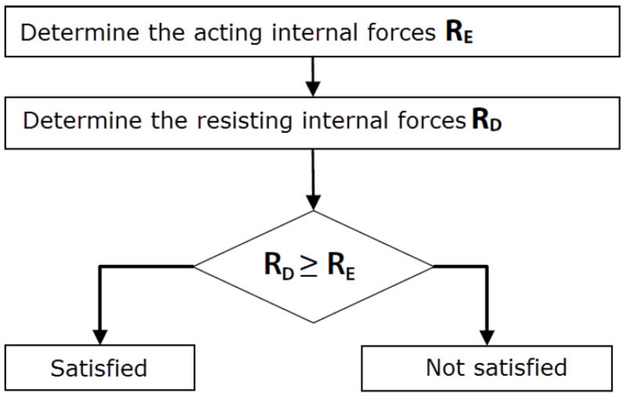
These tensile forces in the reinforcement and the compressive force in the concrete represent the design internal forces.

Once the design internal forces are found, the design can proceed like the design of a one-dimensional structural component.

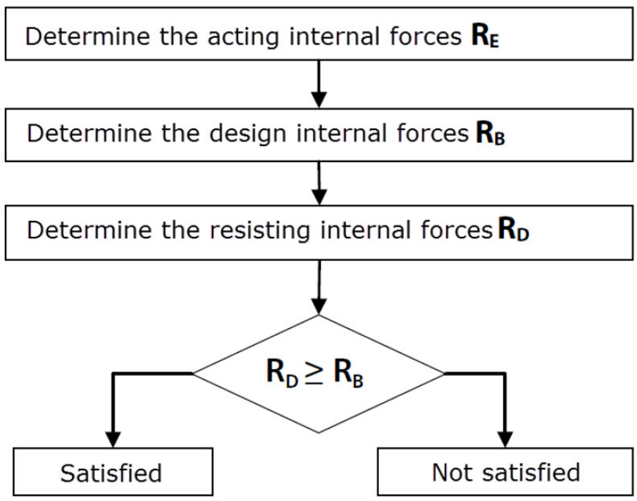
Thus, the main feature for the design of two-dimensional structural components is the transformation of the acting internal forces (principal internal forces) into design internal forces whose direction allows for dimensioning the reinforcement and checking the concrete's load-bearing capacity.

The following graphic illustrates the main difference between the design of one-dimensional and two-dimensional structural components.

One-dimensional structural component



Two-dimensional structural component



2.3 Walls (Diaphragms)

2.3.1 Design Internal Forces

Determining the design internal forces for walls and diaphragms is carried out according to Baumann's [1] method of transformation. In this method, the equations for determining the design internal forces are derived for the general case of a reinforcement with three arbitrary directions. Then these forces can be applied to simpler cases such as orthogonal reinforcement meshes with two reinforcement directions.

Baumann analyzes the equilibrium conditions with the following wall element.

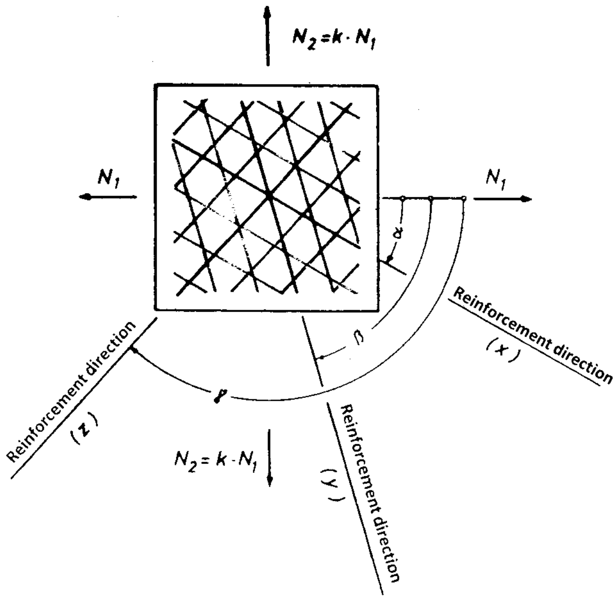


Figure 2.8 Equilibrium conditions according to Baumann

Figure 2.8 shows a rectangular segment of a wall. It is subjected to the principal axial forces N_1 and N_2 (tensile forces). The principal axial force N_2 is expressed by means of the factor k as a multiple of the principal axial force N_1.

$$N_2 = k \cdot N_1$$

Equation 2.1

Three reinforcement directions are applied in the wall. The reinforcement directions are labelled x, y, and z. The angle included in clockwise direction by the first principal axial force N_1 and the reinforcement direction x is labelled α. The angle between the first principal axial force N_1 and the reinforcement direction y is called β; the angle to the remaining reinforcement set is called γ.

Baumann writes in his thesis: If the shear and tension stresses in the concrete are neglected, the external loading (N_1, $N_2 = k \cdot N_1$) of a wall element can generally be resisted by three internal forces oriented in any direction. In a reinforcement mesh with three reinforcement directions, these forces correspond to the three reinforcement directions (x), (y), and (z), which form the angles α, β, γ with the larger main tensile force N_1 and are labelled Z_x, Z_y, Z_z (positive as tensile forces).

To determine these forces Z_x, Z_y (and Z_z in case of a third reinforcement direction), we first have to define a section parallel to the third reinforcement direction.

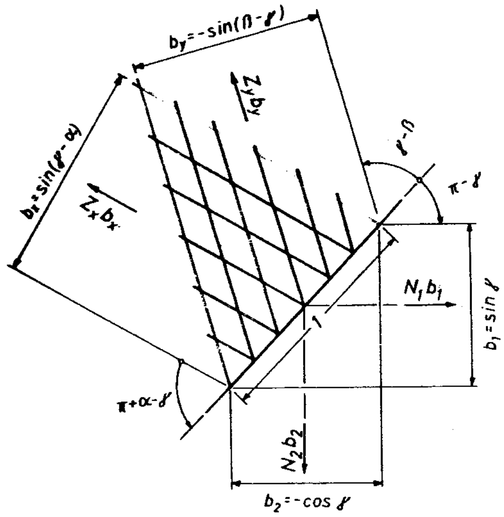


Figure 2.9 Section parallel to the third reinforcement direction z

The value of the section length is applied as 1. With this section length, we can determine the projected section lengths that run perpendicular to the respective force. In the case of the external forces, these are the projected section lengths b_1 (perpendicular to force N_1) and b_2 (perpendicular to force N_2). In the case of the tensile forces in the reinforcement, these are the projected section lengths b_x (perpendicular to the tension force Z_x) and b_y (perpendicular to the tension force Z_y).

The product of the respective force and the corresponding projected section length then results in the force that can be used to establish an equilibrium of forces.

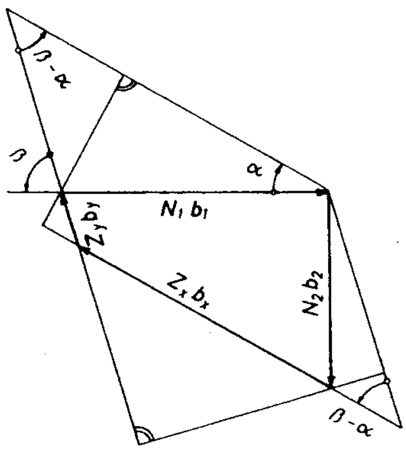


Figure 2.10 Equilibrium of forces in a section parallel to reinforcement in direction z

The equilibrium between the external forces (N_1, N_2) and the internal forces (Z_x, Z_y) can thus be expressed as follows.

$$Z_x \cdot b_x = \frac{1}{\sin(\beta - \alpha)} \cdot (N_1 \cdot b_1 \cdot \sin\beta - N_2 \cdot b_2 \cdot \cos\beta)$$

Equation 2.2

$$Z_y \cdot b_y = \frac{1}{\sin(\beta - \alpha)} \cdot (-N_1 \cdot b_1 \cdot \sin\alpha - N_2 \cdot b_2 \cdot \cos\alpha)$$

Equation 2.3

To determine the equilibrium between the external forces (N_1, N_2) and the internal force Z_z in the reinforcement direction z, we define a section parallel to the reinforcement direction x.

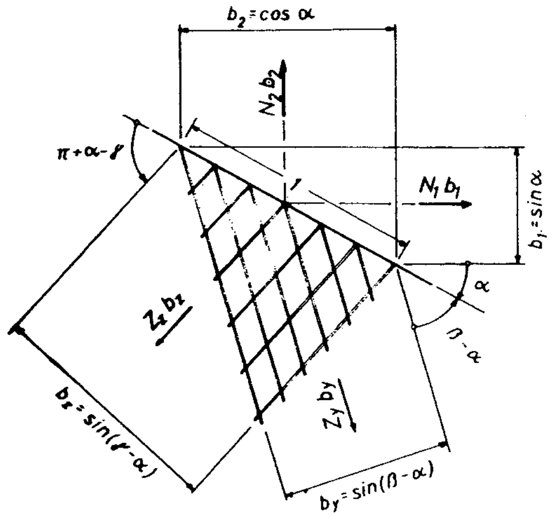


Figure 2.11 Section parallel to the reinforcement direction x

Graphically, we can determine the following equilibrium.

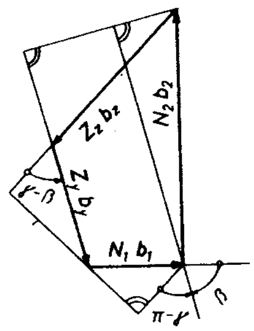


Figure 2.12 Equilibrium of forces in a section parallel to reinforcement in direction x

The equilibrium between the external forces (N_1, N_2) and the internal forces Z_z can then be expressed as follows.

$$Z_z \cdot b_z = \frac{1}{\sin(\beta - \gamma)} \cdot (N_1 \cdot b_1 \cdot \sin\beta - N_2 \cdot b_2 \cdot \cos\beta)$$

Equation 2.4

If you replace the projected section lengths b_1, b_2, b_x, b_y, b_z with the values shown in the figure and use *k* as the quotient of the principal axial force N_2 divided by N_1, you get the following equations.

$$\frac{Z_x}{N_1} = \frac{\sin\beta \cdot \sin\gamma + k \cdot \cos\beta \cdot \cos\gamma}{\sin(\beta - \alpha) \cdot \sin(\gamma - \alpha)}$$

Equation 2.5

$$\frac{Z_y}{N_1} = \frac{\sin\alpha \cdot \sin\gamma + k \cdot \cos\alpha \cdot \cos\gamma}{\sin(\beta - \alpha) \cdot \sin(\beta - \gamma)}$$

Equation 2.6

$$\frac{Z_z}{N_1} = \frac{-\sin\alpha \cdot \sin\beta + k \cdot \cos\alpha \cdot \cos\beta}{\sin(\beta - \gamma) \cdot \sin(\gamma - \alpha)}$$

Equation 2.7

These equations are the core of the design algorithm for RF-CONCRETE Surfaces. You can thus determine the design internal forces Z_x, Z_y, and Z_z for the respective reinforcement directions from the acting internal forces N_1 and N_2.

By adding up Equation 2.5, Equation 2.6, and Equation 2.7, you get:

$$\frac{Z_x}{N_1} + \frac{Z_y}{N_1} + \frac{Z_z}{N_1} = 1 + k$$

Equation 2.8

By multiplying Equation 2.8 with N_1 and substituting k for N_2 / N_1, you get the following equation that clarifies the equilibrium of the internal and external forces.

$$Z_x + Z_y + Z_z = N_1 + N_2$$

Equation 2.9

2.3.2 Two-Directional Reinforcement Meshes with k > 0

For a reinforcement with two reinforcement directions subjected to two positive principal axial forces N_1 und N_2, choose the direction of the concrete compressive strut as follows.

$$\gamma = \frac{\alpha + \beta}{2}$$

Equation 2.10

There are basically two ways to arrange a compression strut exactly at the center between two crossing reinforcement directions.

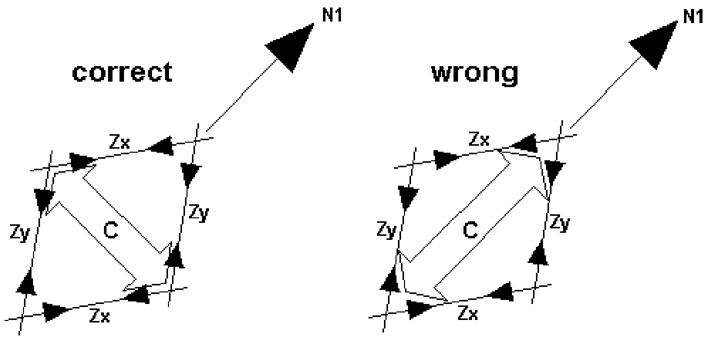


Figure 2.13 Correct and incorrect arrangement of the stiffening concrete compression strut

In the figure on the left, the stiffening concrete compression strut divides the obtuse angle between the crossing reinforcement directions; in the figure on the right, it divides the acute angle. The strut on the left stiffens the reinforcement mesh in the desired way, whereas the concrete compression strut shown in the figure on the right allows the reinforcement mesh to be arbitrarily deformed by the force N_1.

To ensure that the compression strut divides the correct angle, the design forces Z_x, Z_y, and Z_z are determined using Equation 2.5, Equation 2.6, and Equation 2.7 for both geometrically possible directions of the compression strut. A wrong direction of the compression strut would result in a tensile force.

Therefore, the following directions of the concrete compression strut are analyzed:

$$\gamma_{1a} = \frac{\alpha + \beta}{2} \quad \text{and} \quad \gamma_{1b} = \frac{\alpha + \beta}{2} + 90°$$

Equation 2.11

To distinguish the analyzed directions, the index "1a" is assigned to the simple arithmetic mean value and the index "1b" is assigned to the direction of the compression strut that is rotated by 90°.

The following graph shows that for the equilibrium of forces, a tension force is respectively obtained in the two reinforcement directions and a compression force in the selected direction of the compression strut.

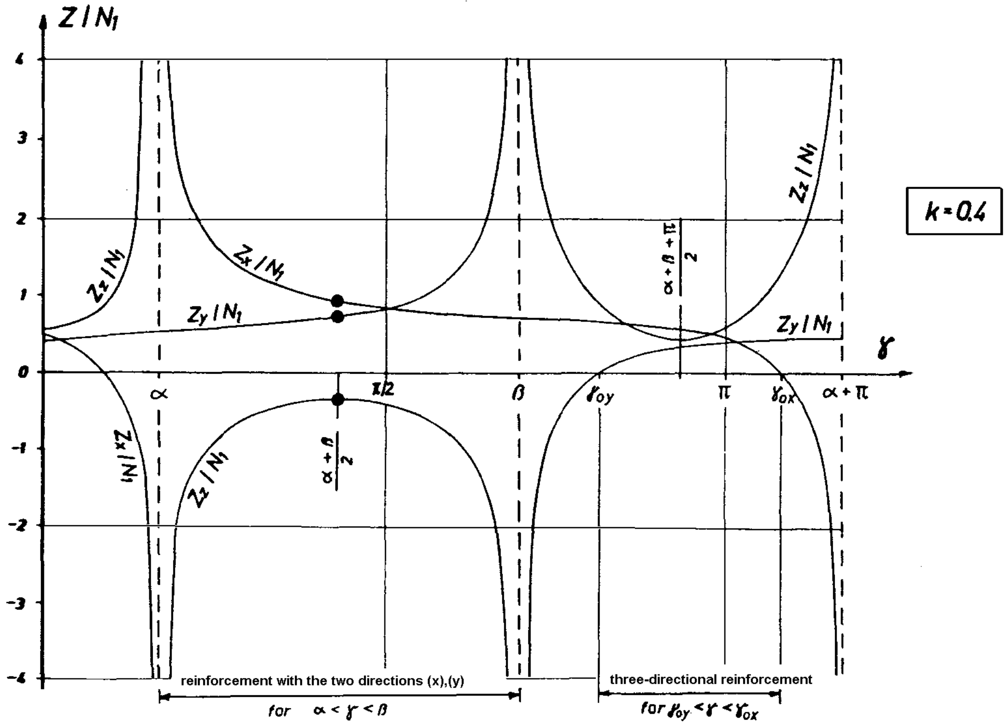


Figure 2.14 Two-directional reinforcement for pure tension

In his studies, Baumann [1] assumed certain ranges of values for the different angles. The angle α (between the principal axial force N_1 and the reinforcement direction closest to it) thus has to be between 0 and $\pi/4$. The angle β must be greater than $\alpha + \pi/2$.

[1] provides Table IV with the possible states of equilibrium (see Figure 2.15). Rows 1 through 4 of this table show the possible states of equilibrium for walls that are only subjected to tension. Row 4 shows the state of equilibrium with two directions of reinforcement subjected to tension and a compression strut. Rows 5 to 7 show walls for which the principal axial forces have different algebraic signs.

Table IV:

Appropriate selection of angle β and γ for a given direction $0 < \alpha < \pi/4$

of the reinforcement direction (x)

Row	Ratio $k = N_2/N_1$ of internal forces	Number of required directions of reinforcement	Direction β of reinforcement direction (y)	Direction γ of direction Z_z	Tension forces (to be resisted by the reinforcement) in the direction of	Tension force (to be resisted by the concrete) in the direction of	Reinforcement layout 1) ($N_2 = k \cdot N_1$, N_1, N_2, $\alpha < \pi/4$, (x))	Row
1	$0 < k < 1$	3	$\alpha < \beta < \gamma_{oy}$	$\gamma_{oy} < \gamma < \gamma_{ox}$	(x), (y), (z)	-	(z) (y) (x)	1
2			$\gamma_{oy} < \beta < \pi$	$\gamma_{ox} < \gamma < \gamma_{oy}$	(x), (y), (z)	-	(y) (z) (x)	2
3		2	—	$\gamma = \gamma_{oy}$	(x), (z)	-	(z) (x)	3
4			$\alpha + \pi/2 < \beta < \gamma_{oy}$	$\gamma = \frac{\alpha + \beta}{2}$	(x), (y)	(z)	(y) (z) (x)	4
5	$-tg^2\alpha \leq k \leq 0$	2	$\alpha + \pi/2 < \beta < \pi - \alpha$	$\gamma = \frac{\alpha + \beta}{2}$	(x), (y)	(z)	(y) (z) (x)	5
6	$k < -tg^2\alpha$	2	$\alpha + \pi/2 < \beta < \pi - \alpha$ und $\beta > 2\gamma_{oy} - \alpha$	$\gamma = \frac{\alpha + \beta}{2}$	(x), (y)	(z)	(y) (z) (x)	6
7		1	—	$\gamma = \gamma_{oy}$	(x)	(z)	(z) (x)	7

1) Reinforcement directions are indicated by continuous lines, concrete compression forces by dotted lines.

Figure 2.15 Possible states of equilibrium according to [1]

The second column of this table defines the value range of the loading.

The third column indicates the number of reinforcement directions subjected to a tension force for this state of equilibrium.

The fourth column (β) shows the value range of the reinforcement direction β. In RF-CONCRETE Surfaces, this range is not available as it results from the directions of reinforcement specified in the input data.

The fifth column (γ) shows the direction of the internal force Z_z. In most cases, this is the direction of the compression strut computed by the program; however, it can also be a user-defined third reinforcement direction to which a tension force is actually assigned.

The seventh column indicates whether or not the force in the direction γ is indeed a compression force.

The penultimate column shows the required internal forces together with their directions. Reinforcement directions with a tension force are represented by simple lines whereas possible compression struts are indicated by dashed lines.

2.3.3 Two-Directional Reinforcement Meshes with k < 0

If the main axial forces N_1 and N_2 have different signs in a two-directional reinforcement mesh, a tension force is respectively obtained for the equilibrium of forces in the two reinforcement directions, as well as a compression force in the selected direction of the compressive strut.

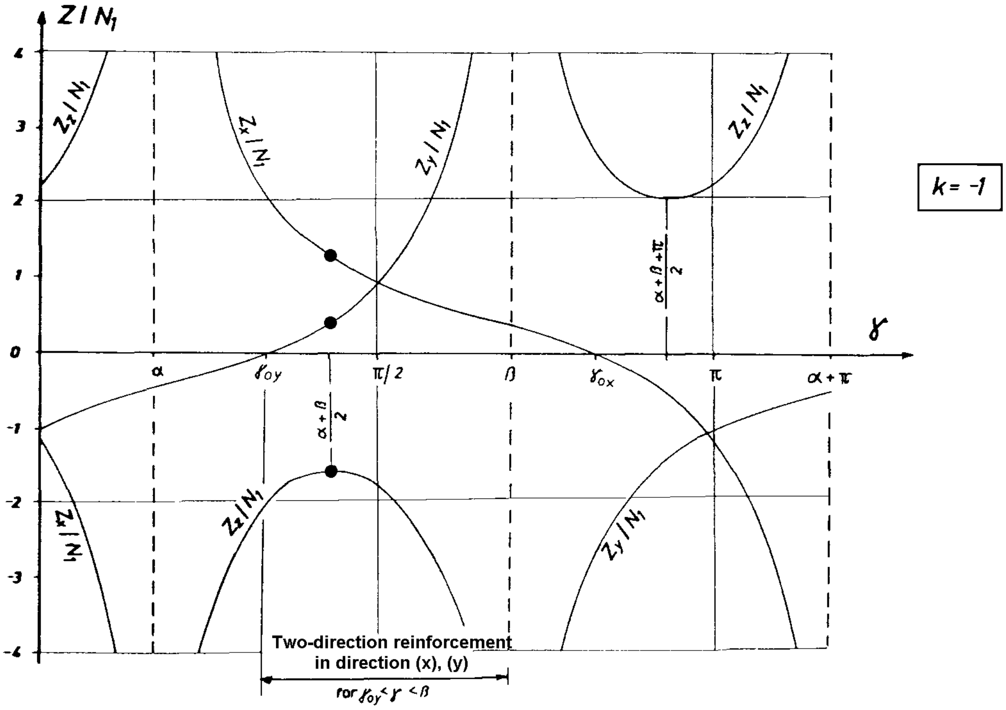


Figure 2.16 Two-directional reinforcement in tension and compression

Rows 5 and 6 of Table IV (Figure 2.15) provide examples for this possible state of equilibrium.

However, for a wall subjected to both tension and compression, a compression strut may expectedly result in the direction γ and another one in the direction β for the selected direction of the concrete compression strut (arithmetic mean between the two reinforcement directions). This is exactly the case when the arithmetic mean in the diagram above is to the left of the zero crossing of the force distribution of Z_y. However, this kind of equilibrium is not possible. The reinforcement of the conjugated direction is determined, that is, the value γ_{0y} is used for the compression strut direction γ.

$$\tan \gamma_{0y} = -k \cdot \cot \alpha$$

Equation 2.12

This means that no force occurs in the second reinforcement direction y under the angle β. Row 7 in Table IV (Figure 2.15) shows an example of this equilibrium of forces. In the add-on module RF-CONCRETE Surfaces, such a state of equilibrium is reached when a compression force in the reinforcement direction y results for the routinely assumed direction of the compression strut (arithmetic mean between the directions of both reinforcement sets).

We have thus described all possible states of equilibrium for two-directional reinforcements.

2.3.4 Possible Load Situations

The load is obtained by applying the principal axial forces n_1 and n_2, with the principal axial force n_1 always being greater than the principal axial force n_2 when taking the algebraic sign into account.

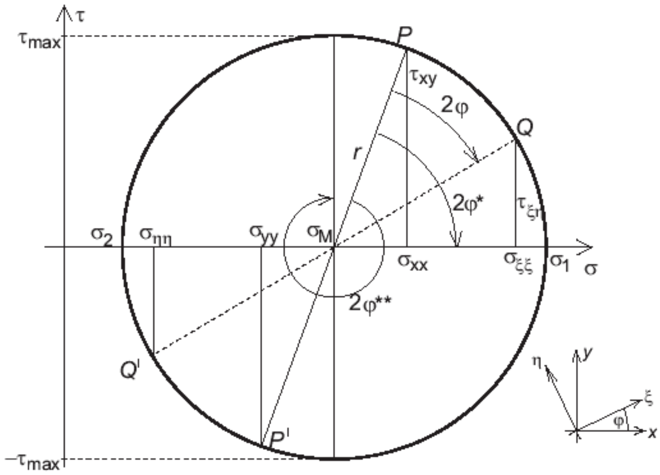


Figure 2.17 Mohr's circle

There are different load situations depending on the algebraic signs of the principal axial forces.

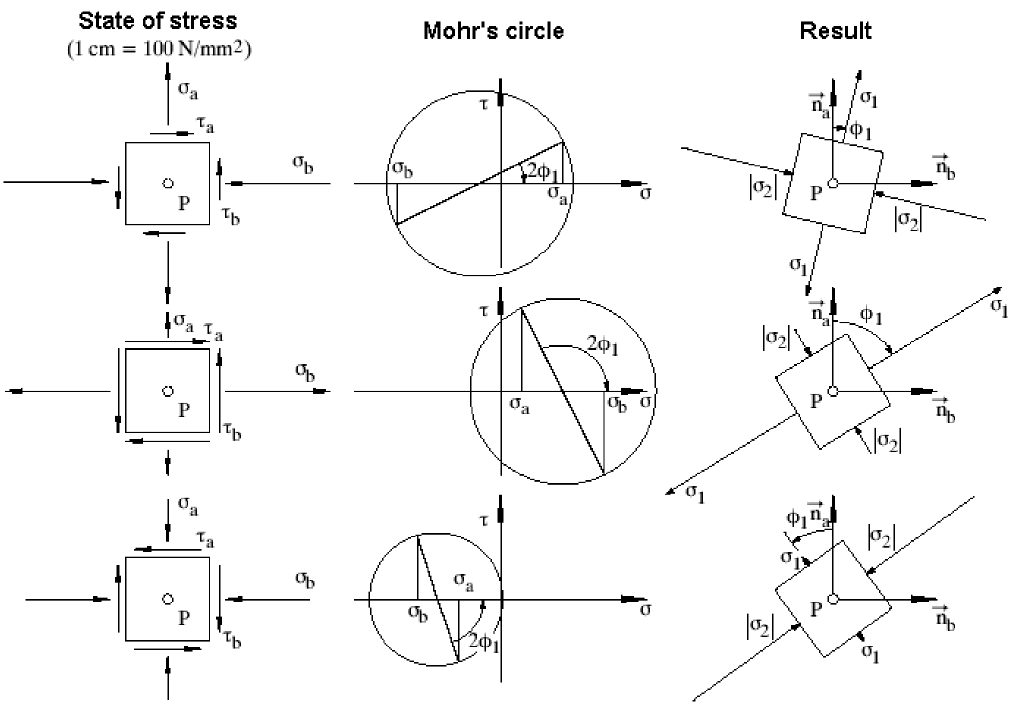


Figure 2.18 Load situations

In a matrix of principal axial forces, you get the following designations of the individual load situations (n_1 is called n_I, n_2 is n_{II}):

n_{II} \ n_I	$n_I > 0$	$n_I = 0$	$n_I < 0$
$n_{II} > 0$	Eliptical tension *(tension - tension)*	*Not possible*	*Not possible*
$n_{II} = 0$	Parabolic tension	*No loading*	*Not possible*
$n_{II} < 0$	Hyperbolic state *(tension - compression)*	Parabolic compression	Eliptical compression *(compression - compression)*

Figure 2.19 Matrix of principal axial forces for load situations

Determining the design axial forces using Equation 2.5 through Equation 2.7 is described in the previous paragraphs for the load situations *Elliptical tension* and *Hyperbolic state*. For the load situation *Parabolic tension*, the design axial forces are obtained in the same way. The value *k* is to be applied with zero in Equation 2.5 through Equation 2.7.

Now we will explain the design axial forces for the following design situations.

Elliptical compression in a mesh with 3 reinforcement directions

Equations 2.5 to 2.7 are applied without changes, even if the two principal axial forces n_1 and n_2 are negative. If a negative design axial force results for each of the three reinforcement directions, none of the three provided reinforcement directions is activated. The concrete is able to transfer the principal axial forces by itself, that is, without the use of a reinforcement mesh in tension stiffened by a concrete compression strut.

The assumption that concrete compression forces in the direction of the provided reinforcement are introduced to resist the principal axial forces is purely hypothetical. It is based on the wish to obtain a distribution of the principal compression forces in the direction of the individual reinforcement directions in order to be able to determine the minimum compression reinforcement that is required, for example, in EN 1992-1-1, clause 9.2.1.1. To this end, a statically required concrete cross-section is needed, which can only be determined by using the previously determined concrete compression forces in the direction of the provided reinforcement.

When determining the minimum compressive reinforcement, other standards manage without a statically required concrete cross-section that results from the principal axial force, transformed into a design axial force. However, for a unified transformation method across different standards, the principal compressive forces are transformed in the defined reinforcement directions for these standards as well. Studies have shown that the design with transformed compressive forces is the safe choice. The concrete compressions that occur in the direction of the individual reinforcement directions are designed.

However, if at least one of the design axial forces is positive after the transformation, the reinforcement mesh is activated for this load situation. Then, as described in chapter 2.3.2 and chapter 2.3.3, an internal equilibrium of forces in the form of two reinforcement directions and one selected concrete compression strut has to be established.

Elliptical compression in a two-directional mesh

Equations 2.5 through 2.7 are used without alterations. If the direction of the two main axial forces is identical with the direction of both reinforcement directions, the design axial forces are equal to the principal axial forces.

If the principal axial forces deviate from the reinforcement directions, the equilibrium between a compression strut in the concrete and the design axial forces in the reinforcement directions is sought again. For the direction of the compression strut, the two intermediate angles between the reinforcement directions are analyzed again. As with elliptical tension, the following applies: The assumption of a compression strut direction is deemed to be correct if a negative design force is actually assigned to the compression strut. If allowable solutions are found for both compression strut directions, the smallest value of all design axial forces determines which solution is chosen.

If the design axial force for a reinforcement direction is a compressive force, the program first checks whether the concrete can resist this design axial force. If this is not the case, a compression reinforcement is determined.

Parabolic compression in a two-directional mesh

In this load situation, the principal axial force n_1 is zero. Since the quotient $k = n_2 / n_1$ cannot be calculated anymore, you cannot use Equations 2.5 through 2.7 in the usual way. The following modifications are necessary.

$$n_{\alpha} = \frac{n_1 \cdot \sin\beta \cdot \sin\gamma + n_2 \cdot \cos\beta \cdot \cos\gamma}{\sin(\beta - \alpha) \cdot \sin(\gamma - \alpha)}$$

$$n_{\beta} = \frac{n_1 \cdot \sin\alpha \cdot \sin\gamma + n_2 \cdot \cos\alpha \cdot \cos\gamma}{\sin(\beta - \alpha) \cdot \sin(\beta - \gamma)}$$

$$n_{\gamma} = \frac{-n_1 \cdot \sin\alpha \cdot \sin\beta + n_2 \cdot \cos\alpha \cdot \cos\beta}{\sin(\beta - \gamma) \cdot \sin(\gamma - \alpha)}$$

Equation 2.13

With these modified equations, the program searches for the design axial forces in the two reinforcement directions and for a design axial force for the concrete in the same way. If a reinforcement direction is identical to the acting principal axial force, its design axial force is the principal axial force. Otherwise, solutions with a compression strut between the two reinforcement directions are found again.

Parabolic compression in a three-directional mesh

The formulas presented above are used according to Equation 2.13.

If the principal axial force runs in a reinforcement direction, solutions for a compression strut direction between the first and the second reinforcement direction or the first and third reinforcement direction are analyzed (as with parabolic tension). Again, the smallest value of all design axial forces decides which solution is chosen.

2.3.5 Design of the Concrete Compression Strut

The concrete compression force in the selected concrete compression strut direction is one of the design forces. It is analyzed whether the concrete is able to resist the compression force. However, the complete concrete compression stress f_{cd} is not applied; instead, the allowable concrete compression stress is reduced to 80%, analogous to the recommendation by Schlaich/Schäfer ([2], page 373).

With the reduced concrete compression stress $f_{cd,08}$, the magnitude of the resistant axial force $n_{strut,d}$ per meter is determined by multiplying it with a width of one meter and the wall thickness.

$$n_{strut,d} = f_{cd,08} \cdot b \cdot d$$

Equation 2.14

This resistant concrete compression force can now be compared to the acting concrete compression force n_{strut}. The analysis of the concrete compression strut is fulfilled, if

$$n_{strut,d} \geq n_{strut}$$

Equation 2.15

The design of the concrete compression strut is carried out in the same way for all standards available in the program, naturally with the respectively valid material properties.

2.3.6 Determining the Required Reinforcement

To determine the dimension of the necessary reinforcement area, the design axial force to be resisted n_φ is divided in the respective reinforcement direction φ by the steel stress at the yield point.

The steel stress at the yield point is defined differently depending on the standard and type of concrete. Furthermore, for the design, the respective partial safety factor for the reinforcing steel has to be considered.

If the reinforcement is under compression instead of tension, the steel stress for the allowable concrete compression at failure is to be determined. It is the same in all standards and equals 2 ‰. Thus, the steel stress can be determined by using the modulus of elasticity as follows:

$$\sigma = E_s \cdot 0.002$$

Equation 2.16

Should the steel stress be greater than the steel stress at the yield point, the steel stress at the yield point is used. Apart from that, a compression reinforcement is only determined if the resistant axial force $n_{strut,d}$ per meter of the concrete is smaller than the acting, compression-inducing design axial force. The compression reinforcement is then designed for the difference of the two axial forces.

2.3.7 Reinforcement Rules

All standards contain regulations for plate structures regarding the size and direction of the reinforcement to be used. For this purpose, the standard classifies the plate structures into certain structural elements. EN 1992-1-1, for example, gives the following types of structural elements:

- plate (slab)
- wall (diaphragm)
- deep beam

The following graphic illustrates the relation between the user-defined *Type of Model*, the model for the design, and the structural element type according to the standard, which is used to determine the size and direction of the minimum or maximum reinforcement.

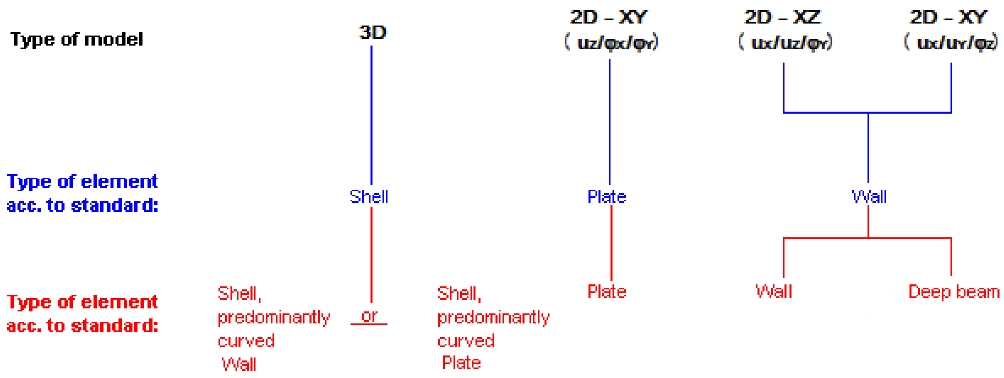


Figure 2.20 Relation between type of model, design model, and structural element type

If *3D* (see Figure 2.1) is selected as the type of model, the structural component is always designed as a shell, independent of whether both axial forces and moments occur in portions of the structural component or if there is only one of these internal forces. A model type defined as 2D - XY ($u_z/\varphi_x/\varphi_y$) is always designed as a plate, while the types 2D - XZ ($u_x/u_y/\varphi_y$) and 2D - XY ($u_x/u_y/\varphi_z$) are designed as walls.

After selecting the structural component type, the regulations of the respective standard are automatically used when determining the required reinforcement. We will now briefly look at these regulations for **EN 1992-1-1**, which distinguishes between *solid plates, walls*, and *deep beams*.

Solid plates

For solid plates, EN 1992-1-1 specifies the following:

- Clause 9.2.1.1 (1):

The minimum area of the longitudinal tension reinforcement must normally correspond to $A_{s,min}$.

$$A_{s,min} = 0.26 \cdot \frac{f_{ctm}}{f_{yk}} \cdot b_t \cdot d \geq 0.0013 \cdot b_t \cdot d$$

Equation 2.17

- Clause 9.2.1.1 (3):

The cross-sectional area of the tension or compression reinforcement may generally not exceed $A_{s,max}$ outside of lap locations. The recommended value is 0.04 A_c.

According to DIN EN 1992-1-1/NA:2010, the sum of the tension and compression reinforcement may not exceed $A_{s,max} = 0.08 \cdot A_c$. This is also true for lap locations.

Walls

For walls, EN 1992-1-1 specifies the following:

- Clause 9.6.2 (1): The area of the vertical reinforcement should normally be between $A_{s,vmin}$ and $A_{s,vmax}$. The recommended values are $A_{s,vmin} = 0.002 \cdot A_c$ and $A_{s,vmax} = 0.04 \cdot A_c$ outside the lap locations.

  DIN EN 1992-1-1/NA:2010 specifies

  - generally: $A_{s,vmin} = 0.15 \mid N_{Ed} \mid \div f_{yd} \geq 0.0015 \cdot A_c$
  - $A_{s,vmax} = 0.04 \cdot A_c$ (this value may be doubled in laps)

  The reinforcement content should be equal at both wall faces.

- Clause 9.6.3 (1): A horizontal reinforcement that runs parallel to the faces of the wall (and to the free edges) should ordinarily be provided at the outer face. Generally, it must not be less than $A_{s,hmin}$. The recommended value is the greater value between 25 % of the vertical reinforcement and $0.001 \cdot A_c$. DIN EN 1992-1-1/NA:2010 specifies

  - generally: $A_{s,hmin} = 0.20 \cdot A_{s,v}$

  The diameter of the horizontal reinforcement must be at least a quarter of the diameter of the perpendicular members.

Deep beam

According to EN 1992-1-1, clause 5.3.1 (3), a beam is considered to be a deep beam if the component's span is less than three times the cross-section depth. In this case, the following applies:

- Clause 9.7 (1): Deep beams should normally be provided with an orthogonal reinforcement mesh with a minimum area of $A_{s,dbmin}$ near each face. The recommended value is $0.001 \cdot A_c$, but not less than 150 mm^2/m per face and direction.

DIN EN 1992-1-1/NA:2010 specifies

- $A_{s,dbmin} = 0.075$ % of $A_c \geq 150\ mm^2/m$

User-defined reinforcement rules across standards

In addition to the normative and therefore unalterable reinforcement specifications, you can specify your own reinforcement rules. These minimum reinforcements can be specified in the *Reinforcement Ratios* tab of window 1.4 *Reinforcement*.

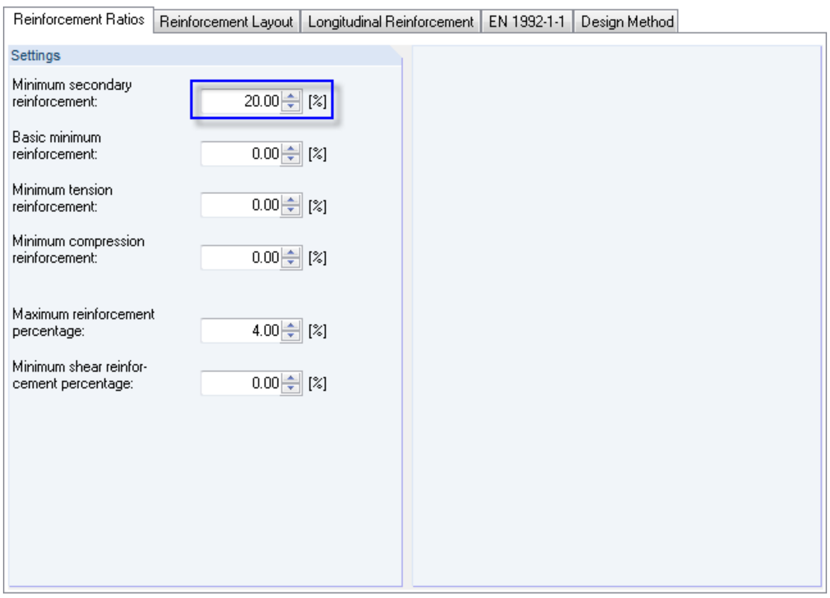
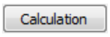
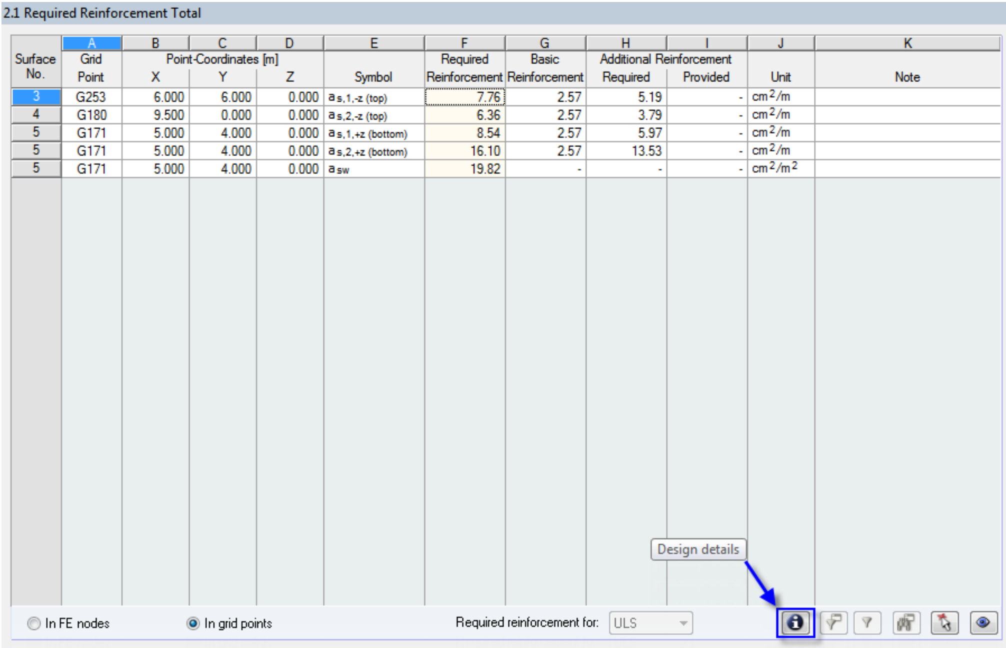


Figure 2.21 Window 1.4 *Reinforcement*, *Reinforcement Ratios* tab



For example, if a minimum secondary reinforcement of 20 % of the largest provided longitudinal reinforcement is specified, the [Calculation] first determines the maximum longitudinal reinforcement. In the result windows, this is shown as the *Required Reinforcement*.



2.1 Required Reinforcement Total

A	B	C	D	E	F	G	H	I	J	K
Surface No.	Grid Point	Point-Coordinates [m] X	Y	Z	Symbol	Required Reinforcement	Basic Reinforcement	Additional Reinforcement Required	Provided	Unit	Note
3	G253	6.000	6.000	0.000	$a_{s,1,-z}$ (top)	7.76	2.57	5.19	-	cm²/m	
4	G180	9.500	0.000	0.000	$a_{s,2,-z}$ (top)	6.36	2.57	3.79	-	cm²/m	
5	G171	5.000	4.000	0.000	$a_{s,1,+z}$ (bottom)	8.54	2.57	5.97	-	cm²/m	
5	G171	5.000	4.000	0.000	$a_{s,2,+z}$ (bottom)	16.10	2.57	13.53	-	cm²/m	
5	G171	5.000	4.000	0.000	a_{sw}	19.82	-	-	-	cm²/m²	

Figure 2.22 Required longitudinal reinforcement and [Design details] button

You can check the minimum secondary reinforcement by clicking the [Design details] button.

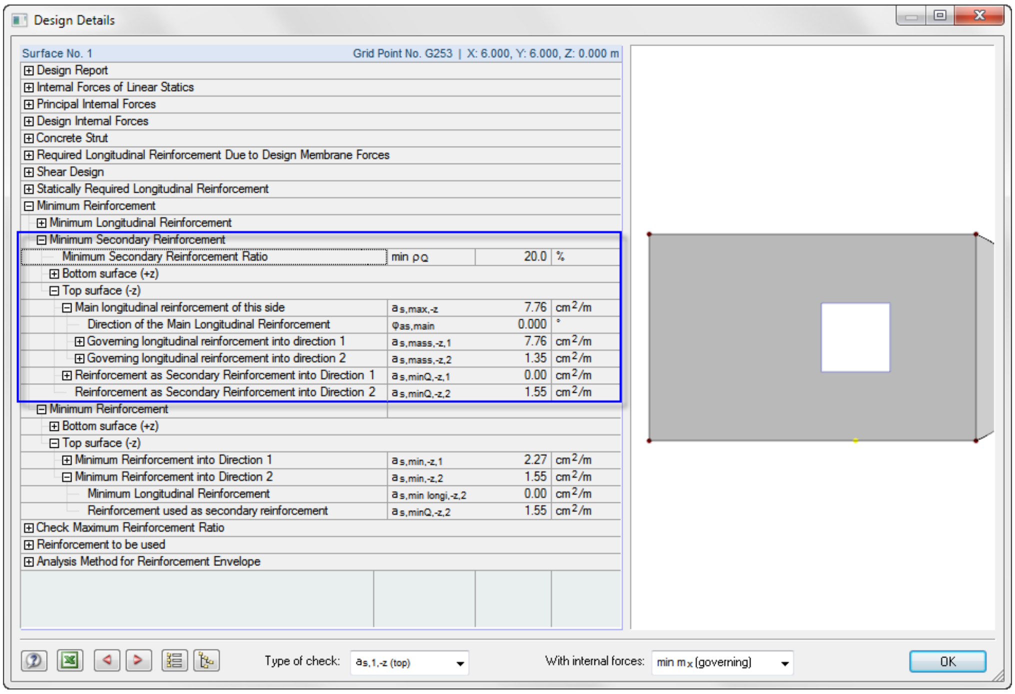


Figure 2.23 *Design Details* dialog box for checking the minimum reinforcement

In the example above, the *Secondary Reinforcement into Direction 2* is 20 % of the reinforcement provided in reinforcement direction 1 (here main direction): $7.76\ \text{cm}^2/\text{m} \cdot 0.2 = 1.55\ \text{cm}^2/\text{m}$. Since this value is greater than the *Governing longitudinal reinforcement into direction 2* of $1.35\ \text{cm}^2/\text{m}$, the secondary reinforcement is decisive.

2.4 Plates

2.4.1 Design Internal Forces

The most important formulas for determining the design axial forces from the principal axial forces are presented in Equation 2.5 through Equation 2.7 in chapter 2.3. According to Baumann [1], these formulas can also be used for moments, because they are nothing more than a force couple with the same absolute value, situated at a certain distance from each other and with a diametrical direction.

Among other things, plates differ from walls in that the actions result in stresses with different algebraic signs on two opposing sides of the plate. It would therefore make sense to provide plates with reinforcement meshes with different directions for both sides of the plates. Since the principal moments m_1 and m_2 are determined in the centroidal plane of the surface, they must be distributed to the plate sides in order to be able to determine the design moments for the reinforcement of the respective plate side.

We look at a plate element with its loading. The surface's local coordinate system is located in the centroidal plane of the plate.

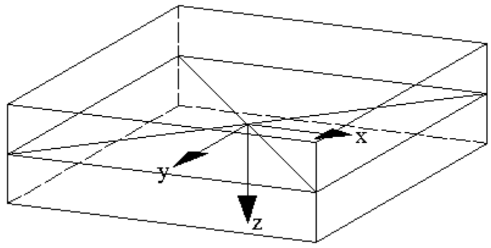


Figure 2.24 Plate element with local surface coordinate system in centroidal plane of the plate

Top and bottom side

In RFEM, the surface's bottom side always lies in the direction of the positive local surface axis z, the top side accordingly in the direction of the negative local z-axis. The surface axes can be switched on in the *Display* navigator by selecting *Model* → *Surfaces* → *Surface Axis Systems x,y,z* or in the shortcut menu of surfaces (see Figure 3.28).

In RFEM, the principal internal forces $\mathbf{m_1}$ and $\mathbf{m_2}$ are determined for the centroidal plane of the plate.

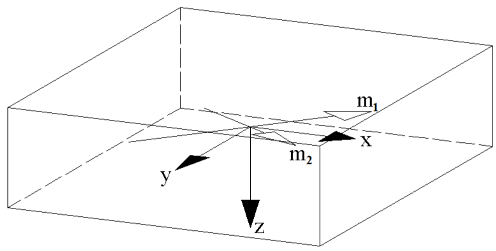


Figure 2.25 Principal moments m_1 and m_2 in centroidal plane of the plate

The principal moments are displayed as simple arrows. They are oriented like the reinforcement that would be required to resist them.

To obtain design moments for the reinforcement mesh at the bottom surface of the plate from these principal moments, the principal moments are shifted to the plate's bottom surface without alteration. For the design, they are described with Roman indexes as $\mathbf{m_I}$ and $\mathbf{m_{II}}$.

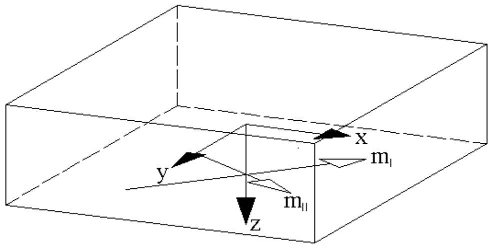


Figure 2.26 Principal moments shifted to the bottom surface of the plate

To obtain the principal moments for determining the design moments for the reinforcement mesh at the top side of the plate, the principal moments are shifted to the plate's top surface. At the same time, their direction is rotated by 180°.

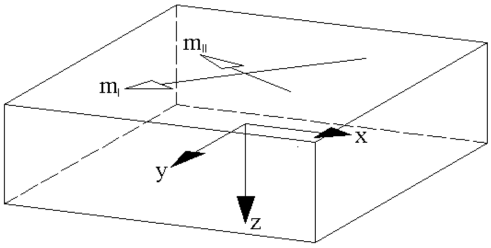


Figure 2.27 Principal moments shifted to the top surface of the plate

Since the principal moment is usually referred to as m_1, which is larger with respect to the algebraic sign (see Figure 2.27), the designations of the principal moments at the top side of the plate have to be reversed.

Thus, the principal moments for determining the design moments at both plate sides are represented as follows:

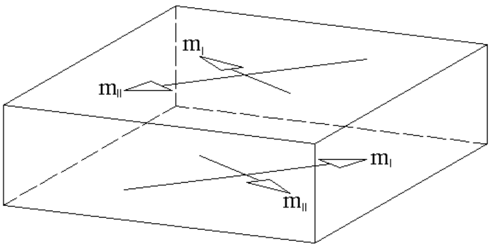


Figure 2.28 Final principal moments at bottom and top side of the plate

If the principal moments for both plate sides are known, the design moments can be determined. To that end, the first step is to determine the differential angle of the reinforcement directions to the direction of the principal moment at each plate side.

The smallest differential angle specifies the positive (clockwise) direction. All other angles are determined in this positive direction and then sorted by size. In RF-CONCRETE Surfaces, they are designated as $\alpha_{m,+z}$, $\beta_{m,+z}$, and $\gamma_{m,+z}$ as shown in the following example. The index +z indicates the bottom surface.

⊞ Design Report			
⊞ Internal Forces of Linear Statics			
⊞ Principal Internal Forces			
⊟ Design Internal Forces			
⊟ Bottom surface (+z)			
⊟ Design Bending Moments			
⊞ Principal Moments			
⊟ Differential Angle Between α,+z,+z and			
⊞ Reinforcement Direction 1	$\Delta\Phi_{+z,1,b}$	0.248	°
⊞ Reinforcement Direction 2	$\Delta\Phi_{+z,2,b}$	90.248	°
⊞ Reinforcement Direction 3	$\Delta\Phi_{+z,3,b}$	45.248	°
⊟ Differential Angle According to Baumann			
1st Differential Angle	$\alpha_{m,+z}$	0.248	°
2nd Differential Angle	$\beta_{m,+z}$	45.248	°
3rd Differential Angle	$\gamma_{m,+z}$	90.248	°

Figure 2.29 Differential angle according to [1] for bottom surface of plate (here for 3 reinforcement directions)

Then, Equations 2.5 through 2.7 are used according to Baumann [1] in order to determine the design moments:

$$m_\alpha = m_I \cdot \frac{\sin\beta \cdot \sin\gamma + k \cdot \cos\beta \cdot \cos\gamma}{\sin(\beta - \alpha) \cdot \sin(\gamma - \alpha)}$$

$$m_\beta = m_I \cdot \frac{\sin\alpha \cdot \sin\gamma + k \cdot \cos\alpha \cdot \cos\gamma}{\sin(\beta - \alpha) \cdot \sin(\beta - \gamma)}$$

$$m_\gamma = m_I \cdot \frac{-\sin\alpha \cdot \sin\beta + k \cdot \cos\alpha \cdot \cos\beta}{\sin(\beta - \gamma) \cdot \sin(\gamma - \alpha)}$$

Equation 2.18

In RF-CONCRETE Surfaces, the design moments $m_{\alpha,+z}$, $m_{\beta,+z}$, and $m_{\gamma,+z}$ for the bottom surface of the plate are output as follows:

⊞ Design Report			
⊞ Internal Forces from Linear Analysis			
⊞ Principal Internal Forces			
⊟ Design Internal Forces			
⊟ Bottom surface (+z)			
⊟ Design Bending Moments			
⊞ Principal Moments			
⊞ Differential Angle Between α,+z,+z and			
⊞ Differential Angle According to Baumann			
⊟ Design Bending Moments According to Baumann			
1st Design Bending Moment	$m_{\alpha,+z}$	35.89	kNm/m
2nd Design Bending Moment	$m_{\beta,+z}$	-0.31	kNm/m
3rd Design Bending Moment	$m_{\gamma,+z}$	0.39	kNm/m

Figure 2.30 Design moments according to [1] for the bottom surface of the plate

In this example, one of the design moments is less than zero. The program now searches for a reinforcement mesh consisting of two reinforcement layers that is stiffened by a concrete compression strut.

The first assumed reinforcement mesh consists of the two reinforcement sets in the directions α_m and β_m. The direction γ of the stiffening concrete compression strut (the stiffening moment that produces compression at this side of the plate) is assumed to be exactly between these two reinforcement directions.

$$\gamma_{1,am} = \frac{\alpha_m + \beta_m}{2}$$

Equation 2.19

With the adapted Equations 2.5 through 2.7, the program once more determines the design moments in the selected reinforcement directions of the mesh and the moment that stiffens them. In the example, the result for the plate's bottom side is the following.

⊞ Internal Forces of Linear Statics			
⊞ Principal Internal Forces			
⊟ Design Internal Forces			
⊟ Bottom surface (+z)			
⊟ Design Bending Moments			
⊞ Principal Moments			
⊞ Differential Angle Between $\alpha_{,+z,+z}$ and			
⊞ Differential Angle According to Baumann			
⊟ Design Bending Moments by Baumann			
1st Design Bending Moment	$m_{\alpha,+z}$	35.89	kNm/m
2nd Design Bending Moment	$m_{\beta,+z}$	-0.31	kNm/m
3rd Design Bending Moment	$m_{\gamma,+z}$	0.39	kNm/m
Does design Bending Moment have different sign	$sign_{m,+z}$	Yes	
Determine strut direction?	$strut_{m,+z}$	Yes	
⊟ First Assumption of the Strut Direction γ			
New Differential Angle	$\gamma_{m,+z,1a}$	22.748	°
⊟ Design Bending Moments by Baumann			
1st Design Bending Moment	$m_{\alpha,+z,1a}$	36.82	kNm/m
2nd Design Bending Moment	$m_{\beta,+z,1a}$	1.02	kNm/m
3rd Design Bending Moment	$m_{\gamma,+z,1a}$	-1.87	kNm/m
Strut direction permissible?	$m_{strut,+z,1a}$	Yes	

Figure 2.31 First assumption for the direction γ of the concrete compression strut

The assumption of the reinforcement mesh results in a viable solution, because the direction of the compression strut is valid.

The analysis of additional compression strut directions must show whether it is the energetic minimum with the least required reinforcement. These analyses are carried out analogously.

Once all sensible possibilities for a reinforcement mesh consisting of two reinforcement directions and a stiffening concrete compression strut are analyzed, the sums of the absolute design moments are shown. For the example above, the overview looks as follows.

⊞ Internal Forces from Linear Analysis			
⊞ Principal Internal Forces			
⊟ Design Internal Forces			
⊟ Bottom Surface (+z)			
⊟ Design Bending Moments			
⊞ Principal Moments			
⊞ Differential Angle Between $\alpha_{,+z,+z}$ and			
⊞ Differential Angle According to Baumann			
⊞ Design Bending Moments According to Baumann			
⊞ First Assumption of the Strut Direction γ			
⊞ Second Assumption of the Strut Direction γ			
⊞ First Assumption of the Strut Direction β			
⊞ Second Assumption of the Strut Direction β			
⊟ Energy = Sum of abs(Design Bending Moments)			
⊟ Smallest Energy for All Valid Cases	$\Sigma_{min,+z}$	36.58	kNm/m
Energy for Differential Angle $\gamma_{m,+z,1b}$	$\Sigma_{\gamma m,+z,1b}$	39.71	kNm/m
Energy for Differential Angle $\beta_{m,+z,2a}$	$\Sigma_{\beta m,+z,2a}$	36.58	kNm/m

Figure 2.32 Sum of the absolute design moments

The *Smallest Energy for all Valid Cases* $\sum_{min,+z}$ is given as the minimum absolute sum of the determined design moments. In the example, the reinforcement mesh from the reinforcement layouts for the differential angle $\beta_{m,+z,2a}$ yields the most favorable solution for the bottom side of the plate.

The design details also show the direction of the governing compression strut. This direction is related to the definition of the differential angles according to Baumann. Hence, the program also gives the direction φ_{strut} in relation to the reinforcement direction. In the example, the following compression strut angle is determined for the plate's bottom side:

⊞ Design Report			
⊞ Internal Forces from Linear Analysis			
⊞ Principal Internal Forces			
⊟ Design Internal Forces			
⊟ Bottom Surface (+z)			
⊟ Design Bending Moments			
⊞ Principal Moments			
⊞ Differential Angle Between $\alpha_{,+z,+z}$ and			
⊞ Differential Angle According to Baumann			
⊞ Design Bending Moments According to Baumann			
⊞ First Assumption of the Strut Direction γ			
⊞ Second Assumption of the Strut Direction γ			
⊞ First Assumption of the Strut Direction β			
⊞ Second Assumption of the Strut Direction β			
⊞ Energy = Sum of abs(Design Bending Moments)			
⊟ Governing Strut			
First Assumption for Direction β	$\beta_{m,+z,2a}$	45.248	°
Strut Direction	$\Phi_{strut,m,+z}$	45.000	°

Figure 2.33 Governing compression strut

For an optimized direction of the design moment that stiffens the reinforcement mesh (see Figure 3.47), the design moments according to Baumann are obtained for the example above. These design moments are applied to the defined reinforcement directions as shown in the following figure.

Surface No. 1	FE Mesh Point No. M15 | X: 2.000, Y: 1.000, Z: 0.000 m		
⊞ Design Report			
⊞ Internal Forces of Linear Statics			
⊞ Principal Internal Forces			
⊟ Design Internal Forces			
⊟ Bottom surface (+z)			
⊟ Design Bending Moments			
⊞ Principal Moments			
⊞ Differential Angle Between $\alpha_{,+z,+z}$ and			
⊞ Differential Angle According to Baumann			
⊞ Design Bending Moments by Baumann			
⊞ First Assumption of the Strut Direction γ			
⊞ Second Assumption of the Strut Direction γ			
⊞ First Assumption of the Strut Direction β			
⊞ Second Assumption of the Strut Direction β			
⊞ Energy = Sum of abs(Design Bending Moments)			
⊞ Governing Strut			
⊟ Governing Design Bending Moments			
into Direction 1	$m_{+z,\Phi 1}$	35.89	kNm/m
into Direction 2	$m_{+z,\Phi 2}$	0.39	kNm/m
into Direction 3	$m_{+z,\Phi 3}$	0.00	kNm/m
into Strut Direction	$m_{end,+z,strut}$	-0.31	kNm/m
Find optimal strut direction?	$Strut_{opti,m,+z}$	No	
The strut found runs in a direction defined as the reinforcement direction.			
The strut force determined will be the design force of this reinforcement direction.			
The force for the design of the concrete strut will be set to zero.			
⊟ Design Bending Moments by Baumann			
into Direction 1	$m_{\alpha,+z}$	35.89	kNm/m
into Direction 2	$m_{\beta,+z}$	-0.31	kNm/m
into Direction 3	$m_{\gamma,+z}$	0.39	kNm/m
⊟ Final Design Bending Moments			
into Direction 1	$m_{end,+z,\Phi 1}$	35.89	kNm/m
into Direction 2	$m_{end,+z,\Phi 2}$	0.39	kNm/m
into Direction 3	$m_{end,\Phi,+z,3}$	-0.31	kNm/m
into Strut Direction	$m_{end,+z,strut}$	-0.31	kNm/m

Figure 2.34 Final design moments for bottom side of plate

2.4.2 Design of the Stiffening Moment

After determining the design moments, the program analyzes the concrete compression strut. It is checked whether the moments used to stiffen the reinforcement mesh can be resisted by the plate.

In the design details, this analysis can be found under the *Concrete Strut* entry:

⊞ Design Report			
⊞ Internal Forces of Linear Statics			
⊞ Principal Internal Forces			
⊞ Design Internal Forces			
⊟ Concrete Strut			
⊞ Thickness of Surface		7.00	cm
⊟ Bottom surface (+z)			
Design Membrane Forces in Strut Direction	$ns_{strut,+z}$	0.000	kN/m
No check necessary: force inside the strut is zero.			
⊟ Top surface (-z)			
⊞ Design Membrane Forces in Strut Direction	$ns_{strut,-z}$	-1.921	kN/m
⊟ Concrete Membrane Force Resistance	$n_{strut,d}$	-746.667	kN/m
Width of Surface	b	1.000	m
Thickness of Surface	h_E	7.00	cm
⊟ Applied Concrete Compressive Strength	$f_{cd,08}$	10.67	N/mm^2
Design Unconfined Concrete Compressive Streng	f_{cd}	13.33	N/mm^2
Coefficient of Maximum Utilization	ξ_{fcd}	0.800	
Failure of concrete strut?	$\lvert n_{strut,d}\rvert < \lvert ns_{strut,-z}\rvert$	No	

Figure 2.35 Design of the stiffening moment

For the determined moments, the program performs a normal bending design at the plate's bottom and top sides. However, the design's aim is not to find a reinforcement: Rather, it is to verify that the concrete compression zone is able to yield a resulting concrete compressive force that, multiplied by the lever arm of the internal forces, results in a moment on the side of the resistance that is greater than the acting moment.

The design is not fulfilled if the moment on the side of the resistance is smaller than the governing design moment ns_{strut} even in the case of a maximum allowable bending compressive strain of the concrete and a maximum allowable retraction of an assumed reinforcement.

The current standards regulate the adherence to the allowable strains via the limit of the ratio between the neutral axis depth x and effective depth d. For this, the stress-strain diagrams for concrete and reinforcing steel as well as the limit strains of these standards are used (see the following explanations for EN 1992-1-1).

Stress-strain diagrams for cross-section design

The parabola-rectangle diagram according to Figure 3.3 of EN 1992-1-1 is used as the calculation value of the stress-strain curve.

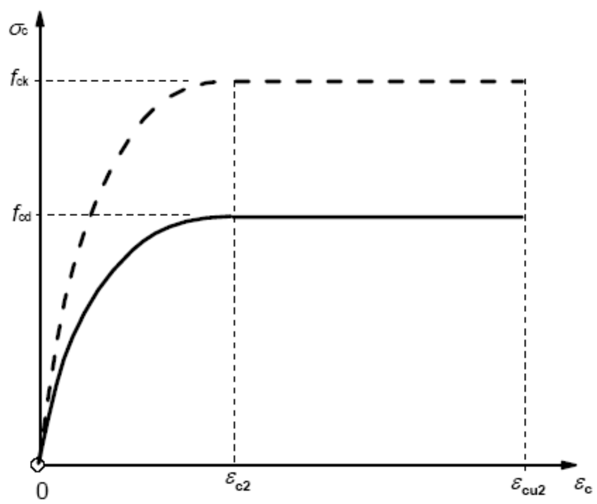


Figure 2.36 Stress-strain diagram for concrete under compression

The stress-strain diagram of the reinforcing steel is shown in Figure 3.8 of EN 1992-1-1.

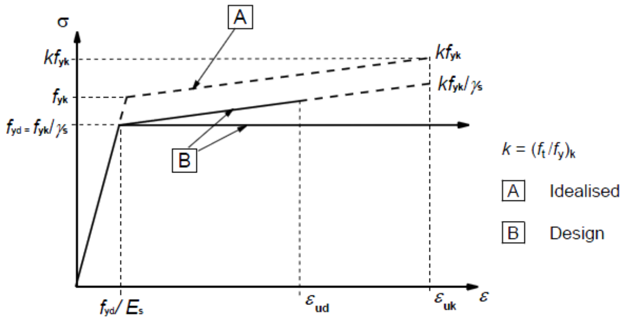


Figure 2.37 Stress-strain diagram for reinforcing steel

The allowable limit deformations are shown in Figure 6.1 of EN 1992-1-1:

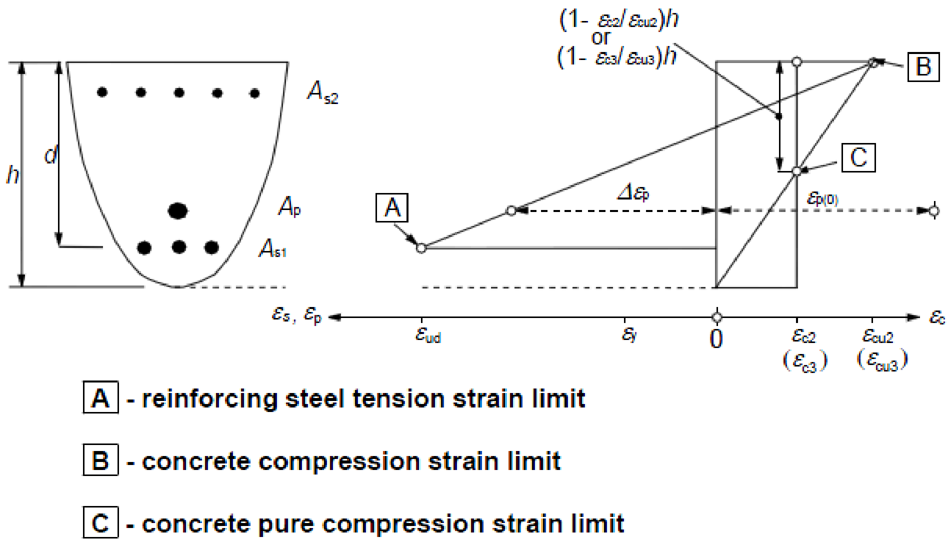


Figure 2.38 Limits of the strain distribution in the ultimate limit state

The ultimate limit state is determined through the limit strains: Either the concrete or the reinforcing steel fails, depending on where the limit strain occurs.

- Failure of concrete, for example C30/37:

  Limit strain for axial compression: ε_{c2} = –2.0 ‰

  Ultimate strain at failure: ε_{c2} = –3.5 ‰
- Failure of reinforcing steel, for example B 500 S (A):

  Steel strain under maximum load: ε_{uk} = 25 ‰
- Simultaneous failure of concrete and reinforcing steel:

  The limit compressive strains of concrete and steel occur simultaneously.

2.4.3 Determining the Statically Required Reinforcement

The stress-strain diagrams for concrete and reinforcing steel described in chapter 2.4.2 together with the allowable range of strain distributions (limit strains) represent the basis for determining the required longitudinal reinforcement for the previously determined design moments. This process is also documented in the design details.

⊞ Design Report			
⊞ Internal Forces of Linear Statics			
⊞ Principal Internal Forces			
⊟ Design Internal Forces			
⊟ Bottom surface (+z)			
⊞ Design Bending Moments			
⊞ Design Axial Forces			
⊞ Design Internal Forces			
⊟ Minimum Lever Arm of the Internal Forces	$z_{min,+z}$	0.150	m
⊞ Due to Design in Reinforcement Direction 1	$z_{+z,\Phi 1}$	0.161	m
⊞ Due to Design in Reinforcement Direction 2	$z_{+z,\Phi 2}$	0.159	m
⊞ Due to Design in Reinforcement Direction 3	$z_{+z,\Phi 3}$	0.150	m
⊞ Membrane Force			
⊞ Design Membrane Forces			
⊞ Top surface (-z)			
⊞ Concrete Strut			
⊟ Required Longitudinal Reinforcement Due to Design Membrane Forces			
⊟ Bottom surface (+z)			
⊟ into Reinforcement Direction 1	$a_{s,dim,+z,1}$	5.29	cm²/m
Design Membrane Force	$ns_{end,+z,\Phi 1}$	240.025	kN/m
⊟ Design Stress	$\sigma_{s,+z,1}$	454.14	N/mm²
Governing Range (see manual)	Range	III	
⊞ Design Internal Forces			
⊟ Strains			
Strain on Top (-z) of Cross-Section	$\varepsilon_{c,-z\,(top),dim,+z,1}$	-3.187	‰
Strain of the Top (-z) Reinforcement	$\varepsilon_{s,-z\,(top),dim,+z,1}$	1.346	‰
Strain of the Bottom (+z) Reinforcement	$\varepsilon_{s,+z\,(bottom),dim,+}$	22.500	‰
Strain on Bottom (+z) of Cross-Section	$\varepsilon_{c,+z\,(bottom),dim,+}$	27.033	‰
⊟ Ratio Depth of Neutral Axis/Effective Depth	$\psi_{dim,+z,1}$	0.124	
Depth of Neutral Axis	$x_{dim,+z,1}$	0.021	m
Effective Depth	$d_{dim,+z,1}$	0.170	m
⊟ Stresses			
Stress on Top (-z) of Cross-Section	$\sigma_{c,-z\,(top),dim,+z,1}$	-13.33	N/mm²
Stress of Top (-z) Longitudinal Reinforcement	$\sigma_{s,-z\,(top),dim,+z,1}$	269.23	N/mm²
Stress of Bottom (+z) Longitudinal Reinforceme	$\sigma_{s,+z\,(bottom),dim,+}$	454.14	N/mm²
Stress on Bottom (+z) of Cross-Section	$\sigma_{c,+z\,(bottom),dim,+}$	0.00	N/mm²
⊞ into Reinforcement Direction 2	$a_{s,dim,+z,2}$	0.06	cm²/m
⊞ into Reinforcement Direction 3	$a_{s,dim,+z,3}$	0.00	cm²/m
⊞ Top surface (-z)			

Figure 2.39 Design details: Required longitudinal reinforcement

The first subentries for the required longitudinal reinforcement are the top and bottom side of the plate. There are main entries for the *Bottom surface (+z)* and *Top surface (-z)* that contain further details for each reinforcement direction.

Figure 2.39 shows that the reinforcement directions 2 and 3 require very little to no reinforcement at the plate's bottom.

Reinforcement Direction 1 is to be designed for the design bending moment $m_{end,+z,\varphi 1}$ = 35.89 kNm/m. The strains provide information about the determination of the longitudinal reinforcement.

The example shown in Figure 2.39 is checked for a dimensionless design procedure by means of a design table. The following input parameters are given:

- Cross-section [cm]: rectangle w/h/d = 100/20/17
- Materials: concrete C20/25 B 500 S (A)
- Design internal forces: $M_{Eds} = ns_{end,+z,\varphi 1} \cdot z_{+z,\varphi 1} = 240.005 \cdot 0.161 = 38.64$ kNm/m
  $N_{Ed} = 0.00$ kNm/m

$$f_{cd} = \frac{\alpha \cdot f_{ck}}{\gamma_c} = \frac{0.85 \cdot 2.0}{1.5} = 1.13 \text{kN/cm}^2$$

$$\mu_{Eds} = \frac{M_{Eds}}{b \cdot d^2 \cdot f_{cd}} = \frac{3864}{100 \cdot 17^2 \cdot 1.13} = 0.1183$$

For μ_{Eds} = 0.1183, the following values can be interpolated from the design tables (e.g. [3] Annex A4):

$$\omega_1 = 0.1170 + \frac{(0.1285 - 0.1170) \cdot (0.1183 - 0.11)}{0.12 - 0.11} = 0.1265$$

$$\sigma_{sd} = 45.24 + \frac{(45.40 - 45.24) \cdot (0.1183 - 0.11)}{0.12 - 0.11} = 45.37\ \text{kN/cm}^2$$

With these values, the required longitudinal reinforcement can be determined:

$$A_{s1} = \frac{\omega_1 \cdot b \cdot d \cdot f_{cd} + N_{Ed}}{\sigma_{sd}} = \frac{0.1265 \cdot 100 \cdot 17 \cdot 1.13 + 0}{45.37} = 5.36\ \text{cm}^2/\text{m}$$

2.4.4 Shear Design

Shear design differs greatly in the individual standards. In the following, it is described for EN 1992-1-1.

The design of the shear force resistance is to be performed only in the ultimate limit state (ULS). The actions and resistances are considered with their design values. The general design requirement is:

$$V_{Ed} \leq V_{Rd}$$

Equation 2.20

where

V_{Ed} : design value of acting shear force (principal shear force determined by the program)

V_{Rd} : design value of shear force resistance

Depending on the failure mechanism, the design value of the shear force resistance is determined by one of the following three values:

$V_{Rd,c}$	design shear resistance of a structural component without shear reinforcement
$V_{Rd,s}$	design shear resistance of a structural component with shear reinforcement; limitation of the resistance by failure of shear reinforcement (failure of tie)
$V_{Rd,max}$	design shear resistance due to the load capacity of the concrete compression strut

If the acting shear force V_{Ed} remains below the value of $V_{Rd,c}$, no calculated shear reinforcement is necessary and the check is verified.

If the acting shear force V_{Ed} is higher than the value of $V_{Rd,c}$, a shear reinforcement must be designed. The shear reinforcement must resist the entire shear force. The load-bearing capacity of the concrete compression strut must additionally be analyzed.

$$V_{Ed} \leq V_{Rd,s}$$
$$V_{Ed} \leq V_{Rd,max}$$

Equation 2.21

2.4.4.1 Shear force resistance without shear reinforcement

$$V_{Rd,c} = \left[C_{Rd,c} \cdot k \cdot (100 \cdot \rho_1 \cdot f_{ck})^{\frac{1}{3}} + k_1 \cdot \sigma_{cp}\right] \cdot b_w \cdot d \qquad (6.2a)$$

Equation 2.22

where

$C_{Rd,c} = 0.18 / \gamma_c$	recommended value; acc. to DIN EN 1992-1-1/NA:2010: $C_{Rd,c} = 0.15 / \gamma_c$
$k = 1 + \sqrt{(200/d)} \leq 2.0$	scaling factor for considering the plate thickness d : mean effective depth in [mm]
$\rho_1 = A_{sl} / (b_w \cdot d) \leq 0.02$	longitudinal reinforcement ratio
	A_{sl} : area of tensile reinforcement that extends beyond the considered cross-section at least by d and is anchored there effectively
	f_{ck} : characteristic value of concrete compressive strength in [N/mm^2]
	b_w : cross-section width
	d : effective depth of bending reinforcement in [mm]
$\sigma_{cp} = N_{Ed} / A_c < 0.2 \cdot f_{cd}$	design value of concrete longitudinal stress in [N/mm^2]
	N_{Ed} : acting axial force in direction of principal shear force

The following minimum value of the shear force resistance $V_{Rd,c}$ may be applied:

$$V_{Rd,c} = (v_{min} + k_1 \cdot \sigma_{cp}) \cdot b_w \cdot d \qquad (6.2b)$$

Equation 2.23

where

$k_1 = 0.15$	recommended value; acc. to DIN EN 1992-1-1/NA:2010: $k_1 = 0.12$
$v_{min} = 0.035 \cdot k^{\frac{3}{2}} \cdot f_{ck}^{\frac{1}{2}}$	recommended value (6.3N)

according to DIN EN 1992-1-1/NA:2010:

$$v_{min} = (0.0525 / \gamma_c) \cdot k^{\frac{3}{2}} \cdot f_{ck}^{\frac{1}{2}}$$ for d ≤ 600 mm (6.3aDE)

$$v_{min} = (0.0375 / \gamma_c) \cdot k^{\frac{3}{2}} \cdot f_{ck}^{\frac{1}{2}}$$ for d > 800 mm (6.3bDE)

for 600 mm < d ≤ 800 mm interpolation possible

These equations are primarily intended for the one-dimensional design case (beam). In it, there is only one provided longitudinal reinforcement from which the ratio of the longitudinal reinforcement is determined. For two-dimensional structural components with up to three reinforcement directions, it is not easy to estimate the magnitude of the longitudinal reinforcement to be applied.

In the *Longitudinal Reinforcement* tab of window 1.4 *Reinforcement*, there are three ways to specify the provided longitudinal reinforcement for the shear design.

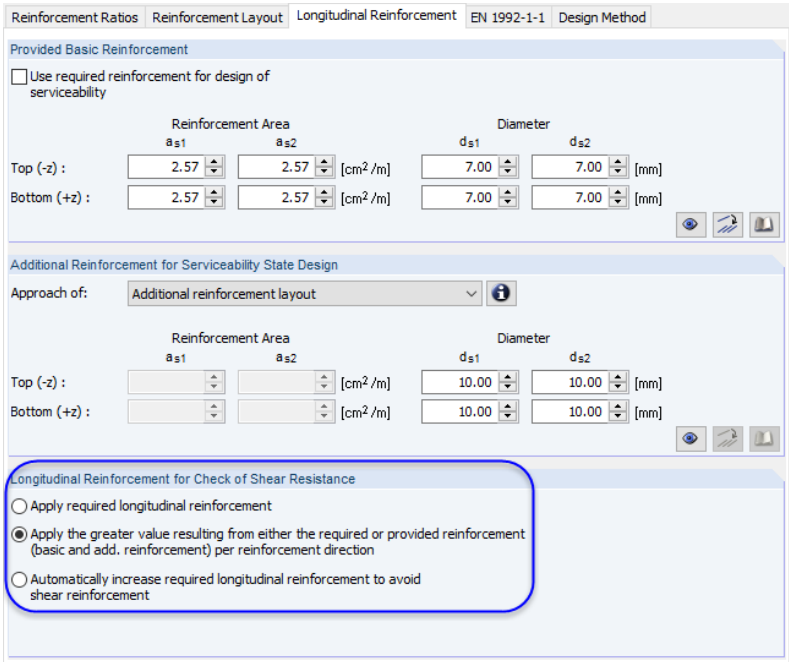


Figure 2.40 Window 1.4 *Reinforcement, Longitudinal Reinforcement* tab

Apply required longitudinal reinforcement

The program first analyzes which of the reinforcement directions at the two plate sides are subjected to tension after the design, including a tension force applied as per clause 6.2.3 (7). According to EN 1992-1-1, the provided longitudinal reinforcement ratio may only be determined from the area of the provided tensile reinforcement.

In order to transform the reinforcement from the different reinforcement directions with tensile forces in the direction β of the maximum shear force, the direction of the maximum shear force is determined as follows.

$$\beta = \arctan \frac{v_y}{v_x}$$

Equation 2.24

With this, the program determines the differential angle $\delta\varphi_i$ between the respective reinforcement direction φ_i and the direction of the maximum shear force.

$$\delta\varphi_i = \beta - \varphi_i$$

Equation 2.25

With the differential angle $\delta\varphi_i$, it is possible to determine the component $a_{sl,i}$ of a specific tensioned longitudinal reinforcement $a_{s,i}$.

$$a_{sl,i} = a_{s,i} \cdot \cos^2(\delta\varphi i)$$

Equation 2.26

In Equation 2.22, the tensile reinforcement a_{sl} to be applied for the determination of $V_{Rd,c}$ is the sum of the components from the individual reinforcement directions to which tension is assigned.

$$a_{sl} = \sum a_{s,i} \cdot \cos^2(\delta\varphi i)$$

Equation 2.27

Apply the greater value resulting from either the required or provided longitudinal reinforcement

The second option shown in Figure 2.40 determines the applied tension reinforcement a_{sl} as described above. The program first checks if a tension force is assigned to the required longitudinal reinforcement. The provided longitudinal reinforcement a_{sl} is then determined according to Equation 2.26 and Equation 2.27.

The design shear resistance $V_{Rd,c}$ is subsequently determined without shear reinforcement. It might turn out that the shear design is possible without shear reinforcement. If the shear force resistance $V_{Rd,ct}$ is negative or insufficient, it is analyzed whether the statically required longitudinal reinforcement $a_{s,dim}$ or the user-defined basic reinforcement $a_{s,def}$ is the greater reinforcement $a_{s,max}$ for a reinforcement direction.

With this larger reinforcement $a_{s,max}$, the provided longitudinal reinforcement a_{sl} is once more determined according to Equation 2.26 and Equation 2.27. Then the shear force resistance $V_{Rd,c}$ is in turn determined without shear reinforcement.

If it is apparent that the shear resistance $V_{Rd,c}$ without shear reinforcement with the respective larger one among statically required and user-defined longitudinal reinforcement is sufficient, the shear design is fulfilled. If, despite this longitudinal reinforcement, the cross-section still cannot be designed because it is fully cracked, a corresponding message appears.

If a shear reinforcement cannot be avoided in spite of the respective greater reinforcement (statically required or user-defined longitudinal reinforcement) being applied, the shear resistance $V_{Rd,c}$ is once more determined with the statically required longitudinal reinforcement. It would make little sense to apply the user-defined longitudinal reinforcement and thus output it later than required, if in doing so a shear reinforcement cannot be avoided anyway.

The shear force design comprises the check of the shear resistance $V_{Rd,max}$ of the concrete compression strut and shear force resistance $V_{Rd,s}$ of the shear reinforcement, as well as the determination of the required shear reinforcement.

Automatically increase required longitudinal reinforcement to avoid shear reinforcement

In the third option shown in Figure 2.40, Equation 2.22 is solved for the longitudinal reinforcement ratio ρ_l for $V_{Rd,c}$. $V_{Rd,c}$ is applied with the acting shear force V_{Ed}.

$$\rho_l = \frac{\left(\frac{V_{Ed} \cdot \gamma_c}{d \cdot b_w \cdot 0.15 \cdot \kappa \cdot \eta_1} + \frac{0.12 \cdot \gamma_c \cdot \sigma_{cd}}{0.15 \cdot \kappa \cdot \eta_1} \right)^3}{100 \cdot f_{ck}}$$

Equation 2.28

Thus, if the longitudinal reinforcement ratio is high enough, a shear reinforcement can be dispensed with.

Again, RF-CONCRETE Surfaces first checks the design shear resistance $V_{Rd,c}$ with the statically required longitudinal reinforcement. If this first design shear resistance is insufficient, the longitudinal reinforcement a_{sl} in the direction of the principal shear force is increased. However, the longitudinal reinforcement a_{sl} cannot be increased indefinitely.

The following flowchart shows when shear reinforcement can be avoided and when shear reinforcement must be used with the statically required longitudinal reinforcement from the design.

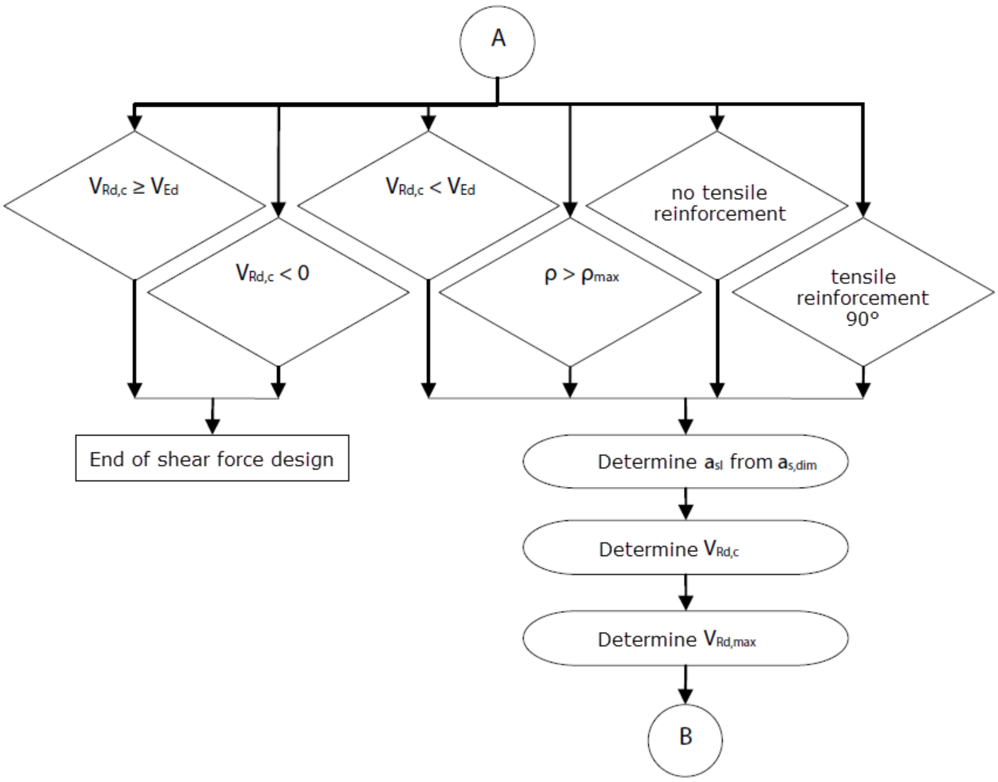


Figure 2.41 Flowchart for increasing the longitudinal reinforcement to avoid shear reinforcement

The two paths on the left ($V_{Rd,c} \geq V_{Ed}$, $V_{Rd,c} < 0$) show the successful prevention of shear reinforcement as well as the possibility that even if the longitudinal reinforcement is increased, the shear force resistance $V_{Rd,c}$ remains negative and therefore no shear design is possible for the fully cracked cross-section.

The other four paths ($V_{Rd,c} < V_{Ed}$, $\rho > \rho_{max}$, no tensile reinforcement, tensile reinforcement 90°) show the reasons why it is not possible to increase the longitudinal reinforcement. For example, despite the maximum longitudinal reinforcement ratio, shear reinforcement is unavoidable or the allowed longitudinal reinforcement ratio in the individual directions of reinforcement is exceeded. When the longitudinal reinforcement a_{sl} that is increased in the direction of the principal shear force is distributed to the individual reinforcement directions, the program checks for **each** of these reinforcement directions if the user-defined longitudinal reinforcement ratio is adhered to. If this is not the case, the longitudinal reinforcement ratio ρ_l is determined by using the *Apply required longitudinal reinforcement* option.

To better understand the two right paths, we must look at how the longitudinal reinforcement that is increased in the direction of the principal shear force is distributed to the individual reinforcement directions. If the determined longitudinal reinforcement ratio ρ_l is smaller than 0.02, the required longitudinal reinforcement a_{sl} per meter is determined as follows.

$$a_{sl} = \rho_1 \cdot d$$

Equation 2.29

This required longitudinal reinforcement is now applied to the reinforcement directions to which tension is applied. To this end, the program once more determines the angle deviation $\delta\varphi_i$ between the direction of the maximum shear force and the reinforcement direction with tension.

$$\delta\varphi_i = \beta - \varphi_i$$

Equation 2.30

The angle deviations $\delta\varphi_i$ are raised to the third power of the cosine and summed up as $\sum(\cos^3)$.

The portion $a_{sl,i}$ of the required longitudinal reinforcement a_{sl} is therefore obtained as per Equation 2.31.

$$a_{sl,i} = a_{sl} \cdot \frac{\cos(\delta\varphi_i)}{\sum \cos^3(\delta\varphi_i)}$$

Equation 2.31

This proportionate required reinforcement $a_{sl,i}$ is compared with the longitudinal reinforcement determined in the design. The greater reinforcement is governing.

In Equation 2.31, we can see that the denominator can become problematic. This is the case if there are no reinforcement directions with tension (the sum of the third power of the angle deviations is only calculated with the tensioned directions) or because even though there are reinforcement directions with tension, they run below 90° to the principal shear force direction and thus their cosine also yields the value zero. These possibilities are represented in the two right paths of the flowchart.

In all cases where no solution is possible, the longitudinal reinforcement is not increased and the *Apply required longitudinal reinforcement* option is used. This includes determining the design shear resistance $V_{Rd,s}$ with shear reinforcement.

2.4.4.2 Shear force resistance with shear reinforcement

The following applies for structural components with shear reinforcement perpendicular to the component's axis (α = 90 °):

$$V_{Rd,s} = \left(\frac{A_{sw}}{s}\right) \cdot z \cdot f_{ywd} \cdot \cot\theta \quad (6.8)$$

Equation 2.32

where

A_{sw}	cross-sectional area of shear reinforcement
s	spacing of links
z	lever arm of internal forces
f_{ywd}	design yield strength of shear reinforcement
θ	inclination of concrete compression strut

The inclination of the concrete compression strut θ may be selected within certain limits depending on the loading. This takes into account the fact that a part of the shear force is resisted by the crack friction and thus does not stress the virtual truss. These limits are specified in EN 1992-1-1, Equation (6.7N).

$$1.00 \leq \cot\theta \leq 2.5 \quad (6.7N)$$

Equation 2.33

Thus, the compression strut inclination θ can be between the following values:

	Minimum inclination	**Maximum inclination**
θ	21.8°	45.0°
cot θ	2.5	1.0

Table 2.1 Limits of the compression strut inclination according to EN 1992-1-1

DIN EN 1992-1-1/NA:2010 specifies the following:

$$1.00 \leq \cot\theta \leq \left(1.2 + 1.4 \cdot \frac{\sigma_{cd}}{f_{cd}}\right) / \left(1 - \frac{V_{Rd,cc}}{V_{Ed}}\right) \leq 3.0 \quad (6.7aDE)$$

Equation 2.34

where

$$V_{Rd,cc} = c \cdot 0.48 \cdot f_{ck}^{\frac{1}{3}} \cdot \left(1 - 1.2 \cdot \frac{\sigma_{cd}}{f_{cd}}\right) \cdot b_w \cdot z \quad (6.7bDE)$$

c = 0.5

$$\sigma_{cd} = \frac{N_{Ed}}{A_c}$$

N_{Ed} : design value of the longitudinal force in the cross-section due to external actions (N_{Ed} > 0 as longitudinal compressive force)

Thus, the compression strut inclination θ can be between the following values:

	Minimum inclination	**Maximum inclination**
θ	18.4°	45.0°
cot θ	3.0	1.0

Table 2.2 Limits of the compression strut inclination according to DIN EN 1992-1-1/NA:2010

A flatter concrete compression strut means smaller tension forces within the shear reinforcement and thus a smaller required reinforcement area. In RF-CONCRETE Surfaces, the inclination of the compression strut is controlled in the *EN 1992-1-1* tab of window 1.4 *Reinforcement*.

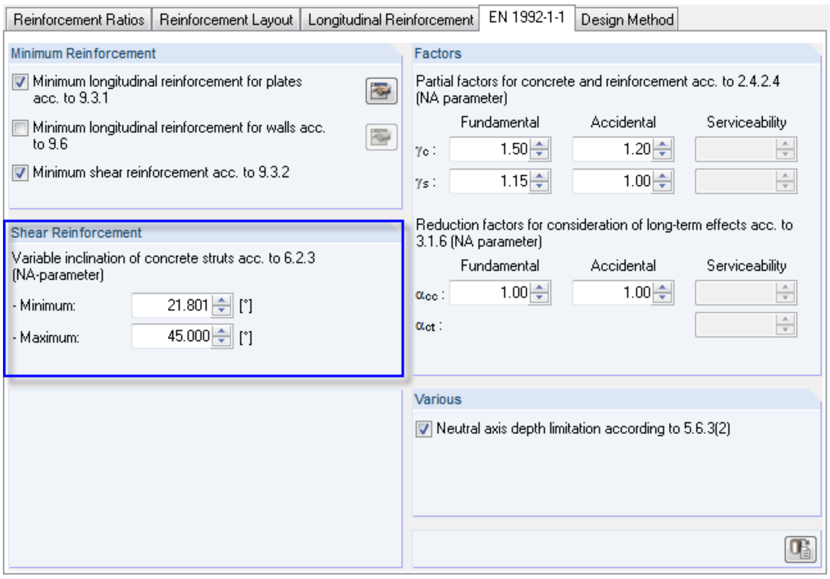


Figure 2.42 Window 1.4 *Reinforcement*, *EN 1992-1-1* tab with limits of the variable compression strut inclination

The size of the minimum compression strut inclination angle θ also depends on the applied internal forces V_{Ed} that can only be taken into account during the calculation. If the minimum compression strut angle is too small, a corresponding message is displayed.

In the calculation, the specified minimum value of the concrete compression strut inclination is first used to determine the shear resistance $V_{Rd,max}$ of the concrete compression strut (see Equation 2.37). If it is smaller than the acting shear force V_{Ed}, a steeper strut inclination must be chosen. Then, the strut inclination θ is increased until the following applies:

$$V_{Ed} \leq V_{Rd,max}$$

Equation 2.35

This compression strut inclination angle leads to the smallest shear reinforcement.

2.4.4.3 Shear force resistance of the concrete compression strut

For structural components with **shear reinforcement perpendicular to the component's axis** (α = 90°), the shear resistance V_{Rd} is the smaller value from:

$$V_{Rd,s} = \left(\frac{A_{sw}}{s}\right) \cdot z \cdot f_{ywd} \cdot \cot\theta \quad (6.8)$$

Equation 2.36

$$V_{Rd,max} = \alpha_{cw} \cdot b_w \cdot z \cdot \nu_1 \cdot \frac{f_{cd}}{(\cot\theta + \tan\theta)} \quad (6.9)$$

Equation 2.37

where

A_{sw}	cross-sectional area of the shear reinforcement
s	spacing of links
f_{ywd}	design yield strength of the shear reinforcement
ν_1	reduction factor for the concrete strength in case of shear cracks
α_{cw}	coefficient for considering the stress state in the compression chord

For structural components with **inclined shear reinforcement**, the shear force resistance is the smaller value from:

$$V_{Rd,s} = \left(\frac{A_{sw}}{s}\right) \cdot z \cdot f_{ywd} \cdot (\cot\theta + \cot\alpha) \cdot \sin\alpha \quad (6.13)$$

Equation 2.38

$$V_{Rd,max} = \alpha_{cw} \cdot b_w \cdot z \cdot \nu_1 \cdot f_{cd} \cdot \frac{(\cot\theta + \cot\alpha)}{(1 + \cot^2\theta)} \quad (6.14)$$

Equation 2.39

2.4.4.4 Example of shear design

The shear design of a plate according to EN 1992-1-1 is presented by means of the design details (see example for statically required reinforcement, Figure 2.39).

In the detailed results, the shear forces determined in RFEM are shown first.

⊞ Design Report			
⊟ Internal Forces of Linear Statics			
⊟ Bending Moment			
x-Axis	m_x	-29.54	kNm/m
y-Axis	m_y	4.14	kNm/m
Differential Moment	m_{xy}	-1.03	kNm/m
⊞ Axial force with axial force vector in direction of			
⊟ Shear Force with Shear Force Vector in Direction of			
x-Axis	v_x	-6.932	kN/m
y-Axis	v_y	-72.968	kN/m
Principal Shear Force	v_{max}	73.297	kN/m
⊟ Principal Internal Forces			
⊞ Principal Moments			
⊞ Principal Axial Forces			
⊟ Principal Shear Force			
Principal Shear Force	v_{max}	73.297	kN/m
Direction	β_m	84.573	°

Figure 2.43 Internal forces of linear statics - shear forces

The required longitudinal reinforcement is determined from these internal forces.

⊞ Design Report			
⊞ Internal Forces of Linear Statics			
⊞ Principal Internal Forces			
⊞ Design Internal Forces			
⊞ Concrete Strut			
⊟ Required Longitudinal Reinforcement Due to Design Membrane Forces			
⊟ Bottom surface (+z)			
⊟ into Reinforcement Direction 1	$a_{s,dim,+z,1}$	0.00	cm²/m
Design Membrane Force	$ns_{end,+z,\Phi 1}$	0.000	kN/m
Design membrane forces equal zero. No longitudinal reinforcement is required.			
⊞ into Reinforcement Direction 2	$a_{s,dim,+z,2}$	0.58	cm²/m
⊞ into Reinforcement Direction 3	$a_{s,dim,+z,3}$	0.00	cm²/m
⊟ Top surface (-z)			
⊞ into Reinforcement Direction 1	$a_{s,dim,-z,1}$	4.03	cm²/m
⊞ into Reinforcement Direction 2	$a_{s,dim,-z,2}$	0.00	cm²/m
⊞ into Reinforcement Direction 3	$a_{s,dim,-z,3}$	0.00	cm²/m

Figure 2.44 Required longitudinal reinforcement

The analysis of the shear resistance is shown further below in the details. It starts with determining the allowed tensile reinforcement in the direction of the principal shear force.

⊞ Design Report			
⊞ Internal Forces of Linear Statics			
⊞ Principal Internal Forces			
⊞ Design Internal Forces			
⊞ Concrete Strut			
⊞ Required Longitudinal Reinforcement Due to Design Membrane Forces			
⊟ Shear Design			
Applied tensile reinforcement determined from the required longitudinal reinforcement.			
The shear reinforcement cannot be avoided in spite of the basic reinforcement.			
⊟ Applied Longitudinal Reinforcement	a_{sl}	0.61	cm²/m
⊟ Bottom surface (+z)			
⊞ from reinforcement direction 1	$a_{sl,+z,1}$	0.00	cm²/m
⊟ from reinforcement direction 2	$a_{sl,+z,2}$	0.58	cm²/m
Required Longitudinal Reinforcement	$a_{s,dim+z,2}$	0.58	cm²/m
State of Stress	$Stress_{+z,2}$	Tension	
Differential Angle in Direction of the Principal She	$\Delta\Phi_{+z,2}$	5.427	°
Second Power of Cosine of Differential Angle to I	$\cos^2(\Delta\Phi_{+z,2})$	0.991	
⊟ from reinforcement direction 3	$a_{sl,+z,3}$	0.00	cm²/m
Required Longitudinal Reinforcement	$a_{s,dim+z,3}$	0.00	cm²/m
State of Stress	$Stress_{+z,3}$	No Stress	
No Transformation (No Tension)			
⊟ Top surface (-z)			
⊟ from reinforcement direction 1	$a_{sl,-z,1}$	0.04	cm²/m
Required Longitudinal Reinforcement	$a_{s,dim-z,1}$	4.03	cm²/m
State of Stress	$Stress_{-z,1}$	Tension	
Differential Angle in Direction of the Principal She	$\Delta\Phi_{-z,1}$	84.573	°
Second Power of Cosine of Differential Angle to I	$\cos^2(\Delta\Phi_{-z,1})$	0.009	
⊞ from reinforcement direction 2	$a_{sl,-z,2}$	0.00	cm²/m
⊞ from reinforcement direction 3	$a_{sl,-z,3}$	0.00	cm²/m

Figure 2.45 Shear Design - Applied tensile reinforcement

The second reinforcement direction at the bottom surface of the plate and the first reinforcement direction at the top surface of the plate are the only reinforcement directions to which tension is applied and which run approximately parallel to the direction of the principal shear force.

These yield the *Applied Longitudinal Reinforcement* a_{sl} of 0.61 cm^2/m.

The shear resistance $V_{Rd,c}$ of the plate without shear reinforcement is determined with the following parameters:

$$C_{Rd,c} = \frac{0.18}{\gamma_c} = \frac{0.18}{0.15} = 0.12$$

$$k = 1 + \sqrt{\left(\frac{200}{d}\right)} = 1 + \sqrt{\left(\frac{200}{160}\right)} = 2.11 \leq 2.00 \rightarrow k = 2.00 \quad d \text{ in [mm]}$$

d = 0.160 m

$$\rho_l = \frac{a_{sl}}{(b_w \cdot d)} = \frac{0.613}{(100 \cdot 16)} = 0.000383 \leq 0.02$$

b_w = 1.00 m

f_{ck} = 20.0 N/mm^2 for concrete C20/25

k_1 = 0.15

σ_{cp} = 0.00 N/mm^2 for concrete C20/25

$$V_{Rd,c} = \left[0.12 \cdot 2.00 \cdot (100 \cdot 0.000383 \cdot 20)^{\frac{1}{3}} + 0.15 \cdot 0.00\right] \cdot 1000 \cdot 160 = 35.135 \text{ kN/m}$$

The same result can be found in the design details:

⊞ Design Report			
⊞ Internal Forces of Linear Statics			
⊞ Principal Internal Forces			
⊞ Design Internal Forces			
⊞ Concrete Strut			
⊞ Required Longitudinal Reinforcement Due to Design Membrane Forces			
⊟ Shear Design			
Applied tensile reinforcement determined from the required longitudinal reinforcement.			
The shear reinforcement cannot be avoided in spite of the basic reinforcement.			
⊞ Applied Longitudinal Reinforcement	a_{sl}	0.61	cm^2/m
⊟ Shear Resistance Without Shear Reinforcement			
⊟ Shear Resistance According to Formula (6.2.a)			
Safety Factor	$C_{Rd,c}$	0.120	
⊟ Factor of Size Effects	k	2.000	
Effective Depth	d	0.160	m
⊟ Longitudinal Reinforcement Ratio	ρ_l	0.000	
Applied Longitudinal Reinforcement	a_{sl}	0.61	cm^2/m
Width of Member	b_w	1.000	m
Effective Depth	d	0.160	m
Characteristic Concrete Compressive Strength	f_{ck}	20.00	N/mm^2
Factor of Longitudinal Stress of Concrete	k_1	0.150	
⊟ Longitudinal Reinforcement Ratio	σ_{cp}	0.00	N/mm^2
Axial Force in Direction of Principal Shear Force	n_β	0.000	kN/m
Width of Structural Member	b_w	1.000	m
Depth of Structural Member	h	20.00	cm
Width of Structural Member	b_w	1.000	m
Effective Depth	d	0.160	m
Shear Resistance According to Formula (6.2.a)	$V_{Rd,c,6.2a}$	35.142	kN/m

Figure 2.46 Shear design - Shear resistance without shear reinforcement

The shear resistance $V_{Rd,c}$ of the plate without shear reinforcement is compared to the acting shear force V_{Ed}.

$V_{Rd,c}$ = 35.142 kN/m ≥ V_{Ed} = 29.56 kN/m

It has therefore been determined that the shear resistance of the plate without shear reinforcement is sufficient and no further checks are necessary.

2.4.5 Reinforcement Rules

For plates, the reinforcement rules presented in chapter 2.3.7 apply.

In RF-CONCRETE Surfaces, user-defined specifications can be set in window 1.4 *Reinforcement*. The following tabs are relevant:

- *Reinforcement Layout* tab (see Figure 3.26)
- *EN 1992-1-1* tab (see Figure 3.44)

If there are different specifications for the minimum shear reinforcement in the two tabs, the more unfavorable specification applies.

The user-defined reinforcement specifications can be found in the design details.

⊞ Design Report			
⊞ Internal Forces of Linear Statics			
⊞ Principal Internal Forces			
⊞ Design Internal Forces			
⊞ Concrete Strut			
⊞ Required Longitudinal Reinforcement Due to Design Membrane Forces			
⊞ Shear Design			
⊞ Statically Required Longitudinal Reinforcement			
⊟ Minimum Reinforcement			
⊟ Minimum Longitudinal Reinforcement			
⊞ Minimum Reinforcement Ratio			
⊞ Bottom surface (+z)			
⊞ Top surface (-z)			
⊟ Minimum Secondary Reinforcement			
Minimum Secondary Reinforcement Ratio	min ρ_Q	20.0	%
⊞ Bottom surface (+z)			
⊞ Top surface (-z)			
⊟ Minimum Reinforcement			
⊞ Bottom surface (+z)			
⊞ Top surface (-z)			
⊟ Check Maximum Reinforcement Ratio			
⊞ Existing Longitudinal Reinforcement Ratio	ρ_l	0.207	
⊞ Maximum longitudinal reinforcement ratio	max ρ_l	4.000	
Maximum longitudinal reinforcement ratio exceeded?	(max ρ_l) < (ρ_l)	No	

Figure 2.47 Minimum reinforcement and maximum reinforcement ratio

⊞ Principal Internal Forces			
⊞ Design Internal Forces			
⊞ Concrete Strut			
⊞ Required Longitudinal Reinforcement Due to Design Membrane Forces			
⊞ Shear Design			
⊞ Statically Required Longitudinal Reinforcement			
⊞ Minimum Reinforcement			
⊞ Check Maximum Reinforcement Ratio			
⊟ Reinforcement to be used			
⊟ Bottom surface (+z)			
⊟ into Reinforcement Direction 1	$a_{s,+z,1}$	5.29	cm²/m
Statically Required Reinforcement	$a_{s,stat,+z,1}$	5.29	cm²/m
⊟ Minimum Reinforcement	$a_{s,min,+z,1}$	2.08	cm²/m
Minimum Longitudinal Reinforcement	$a_{s,min\ longi,+z,1}$	2.08	cm²/m
Reinforcement used secondary reinforcement	$a_{s,minQ,+z,1}$	0.00	cm²/m
⊟ into Reinforcement Direction 2	$a_{s,+z,2}$	1.06	cm²/m
Statically Required Reinforcement	$a_{s,stat,+z,2}$	0.06	cm²/m
⊟ Minimum Reinforcement	$a_{s,min,+z,2}$	1.06	cm²/m
Minimum Longitudinal Reinforcement	$a_{s,min\ longi,+z,2}$	0.00	cm²/m
Reinforcement used as secondary reinforcement	$a_{s,minQ,+z,2}$	1.06	cm²/m
⊟ into Reinforcement Direction 3	$a_{s,+z,3}$	1.06	cm²/m
Statically Required Reinforcement	$a_{s,stat,+z,3}$	0.00	cm²/m
⊟ Minimum Reinforcement	$a_{s,min,+z,3}$	1.06	cm²/m
Minimum Longitudinal Reinforcement	$a_{s,min\ longi,+z,3}$	0.00	cm²/m
Reinforcement used as secondary reinforcement	$a_{s,minQ,+z,3}$	1.06	cm²/m
⊟ Top surface (-z)			
⊞ into Reinforcement Direction 1	$a_{s,-z,1}$	0.00	cm²/m
⊞ into Reinforcement Direction 2	$a_{s,-z,2}$	0.00	cm²/m
⊞ into Reinforcement Direction 3	$a_{s,-z,3}$	0.00	cm²/m

Figure 2.48 Reinforcement to be used

The reinforcement to be used is shown for the *Bottom surface (+z)* and the *Top surface (-z)* in separate entries. The individual reinforcements in each direction indicate whether the reinforcement to be used is the statically required reinforcement or the minimum longitudinal reinforcement.

2.5 Shells

2.5.1 Design Concept

In terms of their internal forces, shells are a combination of walls (chapter 2.3) and plates (chapter 2.4) because they contain both axial forces and moments.

All 3D model types (see Figure 2.1) are designed as shells. RF-CONCRETE Surfaces proceeds as follows: First, as shown in chapter 2.3 and chapter 2.4, the design axial forces and design bending moments are determined separately. They are once again based on the principal axial forces and principal bending moments of the linear RFEM plate analysis.

That way, a design axial force and design moment are determined for each reinforcement direction on each side of the surface. One or both of the internal forces can become zero — if searching for the optimal direction of the concrete compression strut when determining the design internal forces results in the reinforcement not being activated in this direction.

When the design internal forces for the respective reinforcement direction are determined, the focus is on the direction of reinforcement for which design **moments** are available. For it, the program now carries out a common one-dimensional design of a beam with a width of one meter. The goal of this design, however, is not to find a required reinforcement but to determine the lever arm of the internal forces.

As soon as all lever arms of the design directions where a design moment occurs have been determined in this preliminary design, the program determines the smallest lever arm for each plate side. With this eccentricity, the moments of the linear plate analysis can now be transformed into membrane forces. To this end, the moments of the linear plate analysis are simply divided by the smallest lever arm z_{min}.

If you now add half of the axial force from the linear plate analysis that is perpendicular to the moment vector of the moment that is divided by the lever arm of the internal forces, you get the final membrane force. This process can be expressed as follows:

$$n_{xs} = \frac{m_x}{z_{min}} + \frac{n_x}{2}$$

$$n_{ys} = \frac{m_y}{z_{min}} + \frac{n_y}{2}$$

$$n_{xys} = \frac{m_{xy}}{z_{min}} + \frac{n_{xy}}{2}$$

Equation 2.40

The moments at the top and bottom surface of the plate are considered with different algebraic signs.

When the moments m_x, m_y, and m_{xy}, as well as the axial forces n_x, n_y, and n_{xy} of the linear plate analysis have been substituted by the membrane forces n_{xs}, n_{ys}, and n_{xys} by means of the lever arm z_{min} from the preliminary design, the *principal membrane forces* n_{Is} and n_{IIs} can be determined from these membrane forces for the bottom and top surface of the plate.

As described in chapter 2.3, the *design membrane forces* n_α, n_β, and n_γ are determined from the principal membrane forces n_{Is} and n_{IIs} according to Equation 2.5 to Equation 2.7. These design membrane forces n_α, n_β, and n_γ are then assigned to the reinforcement directions φ_1, φ_2, and φ_3. This way, the design membrane forces n_1, n_2, and n_3 are obtained in the reinforcement directions.

The required amount of steel can be determined from the design membrane forces by dividing them by the steel stresses σ_s that have resulted during the determination of the minimum lever arm z_{min} in the respective reinforcement direction.

$$a_{s1} = \frac{n_1}{\sigma_s}$$

$$a_{s2} = \frac{n_2}{\sigma_s}$$

$$a_{s3} = \frac{n_3}{\sigma_s}$$

Equation 2.41

If the design membrane force is a compression force, the concrete's resisting axial force n_c is first determined with the concrete neutral axis depth x, which has resulted from determining the lever arm.

$$n_c = f_{cd} \cdot b \cdot x$$

Equation 2.42

If the resisting axial force n_c of the concrete is not sufficient, a compression reinforcement is determined for the differential force between the acting axial force and the resisting axial force. The design stress for this compression reinforcement results from the deformation of the compression reinforcement during the determination of the lever arm z.

If the lever arm was determined under the assumption of the strain range III, no compression reinforcement will be determined because it was not assumed. The strain ranges I through V are described in the following chapter, in the part regarding the determination of the lever arm.

2.5.2 Lever Arm of Internal Forces

A rectangular cross-section with a width of one meter is always designed. The design is carried out directly with the rectangular stress distribution (see EN 1992-1-1, Figure 3.5). An iterative procedure would take too much time because of the high number of necessary designs.

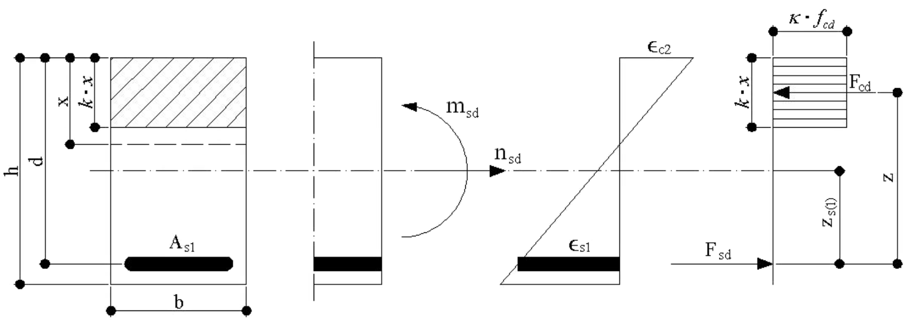


Figure 2.49 Calculation parameters of the design

The desired lever arm z is determined for the figure above as follows.

$$z = d - \frac{k \cdot x}{2}$$

Equation 2.43

Figure 2.49 shows a state of strain that may arise when the moment and axial force act simultaneously. Five states of strain are possible (see Figure 2.50).

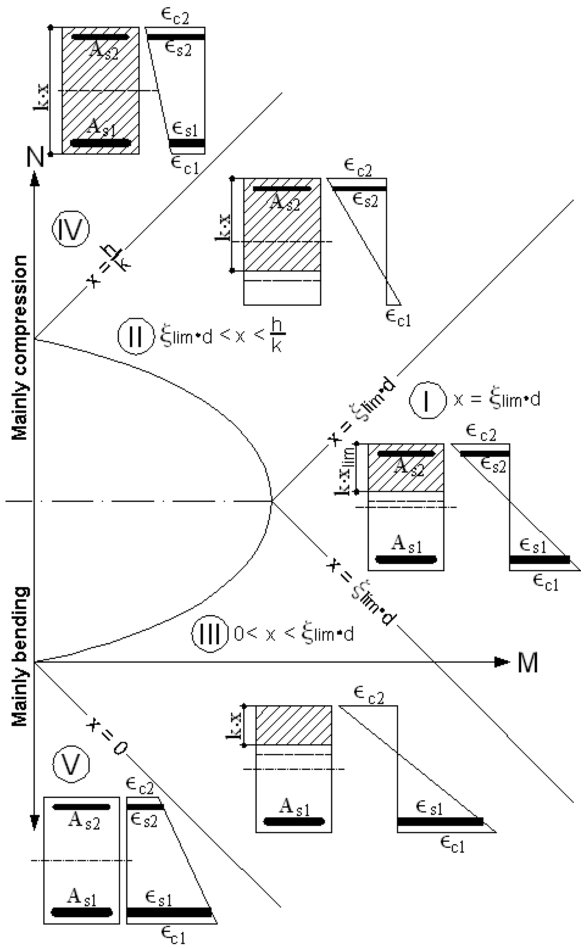


Figure 2.50 Ranges of strain distribution

Range I

This range shows a cross-section strongly subjected to bending. The depth of the neutral axis has reached its maximum value ($x = \xi_{lim} \cdot d$). Another increase of the section modulus is only possible by using a compression reinforcement.

Range II

In this range, compression predominantly occurs. The depth of the neutral axis ranges between the limits $\xi_{lim} \cdot d$ and h/k.

Range III

The applied moment is so small that the concrete compression zone (neutral axis) without compression reinforcement is able to provide a sufficient section modulus. The limits for the neutral axis depth are between 0 and $\xi_{lim} \cdot d$, depending on the applied moment.

Range IV

This range shows a fully compressed cross-section. The depth of the neutral axis is greater than h/k. This range also includes cross-sections that are only subjected to compression forces.

Range V

This state of strain is present if the tension force cracks a cross-section completely. This range also includes cross-sections that are only subjected to tension forces.

The lever arm is determined for each strain range. This makes it possible to divide the moments of the linear plate analysis into membrane forces.

Lever arm for range I

For this range, the depth of the neutral axis is known: The concrete is fully utilized before a compression reinforcement is applied.

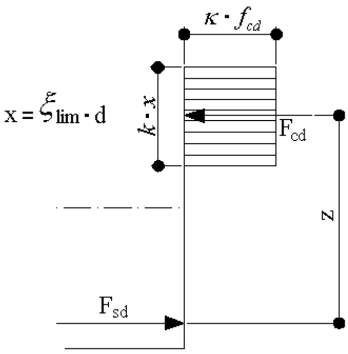


Figure 2.51 Lever arm z for maximum depth of neutral axis of concrete

For the maximum depth of the neutral axis of concrete x, the resisting concrete compressive force F_{cd} is obtained according to the following equation:

$$F_{cd} = \kappa \cdot f_{cd} \cdot k \cdot x_{\lim} \cdot b$$

Equation 2.44

The limit section modulus $m_{sd,lim}$, which can be resisted by the cross-section without compression reinforcement, is determined as follows:

$$m_{sd,\lim} = F_{cd} \cdot \left(d - \frac{k \cdot x_{\lim}}{2} \right)$$

Equation 2.45

With the limit section modulus $m_{sd,lim}$, it is possible to determine the differential moment Δm_{sd} that has to come from the compression reinforcement in order to reach an equilibrium with the applied moment $m_{sd(1)}$.

$$\Delta m_{sd} = m_{sd(1)} \cdot m_{sd,\lim}$$

Equation 2.46

The applied moment $m_{sd(1)}$ relates to the centroid of the tension reinforcement. It results from the applied moment m_{sd}, the acting axial force n_{sd}, and the distance $z_{s(1)}$ between the centroidal axis of the cross-section and the centroidal axis of the tension reinforcement.

$$m_{sd(1)} = m_{sd} - n_{sd} \cdot z_{s(1)}$$

Equation 2.47

With the differential moment Δm_{sd}, you can now determine the required compression force $F_{sd(2)}$ in a compression reinforcement.

$$F_{sd(2)} = \frac{\Delta m_{sd}}{d - d_2}$$

Equation 2.48

Here, d is the effective depth of the tension reinforcement and d_2 is the centroidal distance of the compression reinforcement from the edge of the concrete compression zone.

If you divide the applied moment $m_{sd(1)}$, which is related to the centroid of the tension reinforcement, by the concrete compression force F_{cd} and the force in the compression reinforcement $F_{sd(2)}$, the desired lever arm z is obtained.

$$z = \frac{m_{sd}}{\left| F_{cd} + F_{sd(2)} \right|}$$

Equation 2.49

Lever arm for range II

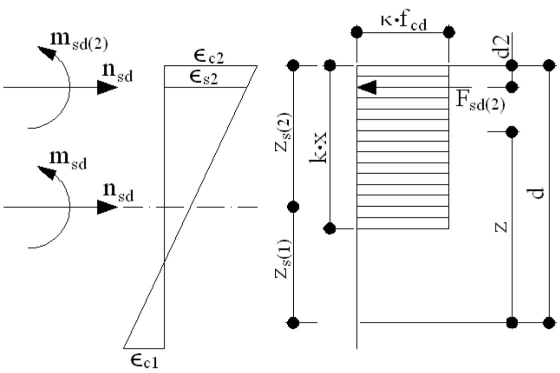


Figure 2.52 Determining the lever arm for range II

In order to be able to determine the concrete's neutral axis depth x, we first determine the design moment $m_{sd(2)}$ about the centroid of the compression reinforcement.

$$m_{sd(2)} = m_{sd} + n_{sd} + z_{s(2)}$$

Equation 2.50

The sum of the moments about the centroid of the compression reinforcement is now calculated. These moments must amount to zero. On the side of the resistance, the moment is created only from the resulting force F_{cd} of the concrete compression zones multiplied by its distance. In range II, there is no reinforcement in tension.

$$\sum m = F_{cd} \cdot \left(\frac{k \cdot x}{2} - d_2 \right) + m_{sd(2)} = 0$$

Equation 2.51

The depth x of the concrete neutral axis is also contained in the resulting concrete compression force F_{cd}.

$$F_{cd} = \kappa \cdot f_{cd} \cdot k \cdot x \cdot b$$

Equation 2.52

Thus, the equation for the determination of x is obtained as:

$$\kappa \cdot f_{cd} \cdot k \cdot x \cdot b \cdot \left(\frac{k \cdot x}{2} - d_2\right) + m_{sd(2)} = \frac{\kappa \cdot f_{cd} \cdot k^2 \cdot x^2}{2} - \kappa \cdot f_{cd} \cdot k \cdot x \cdot b \cdot d_2 + m_{sd(2)} = 0$$

$$x^2 = \frac{2 \cdot d_2 \cdot x}{k} + \frac{2 \cdot m_{sd(2)}}{\kappa \cdot f_{cd} \cdot b \cdot k^2} = 0 \quad \Rightarrow x = \frac{d_2}{k} + \sqrt{\left(\frac{d_2}{k}\right)^2 - \frac{2 \cdot m_{sd(2)}}{\kappa \cdot f_{cd} \cdot b \cdot k^2}}$$

Equation 2.53

With the depth x of the concrete's neutral axis, the lever arm z can be determined by subtracting half of the neutral axis depth x, which is reduced by the factor k, from the effective height d:

$$z = d - \frac{k \cdot x}{2}$$

Equation 2.54

Lever arm for range III

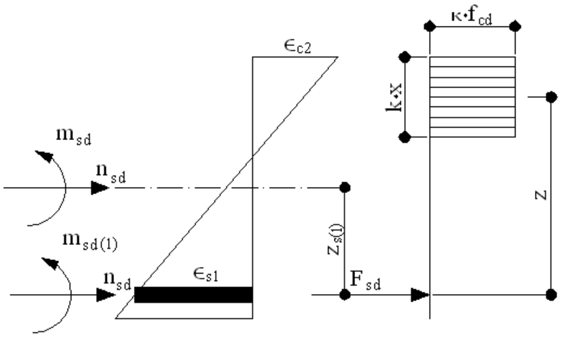


Figure 2.53 Determining the lever arm for range III

To determine the depth x of the neutral axis, we first determine the design moment $m_{sd(1)}$ about the centroid of the tension reinforcement.

$$m_{sd(1)} = m_{sd} + n_{sd} + z_{s(1)}$$

Equation 2.55

The sum of the moments about the tension reinforcement's centroid is now calculated. These moments must amount to zero. On the side of the resistance, the moment is calculated only from the resulting force F_{cd} of the concrete compression zone times its distance. Then the equilibrium of the moments about the position of the tension reinforcement is calculated.

$$\sum m = F_{cd} \cdot \left(d - \frac{k \cdot x}{2}\right) - m_{sd(1)} = 0$$

Equation 2.56

The depth x of the concrete's neutral axis is also contained in the resulting concrete compression force F_{cd} (see Equation 2.52).

$$\kappa \cdot f_{cd} \cdot k \cdot b \cdot d \cdot x - \left(\frac{\kappa \cdot f_{cd} \cdot k^2 \cdot b}{2}\right) - m_{sd(1)} = x^2 - \frac{2d}{k} \cdot x + \frac{2m_{sd(1)}}{\kappa \cdot f_{cd} \cdot k^2 \cdot b} = 0$$

Equation 2.57

This quadratic equation can be solved as follows.

$$x = \frac{d}{k} + \sqrt{\frac{d^2}{k^2} - \frac{2 \cdot m_{sd(1)}}{\kappa \cdot f_{cd} \cdot k^2 \cdot b}} = 0$$

Equation 2.58

With the depth x of the concrete's neutral axis, the lever arm z can be determined by subtracting half of the neutral axis depth x, which is reduced by the factor k, from the effective height d:

$$z = d - \frac{k \cdot x}{2}$$

Equation 2.59

If the steel strain ε_s is greater than the maximum allowable steel strain ε_{ud}, x is calculated iteratively from the equilibrium conditions. The conversion factors κ and k for the concrete neutral axis are directly derived from the concrete's parabola-rectangle diagram.

Lever arm for range IV

In a fully compressed cross-section, the lever arm is assumed as the distance between both reinforcements.

$$z = d - d_2$$

Equation 2.60

For this range, a maximum utilization of the reinforcement is specified, meaning that $\varepsilon_s = \varepsilon_{cu}$.

When the compression is approximately concentric (e_d / h $\leq$ 0.1), the mean compressive strain should be limited to ε_{c2} according to EN 1992-1-1, clause 6.1 (5).

Lever arm for range V

In a fully cracked cross-section, the lever arm is also assumed as the distance between the two reinforcements (see Equation 2.60).

2.5.3 Determining the Design Membrane Forces

The design membrane forces for the abutment of a bridge are determined. For a closer analysis, the grid point No. 1 in surface No. 37 is selected.

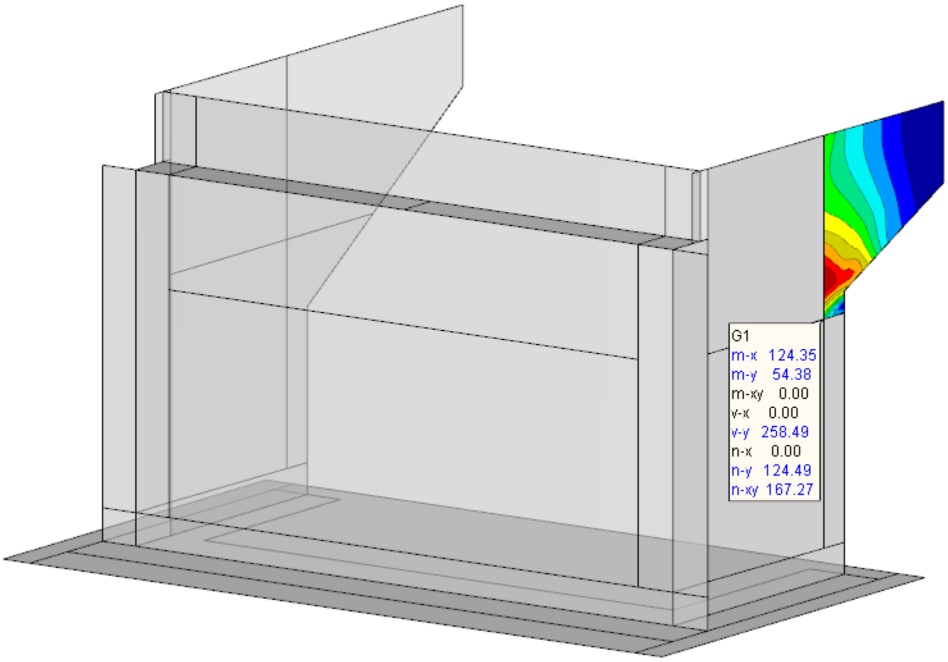


Figure 2.54 Bridge abutment - internal forces in grid point G1

The analyzed surface No. 37 has a thickness of 129 cm.

For the design according to EN 1992-1-1, concrete **C30/37** and reinforcing steel **BSt 500 S (B)** are selected in RF-CONCRETE Surfaces.

Further specifications in window 1.4 *Reinforcement* are:

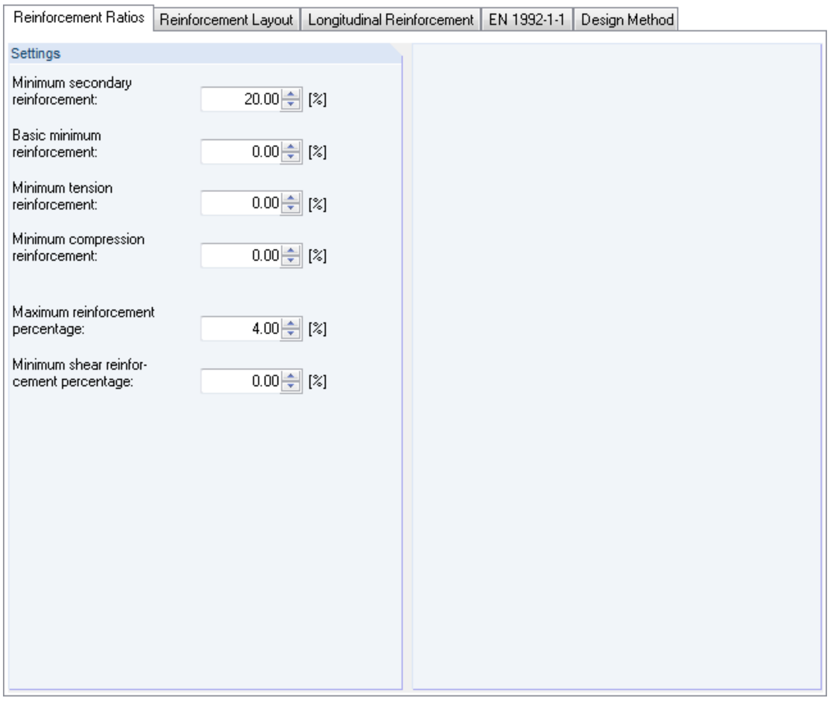


Figure 2.55 Window 1.4 *Reinforcement*, *Reinforcement Ratios* tab

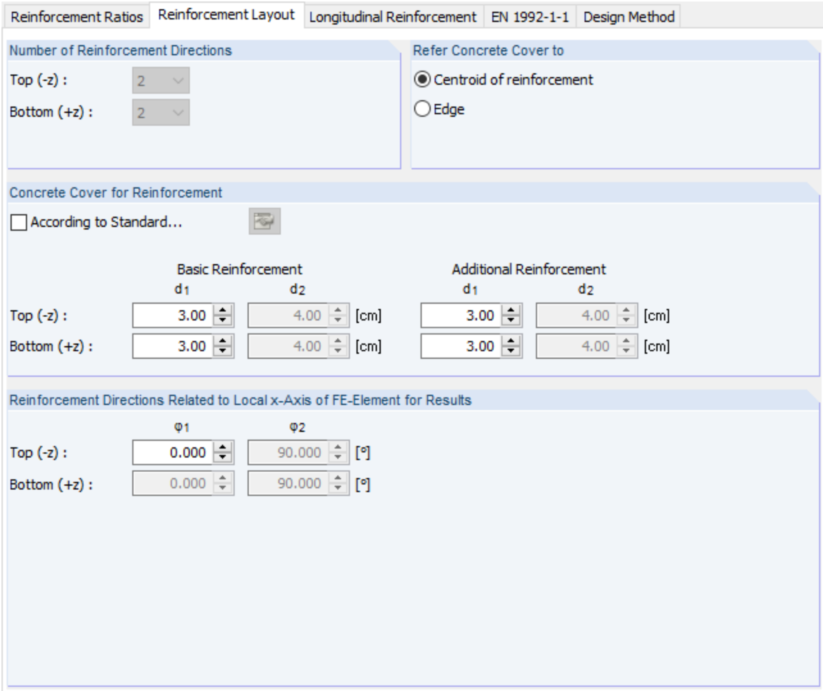


Figure 2.56 Window 1.4 *Reinforcement, Reinforcement Layout* tab

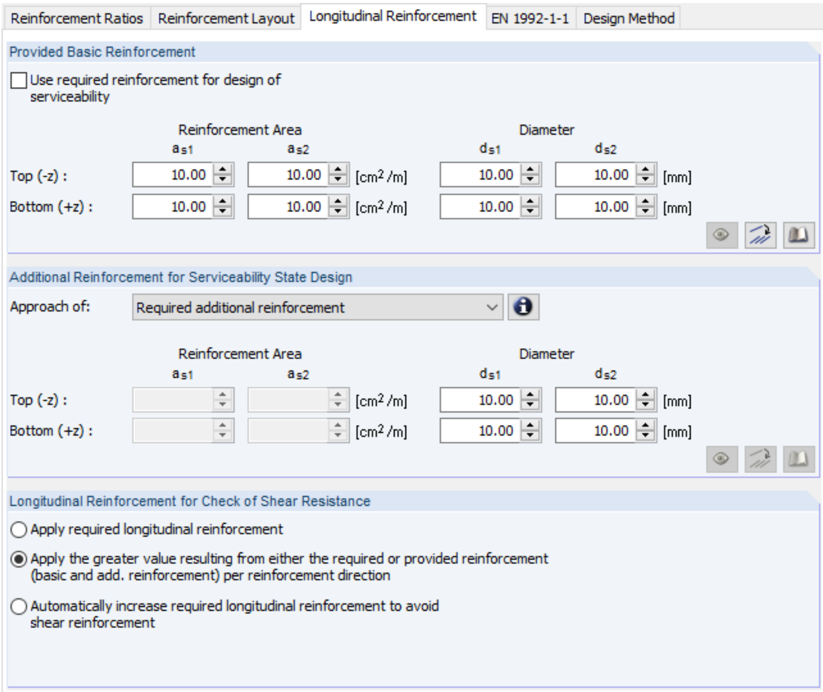


Figure 2.57 Window 1.4 *Reinforcement, Longitudinal Reinforcement* tab

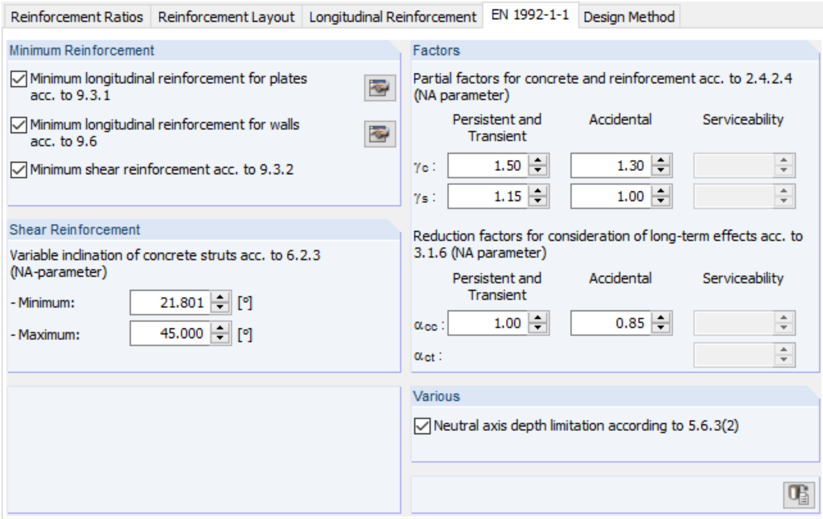


Figure 2.58 Window 1.4 *Reinforcement, EN 1992-1-1* tab

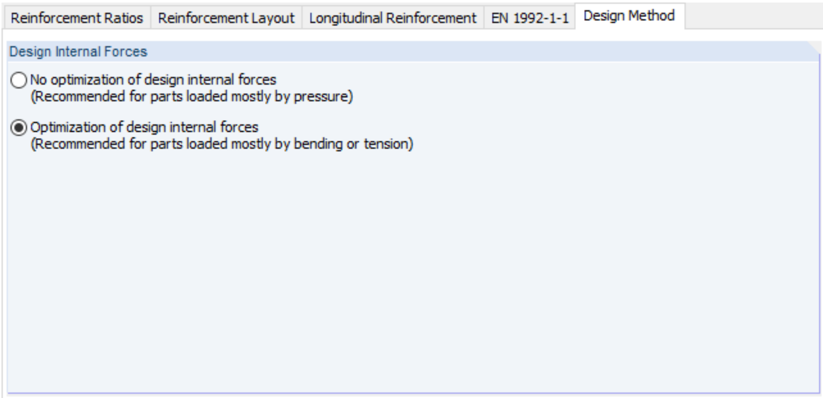
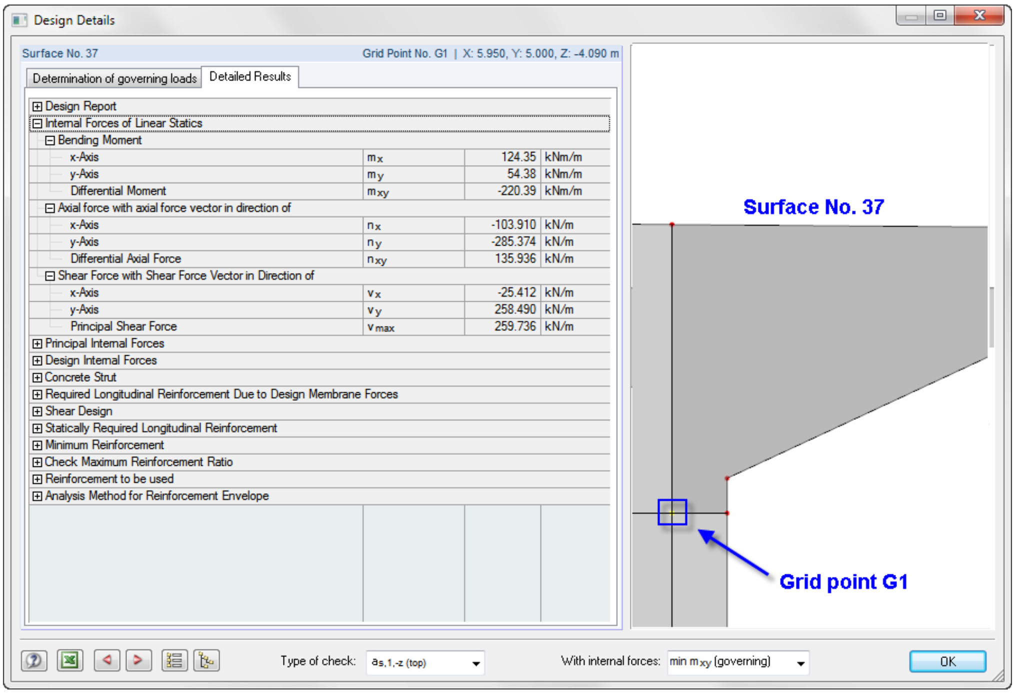


Figure 2.59 Window 1.4 *Reinforcement, Design Method* tab

2.5.3.1 Design moments

The internal forces interpolated from the FE nodes can be found in the design details of the grid point. As the model type *3D* was specified in the general data (see Figure 2.1), the moments m_x, m_y, and m_{xy}, as well as the axial forces n_x, n_y, and n_{xy} exist in the surface.



⊞ Design Report			
⊟ Internal Forces of Linear Statics			
⊟ Bending Moment			
x-Axis	m_x	124.35	kNm/m
y-Axis	m_y	54.38	kNm/m
Differential Moment	m_{xy}	-220.39	kNm/m
⊟ Axial force with axial force vector in direction of			
x-Axis	n_x	-103.910	kN/m
y-Axis	n_y	-285.374	kN/m
Differential Axial Force	n_{xy}	135.936	kN/m
⊟ Shear Force with Shear Force Vector in Direction of			
x-Axis	v_x	-25.412	kN/m
y-Axis	v_y	258.490	kN/m
Principal Shear Force	v_{max}	259.736	kN/m
⊞ Principal Internal Forces			
⊞ Design Internal Forces			
⊞ Concrete Strut			
⊞ Required Longitudinal Reinforcement Due to Design Membrane Forces			
⊞ Shear Design			
⊞ Statically Required Longitudinal Reinforcement			
⊞ Minimum Reinforcement			
⊞ Check Maximum Reinforcement Ratio			
⊞ Reinforcement to be used			
⊞ Analysis Method for Reinforcement Envelope			

Figure 2.60 Internal forces of linear statics

The principal internal forces are determined from the RFEM internal forces of the linear analysis. They are determined according to the equations described in chapter 2.3 and chapter 2.4.

⊞ Design Report			
⊞ Internal Forces of Linear Statics			
⊟ Principal Internal Forces			
⊟ Principal Moments			
⊟ Bottom surface (+z)			
First Principal Moment	$m_{I,+z}$	312.51	kNm/m
Second Principal Moment	$m_{II,+z}$	-133.78	kNm/m
Direction	$\alpha_{m,+z}$	-40.490	°
⊟ Top surface (-z)			
First Principal Moment	$m_{I,-z}$	133.78	kNm/m
Second Principal Moment	$m_{II,-z}$	-312.51	kNm/m
Direction	$\alpha_{m,-z}$	49.510	°
⊟ Principal Axial Forces			
⊟ Bottom surface (+z)			
First Principal Axial Force	$n_{I,+z}$	-31.207	kN/m
Second Principal Axial Force	$n_{II,+z}$	-358.076	kN/m
Direction	$\alpha_{n,+z}$	28.139	°
⊟ Top surface (-z)			
First Principal Axial Force	$n_{I,-z}$	-31.207	kN/m
Second Principal Axial Force	$n_{II,-z}$	-358.076	kN/m
Direction	$\alpha_{n,-z}$	28.139	°
⊟ Principal Shear Force			
Principal Shear Force	v_{max}	259.736	kN/m
Direction	β_m	95.615	°

Figure 2.61 Principal internal forces

For shells, the principal axial forces are shown for both plate sides because they are required for the design as a shell. Unlike the moments, the principal axial forces at the bottom and top surface of the plate are the same.

The design moments are now determined from the principal moments $m_{I,+z}$ and $m_{II,+z}$ at the bottom side of the surface. For this purpose, the program first determines the differential angles $\alpha_{m,+z}$ and $\beta_{m,+z}$ between the direction $\gamma_{m,+z}$ of the first principal axial force $m_{I,+z}$ at the surface's bottom side and the two reinforcement directions φ_1 = 0° and φ_2 = 90°.

⊞ Design Report			
⊞ Internal Forces of Linear Statics			
⊞ Principal Internal Forces			
⊟ Design Internal Forces			
⊟ Bottom surface (+z)			
⊟ Design Bending Moments			
⊞ Principal Moments			
⊟ Differential Angle Between $\alpha_{,+z,+z}$ and			
⊞ Reinforcement Direction 1	$\Delta\Phi_{+z,1,b}$	40.490	°
⊞ Reinforcement Direction 2	$\Delta\Phi_{+z,2,b}$	130.490	°
⊟ Differential Angle According to Baumann			
1st Differential Angle	$\alpha_{m,+z}$	40.490	°
2nd Differential Angle	$\beta_{m,+z}$	130.490	°

Figure 2.62 Differential angles

Now we search for the direction of a moment that stiffens the two-directional reinforcement mesh. As previously shown for walls and plates, only the two angles between the directions of the reinforcement sets qualify as moment directions. The analysis for the surface's bottom side yields these directions for the assumed concrete compression struts:

⊞ Design Report			
⊞ Internal Forces of Linear Statics			
⊞ Principal Internal Forces			
⊟ Design Internal Forces			
⊟ Bottom surface (+z)			
⊟ Design Bending Moments			
⊞ Principal Moments			
⊞ Differential Angle Between $\alpha_{,+z,+z}$ and			
⊞ Differential Angle According to Baumann			
⊟ First Assumption of the Strut Direction γ			
New Differential Angle	$\gamma_{m,+z,1}$	85.490	°
⊟ Design Bending Moments by Baumann			
1st Design Bending Moment	$m_{\alpha,+z,1}$	344.73	kNm/m
2nd Design Bending Moment	$m_{\beta,+z,1}$	274.76	kNm/m
3rd Design Bending Moment	$m_{\gamma,+z,1}$	-440.77	kNm/m
Strut direction permissible?	$m_{strut,+z,1}$	Yes	
⊟ Second Assumption of the Strut Direction γ			
New Differential Angle	$\gamma_{m,+z,2}$	175.490	°
⊟ Design Bending Moments by Baumann			
1st Design Bending Moment	$m_{\alpha,+z,2}$	-96.04	kNm/m
2nd Design Bending Moment	$m_{\beta,+z,2}$	-166.01	kNm/m
3rd Design Bending Moment	$m_{\gamma,+z,2}$	440.77	kNm/m
Strut direction permissible?	$m_{strut,+z,2}$	No	

Figure 2.63 Directions γ of the concrete compression strut

Only the assumption of the direction $\gamma_{m,+z,1}$ of 85.490° proves to be valid. Since no optimization of this angle is carried out anymore, the final design moments $m_{end,+z,\varphi1}$ and $m_{end,+z,\varphi2}$ are obtained in the direction of both reinforcement sets:

⊞ Design Report			
⊞ Internal Forces of Linear Statics			
⊞ Principal Internal Forces			
⊟ Design Internal Forces			
⊟ Bottom surface (+z)			
⊟ Design Bending Moments			
⊞ Principal Moments			
⊞ Differential Angle Between $\alpha_{,+z,+z}$ and			
⊞ Differential Angle According to Baumann			
⊞ First Assumption of the Strut Direction γ			
⊞ Second Assumption of the Strut Direction γ			
⊞ Energy = Sum of abs(Design Bending Moments)			
⊞ Governing Strut			
⊞ Governing Design Bending Moments			
⊟ Final Design Bending Moments			
into Direction 1	$m_{end,+z,\Phi1}$	344.73	kNm/m
into Direction 2	$m_{end,+z,\Phi2}$	274.76	kNm/m
into Strut Direction	$m_{end,+z,strut}$	-440.77	kNm/m

Figure 2.64 Final design moments

2.5.3.2 Design axial forces

The design axial forces $n_{end,+z,\varphi 1}$ and $n_{end,+z,\varphi 2}$ are determined according to the same principle.

⊞ Design Report			
⊞ Internal Forces of Linear Statics			
⊞ Principal Internal Forces			
⊟ Design Internal Forces			
⊟ Bottom surface (+z)			
⊞ Design Bending Moments			
⊟ Design Axial Forces			
⊞ Principal Axial Forces			
⊞ Differential Angle Between $\alpha_{,+z,+z}$ and			
⊞ Differential Angle According to Baumann			
⊞ First Assumption of the Strut Direction γ			
⊞ Second Assumption of the Strut Direction γ			
⊞ Energy = Sum of abs(Design Axial Forces)			
⊞ Governing Strut			
⊟ Governing Design Axial Forces			
into Direction 1	$n_{+z,\varphi 1}$	32.026	kN/m
into Direction 2	$n_{+z,\varphi 2}$	-149.438	kN/m
into Strut Direction	$n_{end,+z,strut}$	-271.872	kN/m
Find optimal strut direction?	$Strut_{opti,n,+z}$	No	
⊟ Final Design Axial Forces			
into Direction 1	$n_{end,+z,\varphi 1}$	32.026	kN/m
into Direction 2	$n_{end,+z,\varphi 2}$	-149.438	kN/m
into Strut Direction	$n_{end,+z,strut}$	-271.872	kN/m

Figure 2.65 Design axial forces

2.5.3.3 Lever arm of the internal forces

With the design internal forces for the reinforcement directions φ_1 = 0° and φ_2 = 90°, you can determine the lever arm of the internal forces.

⊞ Design Report			
⊞ Internal Forces of Linear Statics			
⊞ Principal Internal Forces			
⊟ Design Internal Forces			
⊟ Bottom surface (+z)			
⊞ Design Bending Moments			
⊞ Design Axial Forces			
⊟ Design Internal Forces			
⊟ into Reinforcement Direction 1			
Design Bending Moment	$m_{end,+z,\varphi 1}$	344.73	kNm/m
Design Axial Force	$n_{end,+z,\varphi 1}$	32.026	kN/m
⊟ into Reinforcement Direction 2			
Design Bending Moment	$m_{end,+z,\varphi 2}$	274.76	kNm/m
Design Axial Force	$n_{end,+z,\varphi 2}$	-149.438	kN/m

Figure 2.66 Design internal forces

As described in chapter 2.5.2, a preliminary design is carried out with the determined internal forces for both reinforcement directions. It serves to determine the lever arm of the internal forces. The lever arm is determined from the state of strain due to the design internal forces.

⊞ Design Report			
⊞ Internal Forces of Linear Statics			
⊞ Principal Internal Forces			
⊟ Design Internal Forces			
⊟ Bottom surface (+z)			
⊞ Design Bending Moments			
⊞ Design Axial Forces			
⊞ Design Internal Forces			
⊟ Minimum Lever Arm of the Internal Forces	$z_{min,+z}$	1.239	m
⊞ Due to Design in Reinforcement Direction 1	$z_{+z,\Phi 1}$	1.250	m
⊟ Due to Design in Reinforcement Direction 2	$z_{+z,\Phi 2}$	1.239	m
⊞ Moment about the center of tension reinforcement	$m_{Sd(1),+z,2}$	365.17	kNm/m
⊞ Moment about the center of compression reinforceme	$m_{Sd(2),+z,2}$	184.35	kNm/m
⊞ Maximum Depth of Compression Zone	$x_{lim,+z,2}$	0.563	m
⊞ Limiting Axial Force	$n_{Sd,lim,+z,2}$	-9107.140	kN/m
⊞ Limiting Moment	$m_{Sd,lim,+z,2}$	9253.03	kNm/m
Limiting axial force larger than the design axial force?	$n_{Sd,lim,+z,2} > n_{Sd,+z,2}$	No	
⊞ Radicand (Value Below the Root)	$Radicand_{+z,2}$	22033.60	cm^2
Radicand less than zero?	$Radicand_{+z,2} < 0$	No	
Calculated Depth of the Compression Zone	$x_{calc,+z,2}$	0.031	m
Calculated depth of compression zone negative?	$x_{calc,+z,2} < 0$	No	
⊞ Existing Ratio Depth of the Compression Zone / Effec	$\psi_{+z,2}$	0.025	
Existing ratio greater than maximum ratio of x / d?	$\psi_{+z,2} > \psi_{lim}$	No	
Governing Range (see manual)	Range	III	
⊞ Lever Arm of the Internal Forces	$z_{+z,2}$	1.239	m

Figure 2.67 Lever arm of the internal forces

The value 1.239 m is obtained for the smaller and therefore governing lever arm $z_{min,+z}$.

2.5.3.4 Membrane forces

With the governing lever arm from the preliminary design, it is now possible to transform the internal forces of the linear plate analysis into membrane forces. For this, the equations presented in the design concept (Eq. 2.40) are used.

$$n_{sx,+z} = \frac{m_x}{z_{min,+z}} + \frac{n_x}{2} = \frac{123.35}{1.239} + \frac{-103.911}{2} = 48.408\ \text{kN/m}$$

$$n_{sy,+z} = \frac{m_y}{z_{min,+z}} + \frac{n_y}{2} = \frac{54.36}{1.239} + \frac{-285.386}{2} = -98.819\ \text{kN/m}$$

$$n_{xy,+z} = \frac{m_{xy}}{z_{min,+z}} + \frac{n_{xy}}{2} = \frac{-135.39}{1.239} + \frac{135.935}{2} = -109.910\ \text{kN/m}$$

These membrane forces can also be found in the design details.

⊞ Design Report			
⊞ Internal Forces of Linear Statics			
⊞ Principal Internal Forces			
⊟ Design Internal Forces			
⊟ Bottom surface (+z)			
⊞ Design Bending Moments			
⊞ Design Axial Forces			
⊞ Design Internal Forces			
⊞ Minimum Lever Arm of the Internal Forces	$z_{min,+z}$	1.239	m
⊟ Membrane Force			
⊟ into Direction of the x-Axis	$ns_{x,+z}$	48.414	kN/m
Bending Moment	m_x	124.35	kNm/m
Minimum Lever Arm of the Internal Forces	$z_{min,+z}$	1.239	m
Axial Force	n_x	-103.910	kN/m
⊟ into Direction of the y-Axis	$ns_{y,+z}$	-98.796	kN/m
Moment	m_y	54.38	kNm/m
Minimum Lever Arm of the Internal Forces	$z_{min,+z}$	1.239	m
Axial Force	n_y	-285.374	kN/m
⊟ Differential Membrane Force	$n_{xy,+z}$	-109.923	kN/m
Moment	m_{xy}	-220.39	kNm/m
Minimum Lever Arm of the Internal Forces	$z_{min,+z}$	1.239	m
Axial Force	n_{xy}	135.936	kN/m

Figure 2.68 Membrane forces

2.5.3.5 Design membrane forces

The principal membrane forces $ns_{I,+z}$ and $ns_{II,+z}$ are now determined from the membrane forces $n_{sx,+z}$, $n_{sy,+z}$, and $n_{xy,+z}$ that replace the moments m_x, m_y, m_{xy} and the axial forces n_x, n_y, n_{xy} of the linear plate analysis.

⊞ Design Report			
⊞ Internal Forces of Linear Statics			
⊞ Principal Internal Forces			
⊟ Design Internal Forces			
⊟ Bottom surface (+z)			
⊞ Design Bending Moments			
⊞ Design Axial Forces			
⊞ Design Internal Forces			
⊞ Minimum Lever Arm of the Internal Forces	$z_{min,+z}$	1.239	m
⊞ Membrane Force			
⊟ Design Membrane Forces			
⊟ Principal Membrane Forces			
First Principal Membrane Force	$ns_{I,+z}$	107.100	kN/m
Second Principal Membrane Force	$ns_{II,+z}$	-157.481	kN/m
Direction	$\alpha_{nsI,+z}$	-28.097	°
Quotient k = $ns_{II,+z}/ns_{I,+z}$	$k_{ns,+z}$	-1.470	

Figure 2.69 Design membrane forces

The design membrane forces can be determined from the principal membrane forces according to Equations 2.5 to 2.7. They can be found in the design details.

⊞ Design Report			
⊞ Internal Forces of Linear Statics			
⊞ Principal Internal Forces			
⊟ Design Internal Forces			
⊟ Bottom surface (+z)			
⊞ Design Bending Moments			
⊞ Design Axial Forces			
⊞ Design Internal Forces			
⊞ Minimum Lever Arm of the Internal Forces	$z_{min,+z}$	1.239	m
⊞ Membrane Force			
⊟ Design Membrane Forces			
⊞ Principal Membrane Forces			
⊞ Differential Angle Between $\alpha_{,+z,+z}$ and			
⊞ Differential Angle According to Baumann			
⊟ First Assumption of the Strut Direction γ			
New Differential Angle	$\gamma_{ns,+z,1}$	73.097	°
⊟ Design Membrane Forces by Baumann			
1st Design Membrane Force	$ns_{\alpha,+z,1}$	158.337	kN/m
2nd Design Membrane Force	$ns_{\beta,+z,1}$	11.127	kN/m
3rd Design Membrane Force	$ns_{\gamma,+z,1}$	-219.846	kN/m
Strut direction permissible?	$ns_{strut,+z,1}$	Yes	
⊞ Second Assumption of the Strut Direction γ			
⊞ Energy = Sum of abs(Design Membrane Forces)			
⊞ Governing Strut			
⊞ Governing Design Membrane Forces			
⊟ Final Design Membrane Forces			
into Direction 1	$ns_{end,+z,\Phi 1}$	158.337	kN/m
into Direction 2	$ns_{end,+z,\Phi 2}$	11.127	kN/m
into Strut Direction	$ns_{end,+z,strut}$	-219.846	kN/m

Figure 2.70 Final design membrane forces

With the final design membrane forces $ns_{end,+z,\varphi 1}$ and $ns_{end,+z,\varphi 2}$, the program determines the required reinforcement areas of a two-directional reinforcement mesh for the surface side.

The reinforcement mesh is stiffened by a concrete compression strut. The magnitude of the stiffening strut force $ns_{end,+z,strut}$ is specified under the final design membrane forces. It amounts to −219.846 kN/m.

Analogously, the design membrane forces and stiffening force of the concrete compression strut are determined for the top surface of the plate.

2.5.4 Analysis of the Concrete Compression Struts

To design the concrete compression strut of a shell, it is divided into three surface layers that are subjected to the design membrane forces.

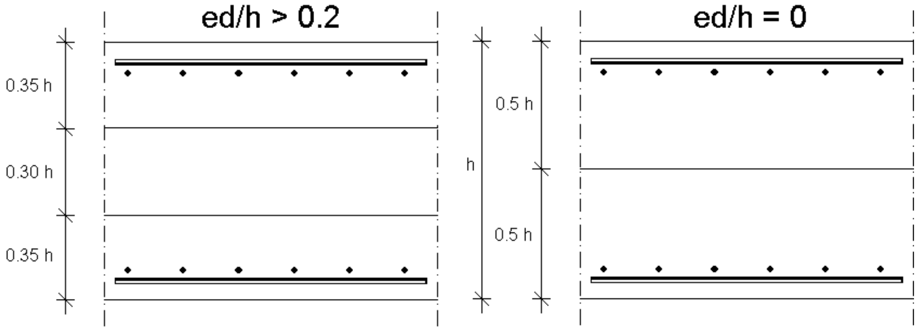


Figure 2.71 Surface layer thicknesses for shells mainly subjected to moment (left) or compression force (right)

For shells where the applied moment is relatively large in relation to the acting axial force ($e_d/h > 0.2$), the thickness h_E of the two outer layers is reduced to $0.35 \cdot d$. For shells subjected to approximately concentric compression, the surface layer thickness h_E is increased to half of the plate thickness h. If the related eccentricity of the axial force e_d/h is between 0 and 0.2, the surface layer thickness is interpolated.

For e_d, the larger value among the quotients of m_x/n_x and m_y/n_y is applied.

For the analysis of the concrete compression strut, the concrete strut's compression force to be resisted $n_{strut,+z}$ is compared with the resistant axial force of the surface layer $n_{strut,d}$.

⊞ Design Report			
⊞ Internal Forces of Linear Statics			
⊞ Principal Internal Forces			
⊞ Design Internal Forces			
⊟ Concrete Strut			
⊟ Thickness of Surface	h_E	45.15	cm
⊞ Related Load Eccentricity	e_d/h	0.928	
Predominant Strain	Under Stress	Compression	
Factor of Surface Thickness	f_{hE}	0.350	
⊟ Bottom surface (+z)			
⊞ Design Membrane Forces in Strut Direction	$ns_{strut,+z}$	-219.846	kN/m
⊟ Concrete Membrane Force Resistance	$n_{strut,d}$	-7224.000	kN/m
Width of Surface	b	1.000	m
Thickness of Surface	h_E	45.15	cm
⊟ Applied Concrete Compressive Strength	$f_{cd,08}$	16.00	N/mm²
Design Unconfined Concrete Compressive Strength	f_{cd}	20.00	N/mm²
Coefficient of Maximum Utilization	ξ_{fcd}	0.800	
Failure of concrete strut?	$\lvert n_{strut,d}\rvert < \lvert ns_{strut,}$	No	

Figure 2.72 Concrete strut and thickness of surface layer

The resistant axial force $n_{strut,d}$ depends on the thickness h_E of the surface layer and the applied concrete strength $f_{cd,08}$.

The first step to determine the thickness of the surface layer is to determine the provided load eccentricities in x- and y-direction from the internal forces of the linear plate analysis:

$$e_{dx} = \left|\frac{m_x}{n_x}\right| = \left|\frac{124.35}{-103.911}\right| = 1.197\ \text{m}$$

$$e_{dy} = \left|\frac{m_y}{n_y}\right| = \left|\frac{54.36}{-285.386}\right| = 0.190\ \text{m}$$

The greater load eccentricity in x-direction is computed as governing. It can be used to determine the relative load eccentricity e_d/h.

$$\frac{e_d}{h} = \frac{1.197}{1.29} = 0.928 > 0.2$$

Since the relative load eccentricity is greater than 0.2, it is a shell that is predominantly subjected to bending. The factor f_{hE} for determining the surface layer thickness is 0.35.

Thus, the thickness h_E of the surface layer is determined as follows:

$$h_E = f_{hE} \cdot h = 0.35 \cdot 129 = 45.15\,\text{cm}$$

The design value of the concrete compressive strength is reduced to 80 % according to the recommendations of Schlaich/Schäfer (in [2], page 378). This recommendation can also be found in EN 1992-1-1, clause 6.5.2, which regulates the design of compression struts in framework models.

$$f_{cd} = \frac{f_{ck}}{\gamma_c} = \frac{30}{1.5} = 20\,\text{N/mm}^2$$

$$f_{cd,08} = 0.8 \cdot 20 = 16\,\text{N/mm}^2$$

This value can also be found in the design details (see Figure 2.72).

With it, you can determine the resisting force of the concrete compression strut $n_{strut,d}$.

$$n_{strut,d} = b \cdot h_E \cdot f_{cd,08} = 100 \cdot 45.15 \cdot 16 = 7\,224.00\ \text{kN/m}$$

The analysis of the concrete compression strut for the top side of the surface is done analogously.

2.5.5 Required Longitudinal Reinforcement

The longitudinal reinforcement to be used at the bottom side of the surface is determined from the design membrane forces. In the design details, the output occurs separately for the two reinforcement directions.

⊞ Design Report			
⊞ Internal Forces of Linear Statics			
⊞ Principal Internal Forces			
⊞ Design Internal Forces			
⊞ Concrete Strut			
⊟ Required Longitudinal Reinforcement Due to Design Membrane Forces			
⊟ Bottom surface (+z)			
⊟ into Reinforcement Direction 1	$a_{s,dim,+z,1}$	3.40	cm²/m
Design Membrane Force	$ns_{end,+z,\Phi 1}$	158.337	kN/m
⊞ Design Stress	$\sigma_{s,+z,1}$	465.93	N/mm²
⊟ into Reinforcement Direction 2	$a_{s,dim,+z,2}$	0.24	cm²/m
Design Membrane Force	$ns_{end,+z,\Phi 2}$	11.127	kN/m
⊞ Design Stress	$\sigma_{s,+z,2}$	465.93	N/mm²

Figure 2.73 Required longitudinal reinforcement

$$a_{s,\dim+z,1} = \frac{ns_{end,+z,\phi_1}}{\sigma_{s,+z,1}} = \frac{158.344}{465.93} = 3.4\,\text{cm}^2/m$$

$$a_{s,\dim+z,2} = \frac{ns_{end,+z,\phi_2}}{\sigma_{s,+z,2}} = \frac{11.116}{465.93} = 0.24\,\text{cm}^2/m$$

The reinforcement for the surface's top side is determined in the same manner.

2.5.6 Shear Design

In the shear design, the applied tensile reinforcement is determined first.

⊟ Shear Design			
Applied tensile reinforcement determined from the required longitudinal reinforcement.			
Application of the basic reinforcement is not necessary because the required longitudinal reinforcement is si			
⊟ Applied Longitudinal Reinforcement	a_{sl}	1.54	cm^2/m
⊟ Bottom surface (+z)			
⊟ from reinforcement direction 1	$a_{sl,+z,1}$	0.03	cm^2/m
Required Longitudinal Reinforcement	$a_{s,dim+z,1}$	3.40	cm^2/m
State of Stress	$Stress_{+z,1}$	Tension	
Differential Angle in Direction of the Principal She	$\Delta\Phi_{+z,1}$	84.385	°
Second Power of Cosine of Differential Angle to I	$cos^2(\Delta\Phi_{+z,1})$	0.010	
⊟ from reinforcement direction 2	$a_{sl,+z,2}$	0.24	cm^2/m
Required Longitudinal Reinforcement	$a_{s,dim+z,2}$	0.24	cm^2/m
State of Stress	$Stress_{+z,2}$	Tension	
Differential Angle in Direction of the Principal She	$\Delta\Phi_{+z,2}$	5.615	°
Second Power of Cosine of Differential Angle to I	$cos^2(\Delta\Phi_{+z,2})$	0.990	
⊟ Top surface (-z)			
⊟ from reinforcement direction 1	$a_{sl,-z,1}$	0.02	cm^2/m
Required Longitudinal Reinforcement	$a_{s,dim-z,1}$	2.00	cm^2/m
State of Stress	$Stress_{-z,1}$	Tension	
Differential Angle in Direction of the Principal She	$\Delta\Phi_{-z,1}$	84.385	°
Second Power of Cosine of Differential Angle to I	$cos^2(\Delta\Phi_{-z,1})$	0.010	
⊟ from reinforcement direction 2	$a_{sl,-z,2}$	1.26	cm^2/m
Required Longitudinal Reinforcement	$a_{s,dim-z,2}$	1.27	cm^2/m
State of Stress	$Stress_{-z,2}$	Tension	
Differential Angle in Direction of the Principal She	$\Delta\Phi_{-z,2}$	5.615	°
Second Power of Cosine of Differential Angle to I	$cos^2(\Delta\Phi_{-z,2})$	0.990	

Figure 2.74 Applied tensile reinforcement

From all reinforcement layers and directions, a total of 1.54 cm^2/m of tension reinforcement can be applied. With it, the shear force $V_{Rd,c}$ that can be resisted without shear reinforcement is determined.

⊟ Shear Design			
Applied tensile reinforcement determined from the required longitudinal reinforcement.			
Application of the basic reinforcement is not necessary because the required longitudinal reinforcement is si			
⊞ Applied Longitudinal Reinforcement	a_{sl}	1.54	cm^2/m
⊟ Shear Resistance Without Shear Reinforcement			
⊟ Design Shear Resistance Without Shear Reinforcement			
Safety Factor	$C_{Rd,c}$	0.120	
⊟ Factor of Size Effects	k	1.399	
Effective Depth	d	1.255	m
⊟ Longitudinal Reinforcement Ratio	ρ_l	0.000	
Applied Longitudinal Reinforcement	a_{sl}	1.54	cm^2/m
Width of Member	b_w	1.000	m
Effective Depth	d	1.255	m
Characteristic Concrete Compressive Strength	f_{ck}	30.00	N/mm^2
Factor of Longitudinal Stress of Concrete	k_1	0.150	
⊟ Longitudinal Concrete Stress	σ_{cp}	0.33	N/mm^2
Axial Force in Direction of Principal Shear Force	n_β	-425.773	kN/m
Width of Structural Member	b_w	1.000	m
Depth of Structural Member	h	129.00	cm
Width of Structural Member	b_w	1.000	m
Effective Depth	d	1.255	m
Design Shear Resistance Without Shear Reinforcer	$V_{Rd,c,6.2a}$	213.271	kN/m
⊟ Minimum shear resistance according to (6.2.b)			
⊟ Factor of Compressive Strength	v_{min}	0.317	
Factor of Size Effects	k	1.399	
Characteristic Concrete Compressive Strength	f_{ck}	30.00	N/mm^2
Factor of Longitudinal Stress of Concrete	k_1	0.150	
⊟ Longitudinal Concrete Stress	σ_{cp}	0.33	N/mm^2
Axial Force in Direction of Principal Shear Force	n_β	-425.773	kN/m
Width of Structural Member	b_w	1.000	m
Depth of Structural Member	h	129.00	cm
Width of Structural Member	b_w	1.000	m
Effective Depth	d	1.255	m
Minimum shear resistance according to (6.2.b)	$V_{Rd,c,6.2b}$	460.326	kN/m
Shear Resistance Without Shear Reinforcement	$V_{Rd,c}$	460.326	kN/m
⊟ Shear reinforcement required?	Check $V_{Rd,c}$	No	
Design Shear Resistance of a Member Without Shear	$V_{Rd,c}$	460.326	kN/m
Design Shear Force	V_{Ed}	259.736	kN/m
Shear capacity without shear reinforcement sufficient. No further checks.			

Figure 2.75 Design shear resistance without shear reinforcement

With the applied tensile reinforcement, the longitudinal reinforcement ratio ρ_l is determined:

$$\rho_l = \frac{A_{sl}}{(b_w \cdot d)} = \frac{1.54}{(100 \cdot 125.5)} = 0.00012 \leq 0.02$$

In a 3D model type (in contrast to a plate), an additional axial force can occur. It must be considered via the corresponding concrete longitudinal stress.

$$\sigma_{cp} = \frac{n_\beta}{(b_w \cdot h)} = \frac{-425.783}{(100 \cdot 129)} = -0.33\,\mathrm{N/m^2}$$

The factor k for considering the plate thickness is calculated as follows:

$$k = 1 + \sqrt{\frac{200}{d}} = 1 + \sqrt{\frac{200}{1255}} = 1.399 \leq 2.0 \quad d \text{ in } [\mathrm{mm}]$$

The following factors are also included in the design:

Factor of concrete longitudinal stress	$k_1 = 0.15$
Concrete compressive strength for C30/37	$f_{ck} = 30.0\ \mathrm{N/mm^2}$
Safety factor	$C_{rd,c} = \frac{0.18}{\gamma_c} = \frac{0.18}{1.5} = 0.12$

Thus, the design shear resistance $V_{Rd,c}$ without shear reinforcement can be determined according to Equation (6.2a):

$$V_{Rd,c} = \left[C_{rd,c} \cdot k\,(100 \cdot \rho_l \cdot f_{ck})^{\frac{1}{3}} + k_1 \cdot \sigma_{cp}\right] \cdot b_w \cdot d =$$

$$= \left[0.12 \cdot 1.399\,(100 \cdot 0.00012 \cdot 30.0)^{\frac{1}{3}} + 0.15 \cdot 0.33\right] \cdot 1000 \cdot 1255 = 212.00\,\mathrm{kN/m}$$

According to Equation (6.2b), the minimum value of the design shear resistance $V_{Rd,c}$ without shear reinforcement is determined from the minimum reinforcement ratio ν_{min}:

$$\nu_{\min} = 0.035 \cdot k^{\frac{3}{2}} \cdot f_{ck}^{\frac{1}{2}} = 0.035 \cdot 1.399^{\frac{3}{2}} \cdot 30.0^{\frac{1}{2}} = 0.317$$

$$V_{Rd,c} = (0.317 \cdot 0.15 \cdot 0.33) \cdot 1000 \cdot 1255 = 459.96\,\mathrm{kN/m}$$

Because the plate's design shear resistance $V_{Rd,c}$ = 459.96 kN/m is greater than the applied shear force V_{Ed} = 259.726 kN/m, no shear reinforcement is required in the example.

Should the plate's shear resistance be insufficient, the program first checks if the maximum shear resistance of the concrete compression strut $V_{Rd,max}$ is sufficient. $V_{Rd,max}$ is determined with the minimum inclination of the compression strut θ. When the design shear resistance of the concrete compression strut is greater than the applied shear force V_{Ed}, the statically required shear reinforcement req a_{sw} can be determined. Then the design for the shear reinforcement $V_{Rd,sy}$ is carried out.

2.5.7 Statically Required Longitudinal Reinforcement

The table of the design details summarizes the statically required longitudinal reinforcement.

⊞ Design Report			
⊞ Internal Forces of Linear Statics			
⊞ Principal Internal Forces			
⊞ Design Internal Forces			
⊞ Concrete Strut			
⊞ Required Longitudinal Reinforcement Due to Design Membrane Forces			
⊞ Shear Design			
⊟ Statically Required Longitudinal Reinforcement			
⊟ Bottom surface (+z)			
⊟ into Reinforcement Direction 1	$a_{s,stat,+z,1}$	3.40	cm^2/m
Due to Design	$a_{s,dim,+z,1}$	3.40	cm^2/m
⊟ into Reinforcement Direction 2	$a_{s,stat,+z,2}$	0.24	cm^2/m
Due to Design	$a_{s,dim,+z,2}$	0.24	cm^2/m
⊟ Top surface (-z)			
⊟ into Reinforcement Direction 1	$a_{s,stat,-z,1}$	2.00	cm^2/m
Due to Design	$a_{s,dim,-z,1}$	2.00	cm^2/m
⊟ into Reinforcement Direction 2	$a_{s,stat,-z,2}$	1.27	cm^2/m
Due to Design	$a_{s,dim,-z,2}$	1.27	cm^2/m

Figure 2.76 Statically required longitudinal reinforcement

For each reinforcement direction, the table shows which design is governing for the statically required reinforcement.

In the example, all longitudinal reinforcements result from the bending design as a shell. In other cases, a required longitudinal reinforcement to avoid shear reinforcement would also be conceivable.

2.5.8 Minimum Longitudinal Reinforcement

The statically required longitudinal reinforcement is now compared to the minimum reinforcement. Unfortunately, none of the standards available in RF-CONCRETE Surfaces provides any regulations on the minimum reinforcement for shells. As a criterium, it is therefore analyzed for which constellation of the moment and axial force the element is more likely to be a wall (mainly subjected to compression) or a plate (mainly subjected to bending). The distinguishing criterion is the related load eccentricity e_d/h in the ultimate limit state (ULS):

$$\frac{e_d}{h} = \frac{m/n}{h}$$

Equation 2.61

where

m	moment of linear plate analysis (ULS)
n	axial force of linear plate analysis (ULS)
h	plate thickness

Since there are moments and axial forces both in x- and y-direction in a design point, the related load eccentricity per design point is the largest quotient from moment over axial force of both directions.

In RF-CONCRETE Surfaces, the following is uniformly specified for all standards:

$$\frac{e_d}{h} > 3.5$$ mainly subjected to bending ⟶ reinforcement rules for **plates**

$$\frac{e_d}{h} \leq 3.5$$ mainly subjected to compression ⟶ reinforcement rules for **walls**

This regulation can be found in EN 1992-1-1, clause 9.3: Solid slabs and clause 9.6: Walls.

The minimum reinforcements are described in chapter 2.3.7 and chapter 2.4.5 in the reinforcement rules for walls and plates.

In our example, where the system is mainly subjected to bending, the following minimum reinforcement is shown in the design details.

⊞ Design Report			
⊞ Internal Forces of Linear Statics			
⊞ Principal Internal Forces			
⊞ Design Internal Forces			
⊞ Concrete Strut			
⊞ Required Longitudinal Reinforcement Due to Design Membrane Forces			
⊞ Shear Design			
⊞ Statically Required Longitudinal Reinforcement			
⊟ Minimum Reinforcement			
⊟ Minimum Longitudinal Reinforcement			
⊟ Minimum Reinforcement Ratio			
Minimum Tension Longitudinal Reinforcement Ratio	min ρ_T	0.0	%
Minimum Compression Longitudinal Reinforcement Ratio	min ρ_C	0.0	%
General Minimum Reinforcement Ratio	min ρ_G	0.0	%
⊞ Referred to Cross-Section	A_c	12900.00	cm^2
⊟ Bottom surface (+z)			
⊟ Main longitudinal reinforcement with tension of this side	$a_{s,max,+z}$	18.93	cm^2/m
Direction of the Main Longitudinal Reinforcement	$\varphi_{as,main}$	0.000	°
⊞ Statically Required Reinforcement	$a_{s,stat,+z,1}$	3.40	cm^2/m
⊞ Statically Required Reinforcement	$a_{s,stat,+z,2}$	0.24	cm^2/m
⊟ Minimum Longitudinal Reinforcement into Direction 1	$a_{s,min\ longi,+z,1}$	18.93	cm^2/m
State of Stress	$Stress_{-z,1}$	Tension	
Minimum Tension Reinforcement	$a_{s,minT,+z,1}$	0.00	cm^2/m
Minimum Compression Reinforcement	$a_{s,minC,+z,1}$	0.00	cm^2/m
General Minimum Reinforcement	$a_{s,minG,+z,1}$	0.00	cm^2/m
Minimum Reinforcement for Walls	$a_{s,minW,+z,1}$	6.45	cm^2/m
⊟ Minimum Longitudinal Reinforcement for Parts with Ductile	$A_{s,min,duc}$	18.93	cm^2/m
⊞ 1st Calculated Value for Minimum Longitudinal Reinforc	$A_{s,min,duc,calc1,-}$	18.93	cm^2/m
⊞ 2nd Calculated Value for Minimum Longitudinal Reinfor	$A_{s,min,duc,calc2,-}$	16.32	cm^2/m
⊞ Minimum Longitudinal Reinforcement into Direction 2	$a_{s,min\ longi,+z,2}$	12.90	cm^2/m

Figure 2.77 Minimum longitudinal reinforcement

2.5.9 Reinforcement to Be Used

The reinforcement to be used is determined from the statically required reinforcement and the minimum reinforcement.

⊞ Design Report			
⊞ Internal Forces of Linear Statics			
⊞ Principal Internal Forces			
⊞ Design Internal Forces			
⊞ Concrete Strut			
⊞ Required Longitudinal Reinforcement Due to Design Membrane Forces			
⊞ Shear Design			
⊞ Statically Required Longitudinal Reinforcement			
⊞ Minimum Reinforcement			
⊞ Check Maximum Reinforcement Ratio			
⊟ Reinforcement to be used			
⊟ Bottom surface (+z)			
⊟ into Reinforcement Direction 1	$a_{s,+z,1}$	18.93	cm^2/m
Statically Required Reinforcement	$a_{s,stat,+z,1}$	3.40	cm^2/m
⊞ Minimum Reinforcement	$a_{s,min,+z,1}$	18.93	cm^2/m
⊟ into Reinforcement Direction 2	$a_{s,+z,2}$	12.90	cm^2/m
Statically Required Reinforcement	$a_{s,stat,+z,2}$	0.24	cm^2/m
⊞ Minimum Reinforcement	$a_{s,min,+z,2}$	12.90	cm^2/m
⊞ into Reinforcement Direction 3	$a_{s,+z,3}$	0.00	cm^2/m
⊟ Top surface (-z)			
⊟ into Reinforcement Direction 1	$a_{s,-z,1}$	6.45	cm^2/m
Statically Required Reinforcement	$a_{s,stat,-z,1}$	2.00	cm^2/m
⊞ Minimum Reinforcement	$a_{s,min,-z,1}$	6.45	cm^2/m
⊟ into Reinforcement Direction 2	$a_{s,-z,2}$	12.90	cm^2/m
Statically Required Reinforcement	$a_{s,stat,-z,2}$	1.27	cm^2/m
⊞ Minimum Reinforcement	$a_{s,min,-z,2}$	12.90	cm^2/m
⊞ into Reinforcement Direction 3	$a_{s,-z,3}$	0.00	cm^2/m

Figure 2.78 Reinforcement to be used

It is also possible to display the reinforcement areas for grid point No. 1 graphically.

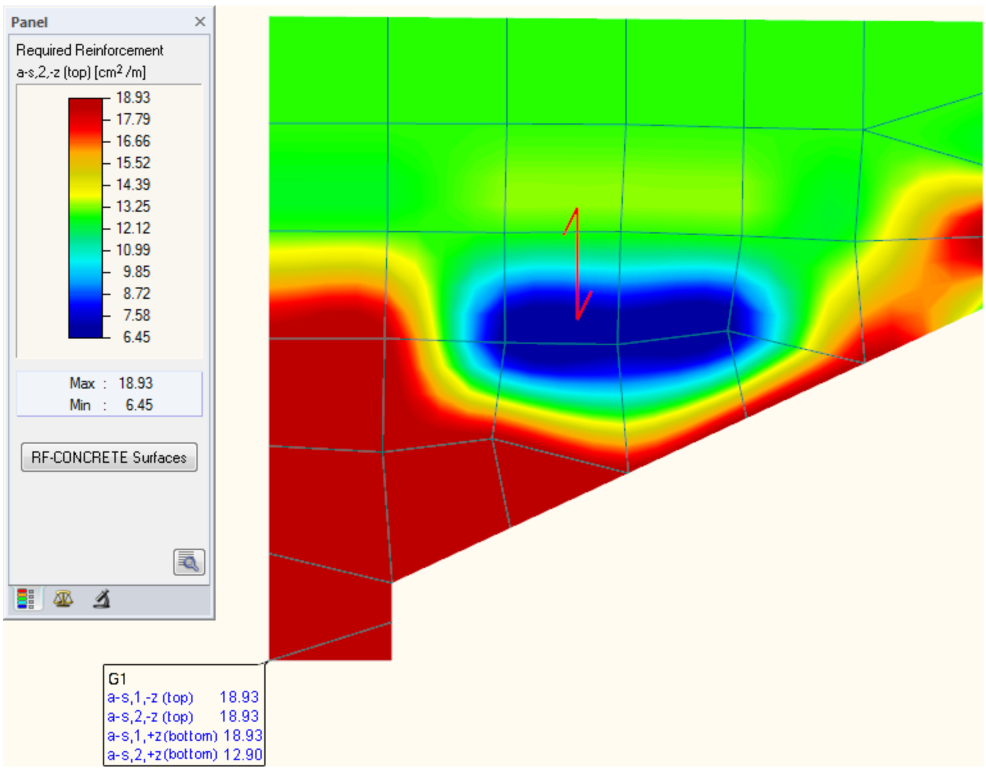


Figure 2.79 Graphic of the reinforcement for surface No. 37

2.6 Serviceability

The serviceability limit state designs consist of various individual designs. The following listing contains the relevant clauses for EN 1992-1-1:

- Stress limitation: clause 7.2
- Crack control: clause 7.3
- Deflection control: clause 7.4

In the reinforced concrete standards, the designs listed above are always described for linear, member-shaped structural elements. As mentioned in the previous part of this manual, the design situation of a surface element is transformed into the design of several linear elements in the direction of the individual reinforcement layers in the ultimate limit state. Such a transformation procedure is used in the serviceability limit state as well.

2.6.1 Design Internal Forces

Unlike the transformation procedure for the ultimate limit state, it is not possible to carry out a purely geometrical division of the principal internal forces into internal forces in the individual reinforcement directions. Such a division assumes a strain ratio of 1.0 for the actually provided reinforcement. For both reinforcement directions to have the same strain, however, corresponding reinforcement areas would have to be respectively provided in these reinforcement directions for different design forces. In the serviceability limit state, though, the design internal forces are sought for a provided reinforcement.

In the serviceability limit state, no required reinforcement is determined; instead, the provided reinforcement is used to determine the actually provided strain ratio. In all cases where the applied reinforcement deviates from the required reinforcement, the actually provided strain ratio of the reinforcements does not equal the value 1.0.

The assumption of an identical strain ratio is therefore invalid. A different strain ratio that confirms the resulting design internal forces must be found. In solving this problem, the geometric relation between the strain ratio and the direction of the concrete compression strut plays an important role.

Baumann [1] writes the following on this point: If you neglect the compression strain of the concrete because it is usually small compared to the strain of the reinforcement, the following is obtained as the compatibility condition from Figure 38:

$$\frac{\varepsilon_y}{\varepsilon_x} = \frac{\sin^2(\beta - \gamma)}{\sin^2(\gamma - \alpha)}$$

Equation 2.62

The following Figure 2.80 shows the mentioned Figure 38 with the compatibility condition of the strains for a two-directional reinforcement mesh.

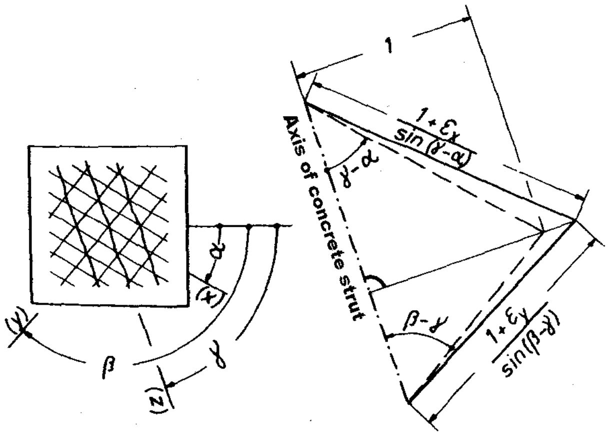


Figure 2.80 Compatibility of the strains

In Equation 2.62, ε_y and ε_x are the strains of two reinforcement directions. The angles α and β represent the intermediate angles between the principal force direction and the direction of the respective reinforcement set. The smaller intermediate angle is named α. The angle γ refers to the differential angle between the direction of the concrete compression strut and the direction of the first principal internal force.

The angles α and β cannot be changed due to the selection of the reinforcement direction. In contrast to this, the angle γ changes if a different direction of the concrete compression strut is necessary to stiffen the reinforcement mesh due to the varyingly stiff reinforcement directions.

The design internal forces in the individual reinforcement directions depend on the selected direction of the concrete compression strut. With these design internal forces, the stresses in the reinforcements of the individual directions can be determined. Based on these stresses, the various standards provide formulas with which you can determine the mean strains of the reinforcement relative to the concrete. In EN-1992-1-1, this is done as per Equation (7.9):

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \cdot \frac{f_{ct,eff}}{\rho_{p,eff}} \cdot (1 + \alpha_e \cdot \rho_{p,eff})}{E_s} \geq 0.6 \cdot \frac{\sigma_s}{E_s}$$

Equation 2.63

Only then can you determine the quotient from the differences in the strains between concrete and reinforcing steel of the second and first reinforcement direction.

$$Q_\varepsilon = \frac{(\varepsilon_{sm} - \varepsilon_{cm})_{\phi_2}}{(\varepsilon_{sm} - \varepsilon_{cm})_{\phi_1}}$$

Equation 2.64

Equation 2.62 also gives a quotient of the strains, derived from the geometric principles.

$$Q_{\varepsilon,geo} = \frac{\varepsilon_{\phi_2}}{\varepsilon_{\phi_1}} = \frac{\sin^2(\beta - \gamma)}{\sin^2(\gamma - \alpha)}$$

Equation 2.65

For both quotients, the strain of the second reinforcement direction is in the numerator. This is based on the assumption that the first reinforcement direction forms the smaller differential angle with the first principal internal force. If the second reinforcement direction formed the smaller differential angle with the first principal internal force, the strains of the first reinforcement direction would be in the numerator.

Both quotients depend on the selected direction of the concrete compression strut. The program now tries to select the direction of the concrete compression strut in such a way that both quotients become identical.

$$Q_{\varepsilon} = Q_{\varepsilon,\, geo}$$

Equation 2.66

If the geometric strain ratio $Q_{\varepsilon,geo}$ does not yet correspond to the actual strain ratio after one calculation run, the program specifies a new compression strut direction and determines the resulting geometric strain ratio. This process is repeated iteratively until a convergence is reached.

Determining the design internal forces by selecting the suitable compression strut direction is the most demanding part of the serviceability limit state design. If the selected provided reinforcement approximately corresponds to the statically required reinforcement for the analyzed service load magnitudes, the design internal forces only marginally differ from the internal forces that would result from an assumed strain ratio of 1.0. Therefore, RF-CONCRETE Surfaces additionally provides the option to determine design internal forces with an assumed strain ratio of 1.0.

Design internal forces for the serviceability limit state design are only determined if the cracking of the concrete leads to an activation of the reinforcement. To this end, the program analyzes the concrete tensile stresses caused by the first principal internal force.

2.6.2 Principal Internal Forces

If the first principal internal force is negative, uncracked concrete is assumed in this area of the analyzed surface element. For walls, only the magnitude of the concrete compression stress is checked in such a case. For plates, no serviceability limit state design is carried out on this surface side, at least.

For a **wall**, if the first principal axial force is a tension force, the provided concrete tensile stress is determined according to the following equation.

$$\sigma_{c,I} = \frac{n_I}{A_c} = \frac{n_I}{b \cdot h}$$

Equation 2.67

For a **plate**, if the first principal moment is a positive moment, the provided concrete tensile stress is determined as follows.

$$\sigma_{c,I} = \frac{m_I}{W} = \frac{m_I \cdot 6}{b \cdot h^2}$$

Equation 2.68

If this linear-elastically determined stress $\sigma_{c,I}$ is greater than the mean value of the axial tensile strength f_{ctm}, cracked concrete is assumed. Only then does RF-CONCRETE Surfaces determine the design internal forces for the individual directions of reinforcement and perform the serviceability limit state designs mentioned at the beginning of chapter 2.6.

2.6.3 Provided Reinforcement

Before the serviceability limit state designs, RF-CONCRETE Surfaces checks the provided reinforcement: The program first uses the internal forces of the serviceability to perform a design similar to the ultimate limit state design. The thus determined statically required reinforcement is compared to the user-defined provided reinforcement.

If the provided reinforcement is smaller than the statically required reinforcement or if the design reveals any non-designable situations, the serviceability limit state designs are not performed. The problematic zones of the surface elements are indicated as being non-designable.

2.6.4 Serviceability Limit State Designs

The following example illustrates how the various serviceability limit state designs are implemented in RF-CONCRETE Surfaces. A rectangular slab is analyzed. The first applied principal moment m_I is greater than zero and the second applied moment m_{II} equals zero.

The design is carried out according to EN 1992-1-1 with the analytical method.

2.6.4.1 Input data for the example

Geometric specifications

Plate thickness:	$d = 20$ cm	
Rectangular reinforcement:	$\varphi_1 = 30°$	$\varphi_2 = 120°$
Centroid of concrete cover:	$d_1 = 3.0$ cm	$d_2 = 4.2$ cm

Material

Concrete:	C30/37
Reinforcing steel:	B 500 S (B)

2.6.4.2 Check of principal internal forces

The program first checks if the concrete cracks under the principal moment in the ULS. In the serviceability design details of the relevant grid point, we can see that this is indeed the case:

⊞ Internal Forces of Linear Statics			
⊟ Check if internal forces cause cracks			
⊟ Bottom surface (+z)			
First Principal Moment	$m_{I,+z}$	-7.16	kNm/m
The first principal moment is negative. No check of this side.			
⊟ Top surface (-z)			
⊟ Tensile Stress of Concrete	$\sigma_{c,I,-z}$	5.05	N/mm²
First Principal Moment	$m_{I,-z}$	33.65	kNm/m
⊟ Section modulus	S	6666.67	cm³
Width of the Element	b_w	1.000	m
Depth of Structural Member	h	0.200	m
Mean Axial Tensile Strength	f_{ctm}	2.90	N/mm²
Concrete cracks on this side. Longitudinal reinforcement is activated.			
Concrete cracks on one side. Longitudinal Reinforcement is activated and check is completed.			

Figure 2.81 Check of principal internal forces

The linear-elastically determined stress $\sigma_{c,I,-z}$ at the upper concrete edge is compared to the mean value of the axial tensile strength f_{ctm} of 2.9 N/mm^2 for concrete C30/37.

$$\sigma_{c,I,-z} = \frac{m_{I,-z}}{W} = \frac{m \cdot 6}{b \cdot h^2} = \frac{33.65 \cdot 6}{1.0 \cdot 0.2^2} = 5.05\,\text{N/mm}^2$$

Thus, the concrete edge stress $\sigma_{c,I,-z}$ = 5.05 N/mm^2 significantly exceeds the tensile strength f_{ctm}. The reinforcement is therefore also activated for the serviceability limit state.

2.6.4.3 Required reinforcement for ULS

The design for the ultimate limit state for the plate's top surface is carried out with the following values:

⊞ Design Report			
⊞ Internal Forces of Linear Statics			
⊞ Principal Internal Forces			
⊟ Design Bending Moments			
⊞ Bottom surface (+z)			
⊟ Top surface (-z)			
⊟ Principal Moments			
First Principal Moment	$m_{I,-z}$	56.08	kNm/m
Second Principal Moment	$m_{II,-z}$	11.93	kNm/m
Direction	$\alpha_{b,-z}$	0.000	°
Quotient k = $m_{II,-z}/m_{I,-z}$	$k_{m,-z}$	0.213	
⊞ Differential Angle Between $\alpha_{,-z,-z}$ and			
⊞ Differential Angle According to Baumann			
⊞ First Assumption of the Strut Direction γ			
⊞ Second Assumption of the Strut Direction γ			
⊞ Energy = Sum of abs(Design Bending Moments)			
⊟ Governing Strut			
First Assumption for Direction γ	$\gamma_{m,-z,1}$	75.000	°
Strut Direction	$\Phi_{strut,m,-z}$	75.000	°
⊟ Governing Design Bending Moments			
into Direction 1	$m_{-z,\Phi1}$	64.16	kNm/m
into Direction 2	$m_{-z,\Phi2}$	42.08	kNm/m
into Strut Direction	$m_{end,-z,strut}$	-38.23	kNm/m
Find optimal strut direction?	$Strut_{opti,m,-z}$	No	
⊟ Final Design Bending Moments			
into Direction 1	$m_{end,-z,\Phi1}$	64.16	kNm/m
into Direction 2	$m_{end,-z,\Phi2}$	42.08	kNm/m
into Strut Direction	$m_{end,-z,strut}$	-38.23	kNm/m

Figure 2.82 Design internal forces ULS

Final design bending moments: $m_{end,-z,\varphi1}$ = 64.16 kNm/m

$m_{end,-z,\varphi2}$ = 42.08 kNm/m

$m_{end,-z,strut}$ = −38.23 kNm/m

Direction of the concrete compression strut: $\varphi_{strut,m,-z}$ = 75.0°

The following required reinforcement for the top surface is obtained from the design internal forces:

2.1 Required Reinforcement Total

	A	B	C	D	E	F	G	H	I	J
Surface No.	Grid Point	Point-Coordinates [m] X	Y	Z	Symbol	Required Reinforcement	Basic Reinforcement	Additional Reinforcement Required	Provided	Unit
1	G4	0.000	0.500	0.000	$a_{s,1,-z\,(top)}$	8.97	11.31	0.00	0.00	cm^2/m
1	G4	0.000	0.500	0.000	$a_{s,2,-z\,(top)}$	6.15	11.31	0.00	0.00	cm^2/m
1	G6	1.000	0.500	0.000	$a_{s,1,+z\,(bottom)}$	0.14	0.00	0.14	0.14	cm^2/m
1	G6	1.000	0.500	0.000	$a_{s,2,+z\,(bottom)}$	0.70	0.00	0.70	0.70	cm^2/m

Figure 2.83 Required reinforcement

2.6.4.4 Specification of a reinforcement

At the top surface of the plate, we select a reinforcement based on rebars with a diameter d_s of 12 mm at a distance l_s of 10.0 cm for both directions.

The following provided reinforcement is thus obtained:

$$\text{prov } a_{s1,-z} = \frac{d_s^{\,2}}{4} \cdot \pi \cdot \frac{100\text{ cm/m}}{l_s} = \frac{(1.2\text{ cm})^2}{4} \cdot \pi \cdot \frac{100\text{ cm/m}}{10.0\text{ cm}} = 11.31\text{ cm}^2/\text{m}$$

These values are entered in the *Longitudinal Reinforcement* tab of window 1.4 *Reinforcement* or selected with the [Rebars] button (see Figure 2.105).

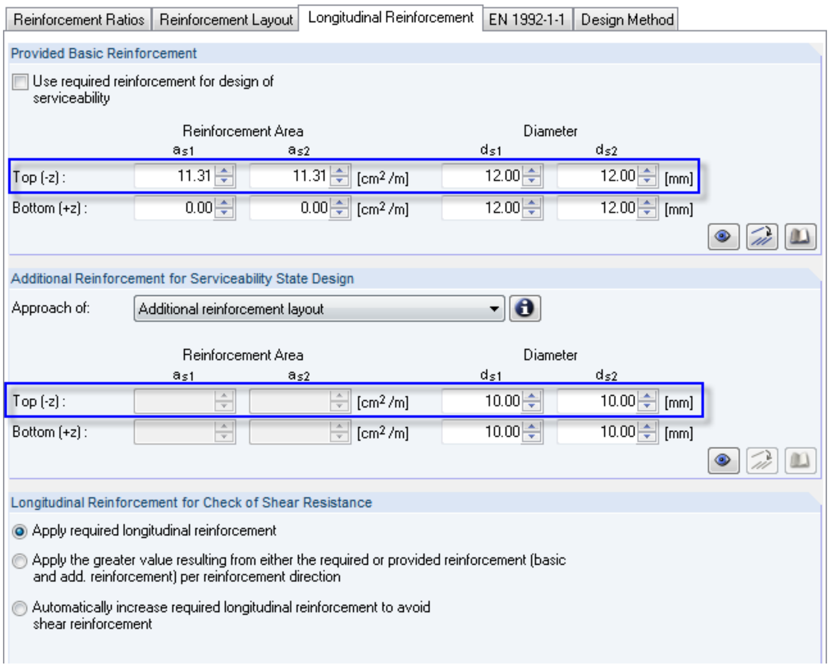


Figure 2.84 Window 1.4 *Reinforcement, Longitudinal Reinforcement* tab for entering basic and additional reinforcement

With this reinforcement diameter, the following centroids of the concrete cover are obtained:

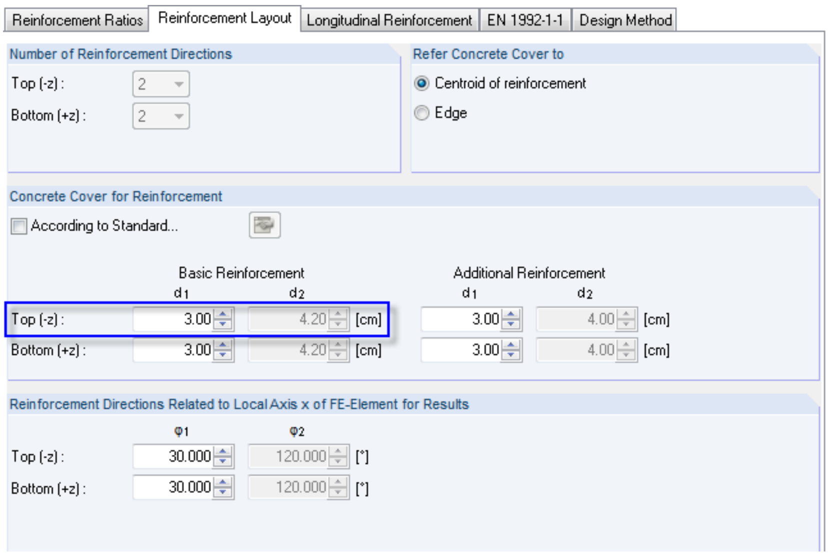


Figure 2.85 Window 1.4 *Reinforcement, Reinforcement Layout* tab

Thus, the effective depth for the individual reinforcement directions is determined as follows:

$$d_{1,-z} = h - d_1 = 20 - 3 = 17\,\text{cm}$$

$$d_{2,-z} = h - d_2 = 20 - 4.2 = 15.8\,\text{cm}$$

2.6.4.5 Check of provided reinforcement for SLS

First, the strain ratio $\varepsilon_{\varphi 2} / \varepsilon_{\varphi 1} = 1.0$ is assumed for the serviceability limit state. With it, the following values are determined:

⊞ Internal Forces of Linear Statics				
⊞ Check if internal forces cause cracks				
⊟ Check Existing Longitudinal Reinforcement				
⊟ Design Bending Moments				
⊞ Bottom surface (+z)				
⊟ Differential Angle According to Baumann				
1st Differential Angle	$\alpha_{m,+z}$	30.000	°	
2nd Differential Angle	$\beta_{m,+z}$	120.000	°	
⊟ Governing Strut				
First Assumption for Direction γ	$\gamma_{m,+z,1a}$	75.000	°	
Strut Direction	$\Phi_{strut,m,+z}$	75.000	°	
⊟ Governing Design Bending Moments				
into Direction 1	$m_{+z,\Phi 1}$	38.49	kNm/m	
into Direction 2	$m_{+z,\Phi 2}$	25.25	kNm/m	
into Strut Direction	$m_{end,+z,strut}$	-22.94	kNm/m	
⊟ Design Bending Moments by Baumann				
into Direction 1	$m_{\alpha,+z}$	38.49	kNm/m	
into Direction 2	$m_{\beta,+z}$	25.25	kNm/m	
into Strut Direction	$m_{\gamma,+z}$	-22.94	kNm/m	
Find optimal strut direction?	$Strut_{opti,m,+z}$	Yes		
⊟ Final Design Bending Moments				
into Direction 1	$m_{end,+z,\Phi 1}$	38.49	kNm/m	
into Direction 2	$m_{end,+z,\Phi 2}$	25.25	kNm/m	
into Strut Direction	$m_{end,+z,strut}$	-22.94	kNm/m	

Figure 2.86 Design moments in SLS for strain ratio of 1.0

Design internal forces: $m_{end,-z,\varphi 1} = 38.49$ kNm/m

$m_{end,-z,\varphi 2} = 25.25$ kNm/m

Stiffening compression moment: $m_{end,-z,strut} = -22.94$ kNm/m

Direction of stiffening compression moment: $\varphi_{strut,m,-z} = 75.0°$

For these design moments, a required reinforcement of $a_{s,dim,-z,1} = 4.33$ cm²/m in the first reinforcement direction and of $a_{s,dim,-z,2} = 3.04$ cm²/m in the second reinforcement direction is determined at the top surface of the plate.

⊞ Internal Forces of Linear Statics				
⊞ Check if internal forces cause cracks				
⊟ Check Existing Longitudinal Reinforcement				
⊞ Design Bending Moments				
⊟ Statically Required Reinforcement				
⊞ Bottom surface (+z)				
⊟ Top surface (-z)				
into Reinforcement Direction 1	$a_{s,dim,-z,1}$	4.33	cm²/m	
into Reinforcement Direction 2	$a_{s,dim,-z,2}$	3.04	cm²/m	
⊟ Existing Longitudinal Reinforcement				
⊞ Bottom surface (+z)				
⊟ Top surface (-z)				
into Reinforcement Direction 1	$a_{s,exist,-z,1}$	11.31	cm²/m	
into Reinforcement Direction 2	$a_{s,exist,-z,2}$	11.31	cm²/m	

Figure 2.87 Statically required reinforcement for internal forces in SLS

The required reinforcement for the internal forces of the serviceability limit state is smaller than the user-defined provided reinforcement. Hence, we can continue with the analysis.

2.6.4.6 Selection of concrete compression strut

With the design internal forces $m_{end,-z,\varphi 1}$ = 38.49 kNm/m and $m_{end,-z,\varphi 2}$ = 25.25 kNm/m, we obtain the strains $\varepsilon_{\varphi 1}$ = 0.735 ‰ in the first reinforcement direction and $\varepsilon_{\varphi 2}$ = 0.527 ‰ in the second reinforcement direction. Thus, there is a strain ratio $R_{s,-z}$ of 0.717.

The assumed strain ratio of 1.00 therefore does not correspond to the actual strain ratio. Hence, the inclination of the stiffening compression moment is increased from 75.0° to 79.746°.

Geometrically, this inclination of the stiffening compression moment can only appear if the geometric ratio $R_{s,geo,-z}$ of the strain in the reinforcement direction φ_2 to the strain in the reinforcement direction φ_1 is approximately 0.717. This is the case in our example.

When determining the crack width w_k, it is shown that with the design moments, strains result in the individual reinforcement directions for an inclination of the stiffening compression moment of 79.746°, which lead to the strain ratio $R_{s,geo,-z}$ of 0.717.

⊞ Internal Forces of Linear Statics			
⊞ Check if internal forces cause cracks			
⊞ Check Existing Longitudinal Reinforcement			
⊟ Design Internal Forces in Serviceability Limit State			
⊞ Bottom surface (+z)			
⊟ Top surface (-z)			
⊟ Concrete Strut Direction	$\gamma_{strut,exist,2,-z}$	79.746	°
Concrete Strut Direction acc. to Baumann	$\gamma_{strut,Bau,exist,2}$	79.746	°
Ratio of Reinforcement Deformation	$R_{s,-z}$	0.717	
Supposed Geometrical Ratio of Deformation	$R_{s,geo,-z}$	0.717	
⊟ Modified design Moments			
into Direction 1	$m_{exist,-z,\Phi 1}$	36.74	kNm/m
into Direction 2	$m_{exist,-z,\Phi 2}$	27.33	kNm/m
into Strut Direction	$m_{exist,strut,-z}$	-23.26	kNm/m
⊞ Check Steel Stress			
⊞ Check Final Crack Spacing			
⊟ Determination of difference in the mean strain			
⊞ Calculation Parameter for All Directions			
⊞ Bottom surface (+z)			
⊟ Top surface (-z)			
⊞ Difference in the mean strain into the direction 1	$(\varepsilon_{sm} - \varepsilon_{cm})_{-z,\Phi 1}$	0.735	‰
⊞ Steel Stress into Reinforcement Direction 1	$\sigma_{s,-z,\Phi 1}$	208.18	N/mm²
⊞ Difference in the mean strain into the direction 2	$(\varepsilon_{sm} - \varepsilon_{cm})_{-z,\Phi 2}$	0.527	‰
⊞ Steel Stress into Reinforcement Direction 2	$\sigma_{s,-z,\Phi 2}$	167.09	N/mm²
⊞ Resulting Difference in the Mean Strain	$(\varepsilon_{sm} - \varepsilon_{cm})_{-z,res}$	1.291	‰

Figure 2.88 Direction of the concrete compression strut and ratios of deformation

The selected inclination of the stiffening compression moment of 79.746° results in modified design moments in the individual reinforcement directions. This corresponds to the method for determining the design internal forces in the serviceability limit state that is used here, which takes into account the deformation ratio of the longitudinal reinforcement that was selected in the *Settings for Analytical Method of Serviceability Limit State Design* dialog box (see Figure 2.89).

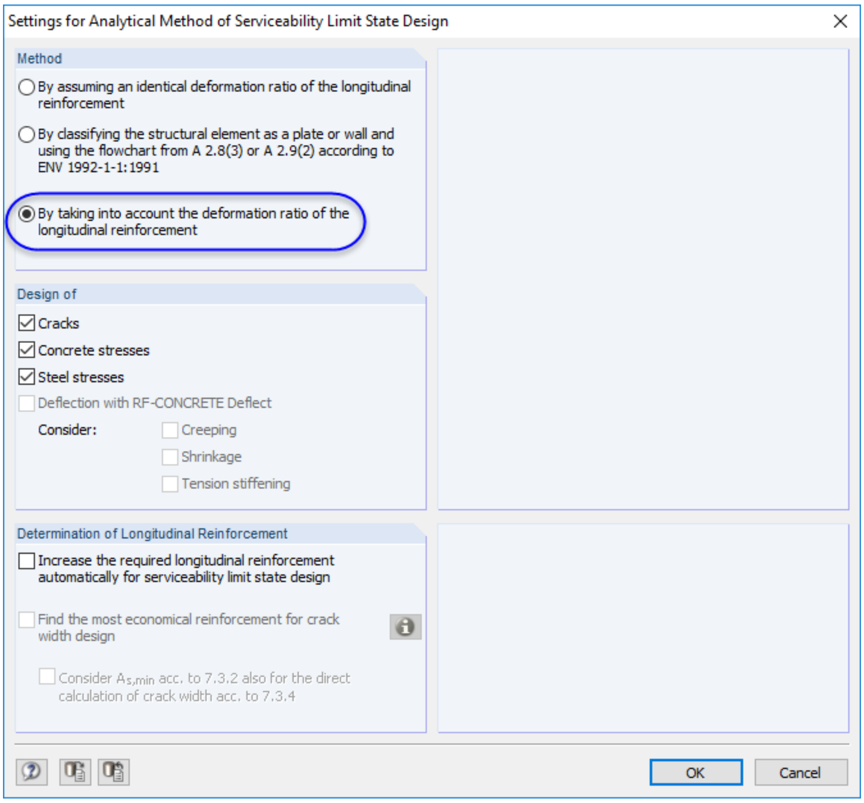


Figure 2.89 *Settings for Analytical Method of Serviceability Limit State Design* dialog box

2.6.4.7 Limitation of concrete compressive stress

In window 1.3 *Surfaces*, the concrete compressive stress is limited to $\sigma_{c,max} = 0.45 \cdot f_{ck}$ and the steel stress to $\sigma_{s,max} = 0.80 \cdot f_{yk}$.

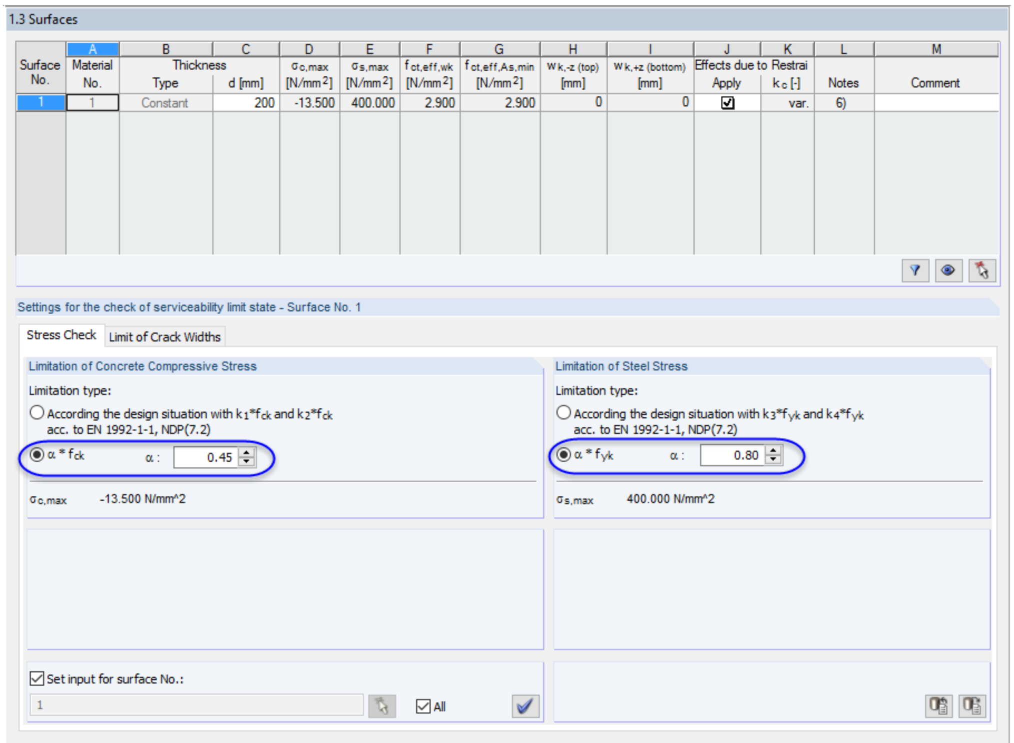


Figure 2.90 Limiting the concrete compressive and steel stresses in the *Stress Check* tab of window 1.3 *Surfaces*

For concrete C30/37, the maximum (negative) concrete stress $\sigma_{c,max}$ is thus determined:

$$\sigma_{c,max} = 0.45 \cdot f_{ck} = 0.45 \cdot (-30.0) = -13.5 \text{ N/mm}^2$$

The provided concrete compressive stress is determined under the assumption of a linear stress distribution because the multitude of iterations for determining the suitable direction of the concrete compression strut would be too time-consuming. A linear distribution is sufficiently accurate because in the serviceability state, there are normally concrete compressive strains of at most 0.3 to 0.5 ‰.

The maximum stress $\sigma_{c,max}$ is to be compared with the provided stress of the concrete compression zone for both reinforcement directions.

The provided concrete compressive stress σ_c is determined as follows:

$$\sigma_c = \frac{m_{Ed}}{I_{i,II}} \cdot x$$

Equation 2.69

where

m_{Ed}	applied moment
$I_{i,II} = \frac{1}{3} \cdot b \cdot x^3 + \alpha_e \cdot a_s \cdot (d - x)^2$	ideal moment of inertia in state II
b	width of element (always 1 m for plates)
α_E	ratio of elastic moduli
a_s	provided tension reinforcement
d	effective depth
$x = \frac{\alpha_E \cdot a_s}{b} \cdot \left(-1.0 + \sqrt{1.0 + \frac{2.0 \cdot b \cdot d}{\alpha_E \cdot a_s}}\right)$	depth of concrete neutral axis

For the reinforcement direction φ_1, the following neutral axis depth $x_{-z,\varphi 1}$ is thus obtained:

$$x_{-z,\phi_1} = \frac{6.061 \cdot 11.31}{100} \cdot \left(-1.0 + \sqrt{1.0 + \frac{2.0 \cdot 100 \cdot 17}{6.061 \cdot 11.31}}\right) = 4.19 \text{ cm}$$

The same value and the related intermediate values can also be found in the details table.

⊞ Internal Forces of Linear Statics			
⊞ Check if internal forces cause cracks			
⊞ Check Existing Longitudinal Reinforcement			
⊞ Design Internal Forces in Serviceability Limit State			
⊟ Determination of concrete compressive stress in particular reinforcement directions			
⊞ Bottom surface (+z)			
⊟ Top surface (-z)			
⊟ Concrete compressive stress in reinforcement direction 1	$\sigma_{c,+z,\Phi 1}$	-11.23	N/mm²
Design Moment in the reinforcement direction 1	$m_{d,-z,\varphi 1}$	36.74	kNm/m
⊟ Ideal Moment of Inertia	$I_{i,II,-z,\Phi 1}$	13701.00	cm⁴
Compression reinforcement from reinforcement of opposite surface is possible.			
Width of the Element	b_w	100.00	cm
⊞ Depth of the Concrete Compression Zone	$x_{-z,\Phi 1}$	4.19	cm
Ratio of Modulus of Elasticity	α_e	6.061	
Tension Reinforcement into Direction 1	$a_{s,exist,-z,1}$	11.31	cm²/m
Effective Depth	$d_{-z,\Phi 1}$	0.170	m
Depth of the Concrete Compression Zone	$x_{-z,\Phi 1}$	4.19	cm
Compression Reinforcement into Direction 1	$a_{s,exist,+z,1}$	0.14	cm²/m
Concrete Cover of Compression Reinforcement	$c_{-z,\Phi 1}$	0.030	m
Depth of the Concrete Compression Zone	$x_{-z,\Phi 1}$	4.19	cm
⊞ Concrete compressive stress in reinforcement direction 2	$\sigma_{c,+z,\Phi 2}$	-9.40	N/mm²

Figure 2.91 Depth of concrete compression zone for reinforcement direction 1

For the reinforcement direction φ_2, the neutral axis depth $x_{-z,\varphi 2}$ is obtained:

$$x_{-z,\phi_2} = \frac{6.061 \cdot 11.31}{100} \cdot \left(-1.0 + \sqrt{1.0 + \frac{2.0 \cdot 100 \cdot 15.8}{6.061 \cdot 11.31}}\right) = 4.02\,\text{cm}$$

This value and the related intermediate values can also be found in the details.

⊞ Internal Forces of Linear Statics			
⊞ Check if internal forces cause cracks			
⊞ Check Existing Longitudinal Reinforcement			
⊞ Design Internal Forces in Serviceability Limit State			
⊟ Determination of concrete compressive stress in particular reinforcement directions			
⊞ Bottom surface (+z)			
⊟ Top surface (-z)			
⊞ Concrete compressive stress in reinforcement direction 1	$\sigma_{c,+z,\Phi 1}$	-11.23	N/mm²
⊟ Concrete compressive stress in reinforcement direction 2	$\sigma_{c,+z,\Phi 2}$	-9.40	N/mm²
Design Moment in the reinforcement direction 2	$m_{d,-z,\varphi 2}$	27.33	kNm/m
⊟ Ideal Moment of Inertia	$I_{i,II,-z,\Phi 2}$	11677.20	cm⁴
Compression reinforcement from reinforcement of opposite surface is possible.			
Width of the Element	b_w	100.00	cm
⊟ Depth of the Concrete Compression Zone	$x_{-z,\Phi 2}$	4.02	cm
⊟ Ratio of Modulus of Elasticity	α_e	6.061	
Modulus of Elasticity of Reinforcement	E_s	200000.00	N/mm²
Mean Secant Modulus of Elasticity	E_{cm}	33000.00	N/mm²
Tension Reinforcement into Direction 2	$a_{s,exist,-z,2}$	11.31	cm²/m
Compression Reinforcement into Direction 2	$a_{s,exist,+z,2}$	0.70	cm²/m
Width of the Element	b_w	100.00	cm
Effective Depth	$d_{-z,\Phi 2}$	0.158	m
Concrete Cover of Compression Reinforcement	$c_{+z,\Phi 2}$	0.040	m
Ratio of Modulus of Elasticity	α_e	6.061	
Tension Reinforcement into Direction 2	$a_{s,exist,-z,2}$	11.31	cm²/m
Effective Depth	$d_{-z,\Phi 2}$	0.158	m
Depth of the Concrete Compression Zone	$x_{-z,\Phi 2}$	4.02	cm
Compression Reinforcement into Direction 2	$a_{s,exist,+z,2}$	0.70	cm²/m
Concrete Cover of Compression Reinforcement	$c_{-z,\Phi 2}$	0.040	m
Depth of the Concrete Compression Zone	$x_{-z,\Phi 2}$	4.02	cm

Figure 2.92 Depth of concrete compression zone for reinforcement direction 2

The ideal moments of inertia $I_{i,II}$ in state II (cracked section) are determined as follows for the two directions of reinforcement:

$$I_{i,II,-z,\phi_1} = \frac{1}{3} \cdot 100.0 \cdot 4.19^3 + 6.061 \cdot 11.31 \cdot (17 - 4.19)^2 = 13701\,\text{cm}^4$$

$$I_{i,II,-z,\phi_2} = \frac{1}{3} \cdot 100.0 \cdot 4.02^3 + 6.061 \cdot 11.31 \cdot (15.8 - 4.02)^2 = 11\,678\,\text{cm}^4$$

Thus, according to Equation 2.69, the following concrete compressive stresses σ_c are obtained in the concrete compression zone (i.e. at the top side of the surface) for the two reinforcement directions φ_1 and φ_2:

$$\sigma_{c,o,\phi_1} = \frac{3\,676 \cdot 4.19}{13\,701} = -11.24\,\text{N/mm}^2$$

$$\sigma_{c,o,\phi_2} = \frac{2\,773 \cdot 4.02}{11\,678} = -9.41\,\text{N/mm}^2$$

These values are also shown in Figure 2.92 (the program takes more decimal places into account).

The existing compressive stresses $\sigma_{c,+z,\varphi1}$ and $\sigma_{c,+z,\varphi2}$ are therefore smaller than the maximum concrete stress $\sigma_{c,max}$ (see Figure 2.90). The governing quotient of existing and allowable concrete compressive stress is available in the reinforcement direction φ_1. The design is fulfilled.

Maximum Concrete Compressive Stress	max σc	-11.23	N/mm²	
⊞ Allowable Concrete Compressive Stress				
⊟ Check				
Maximum Concrete Compressive Stress	max σc	-11.23	N/mm²	
Allowable Concrete Compressive Stress	perm σc	-13.50	N/mm²	
Criterion of Check	Criterion	0.832		

Figure 2.93 Analysis of concrete compressive stress

2.6.4.8 Limitation of reinforcing steel stress

In window 1.3 *Surfaces*, the tension stresses of the reinforcing steel reinforcement are limited to $\sigma_{s,max} = 0.8 \cdot f_{yk}$ according to EN 1992-1-1, clause 7.2(5) (see Figure 2.90). For BSt 500 S (B), the maximum steel stress $\sigma_{s,max}$ is thus determined as:

$$\sigma_{s,max} = 0.8 \cdot f_{yk} = 0.8 \cdot 500 = 400\,\text{N/mm}^2$$

The maximum stress $\sigma_{s,max}$ is to be compared with the provided tension stress for both reinforcement directions.

The provided tension stress σ_s is determined as follows:

$$\sigma_s = \frac{\alpha_E \cdot m_{Ed} \cdot (d - x)}{I_{i,II}}$$

Equation 2.70

where

α_E	relation of elastic moduli (E_s / E_{cm})
m_{Ed}	applied moment
d	effective depth
$x = \frac{\alpha_E \cdot a_s}{b} \cdot \left(-1.0 + \sqrt{1.0 + \frac{2.0 \cdot b \cdot d}{\alpha_E \cdot \alpha_s}}\right)$	depth of concrete neutral axis
b	width of element (always 1 m for plates)

a_s provided tension reinforcement

$$I_{i,II} = \frac{1}{3} \cdot b \cdot x^3 + \alpha_e \cdot \alpha_s \cdot (d - x)^2$$

ideal moment of inertia in state II

With the values calculated in chapter 2.6.4.6, the provided tension stresses $\sigma_{s,u,\varphi1}$ and $\sigma_{s,u,\varphi2}$ in the two reinforcement directions φ_1 and φ_2 can be determined as follows:

$$\sigma_{s,u,\phi_1} = \frac{6.061 \cdot 3\,674 \cdot (17 - 4.19)}{13\,701} = 208.18\ \mathrm{N/mm^2}$$

$$\sigma_{s,u,\phi_2} = \frac{6.061 \cdot 2\,733 \cdot (15.8 - 4.02)}{11\,677} = 167.09\ \mathrm{N/mm^2}$$

⊞ Internal Forces of Linear Statics			
⊞ Check if internal forces cause cracks			
⊞ Check Existing Longitudinal Reinforcement			
⊞ Design Internal Forces in Serviceability Limit State			
⊟ Check Steel Stress			
⊞ Bottom surface (+z)			
⊟ Top surface (-z)			
Concrete cracks. Longitudinal reinforcement is activated.			
⊟ Steel Stress into Reinforcement Direction 1	$\sigma_{s,-z,\Phi1}$	208.18	N/mm²
Design Moment in the reinforcement direction 1	$m_{d,-z,\varphi1}$	36.74	kNm/m
⊞ Ratio of Modulus of Elasticity	α_e	6.061	
Mean Secant Modulus of Elasticity	E_{cm}	33000.00	N/mm²
⊞ Ideal Moment of Inertia	$I_{i,II,-z,\Phi1}$	13701.00	cm⁴
Depth of the Concrete Compression Zone	$x_{-z,\Phi1}$	4.19	cm
⊟ Steel Stress into Reinforcement Direction 2	$\sigma_{s,-z,\Phi2}$	167.09	N/mm²
Design Moment in the reinforcement direction 2	$m_{d,-z,\varphi2}$	27.33	kNm/m
⊞ Ratio of Modulus of Elasticity	α_e	6.061	
Mean Secant Modulus of Elasticity	E_{cm}	33000.00	N/mm²
⊞ Ideal Moment of Inertia	$I_{i,II,-z,\Phi2}$	11677.20	cm⁴
Depth of the Concrete Compression Zone	$x_{-z,\Phi2}$	4.02	cm
Maximum Steel Stress	max σ_s	208.18	N/mm²

Figure 2.94 Maximum steel stresses in reinforcement direction 1 and 2

The existing tension stresses $\sigma_{s,-z,\varphi1}$ and $\sigma_{s,-z,\varphi2}$ are therefore smaller than the maximum steel stress $\sigma_{s,max}$ (see Figure 2.90). The governing quotient of existing to allowable steel stress is available in the reinforcement direction φ_1. The design is fulfilled.

⊞ Design Internal Forces in Serviceability Limit State			
⊞ Check Steel Stress			
⊞ Allowable Steel Stress			
⊟ Check			
Maximum Steel Stress	max σ_s	208.18	N/mm²
Allowable Steel Stress	perm σ_s	400.00	N/mm²
Criterion of Check	Criterion	0.520	

Figure 2.95 Analysis of reinforcing steel stress

2.6.4.9 Minimum reinforcement for crack control

The minimum reinforcement area for crack control is determined according to EN 1992-1-1, clause 7.3.2, Equation (7.1).

$$a_{s,min} = \frac{k_c \cdot k \cdot f_{ct,eff} \cdot A_{ct}}{\sigma_s}$$

Equation 2.71

where

k_c	coefficient for considering the influence of the stress distribution in the cross-section prior to cracking as well as the change of the internal lever arm
k	coefficient for considering non-uniform self-equilibrating stresses, which lead to the reduction of restraint forces
$f_{ct,eff}$	mean value of the effective tensile strength of the concrete, to be expected when the cracks occur
A_{ct}	area of the concrete tension zone (part of the cross-section or partial cross-section that is calculated to be in tension in the uncracked state under the action combination that leads to the formation of the first crack at the gross cross-section)
σ_s	absolute value of the maximum allowable stress in the reinforcement immediately after crack formation

The maximum bar diameter d^*_s is determined according to EN 1992-1-1, clause 7.3.3 (2) depending on the actually provided diameter d_s from the rearranged Equation (7.6N).

$$d_s = d_s^* \cdot \frac{f_{ct,\,eff}}{2.9} \cdot \frac{k_c \cdot h_{cr}}{2 \cdot (h - d)}$$

Equation 2.72

where

d_s	adjusted maximum bar diameter
d^*_s	maximum bar diameter according to EN 1992-1-1, Table 7.2 (see Figure 2.96)
h	overall depth of cross-section
h_{cr}	depth of tensile zone immediately prior to cracking while taking the characteristic values of prestress and axial forces under the quasi-permanent action combination into account
d	effective depth up to centroid of outside reinforcement

Table 7.2N Maximum bar diameters ϕ^*_s for crack control[1]

Steel stress[2] [MPa]	Maximum bar size [mm] w_k= 0,4 mm	w_k= 0,3 mm	w_k= 0,2 mm
160	40	32	25
200	32	25	16
240	20	16	12
280	16	12	8
320	12	10	6
360	10	8	5
400	8	6	4
450	6	5	-

Figure 2.96 Maximum bar diameter of reinforcing bars according to EN 1992-1-1, clause 7.3.3

In our example, determining the minimum reinforcement at the plate's bottom surface is excluded by opening the following dialog box in the *Limit of Crack Width* tab of window 1.3 *Surfaces*. In it, the check boxes for the *Bottom (+z) reinforcement* have to be cleared.

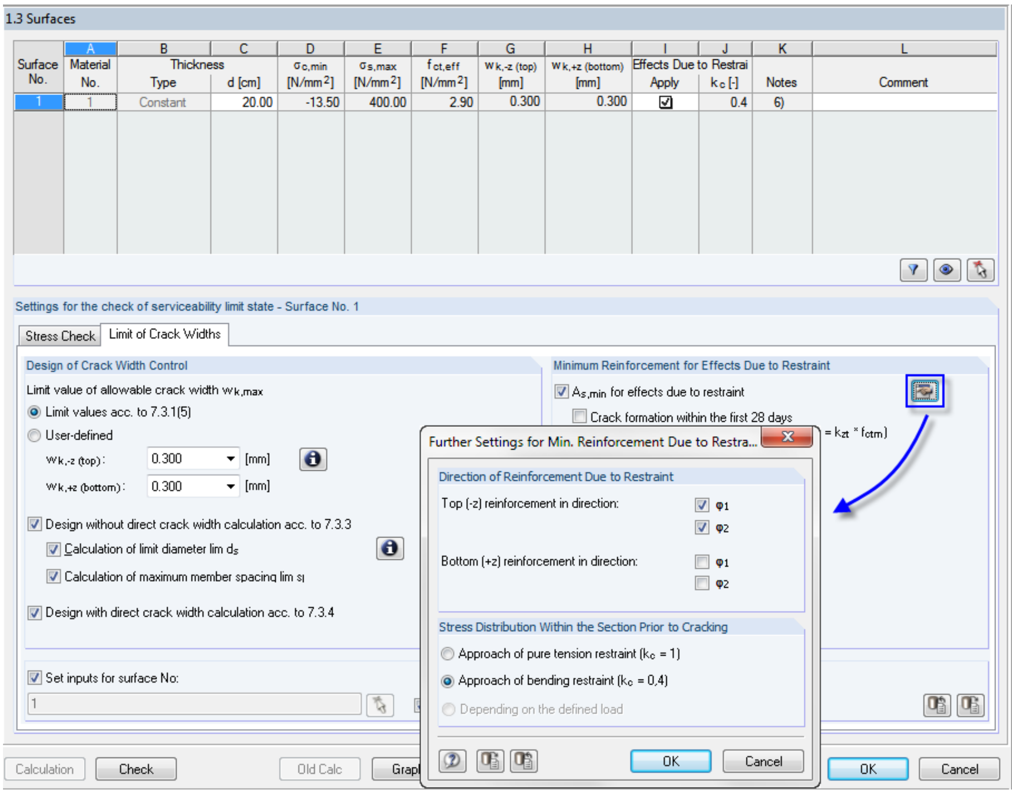


Figure 2.97 *Further Settings for Min. Reinforcement Due to Restraint* dialog box in window 1.3 *Surfaces*

The limit diameter $d^*_{s,-z,\varphi 1}$ for the reinforcement direction φ_1 at the top side of the plate is determined according to Equation 2.72.

$$d^*_{s,-z,\phi_1} = 12 \cdot \frac{2.9}{2.9} \cdot \frac{2 \cdot (200 - 170)}{0.4 \cdot 100} = 18 \text{ mm}$$

⊞ Internal Forces of Linear Statics				
⊞ Check if internal forces cause cracks				
⊞ Check Existing Longitudinal Reinforcement				
⊞ Design Internal Forces in Serviceability Limit State				
⊟ Determination of minimum reinforcement				
⊟ Calculation Parameter for All Directions				
Maximum allowable crack width at the bottom (+z) surface	$w_{k,-z,limit}$	0.300	mm	
Maximum allowable crack width at the top (-z) surface ac	$w_{k,-z,limit}$	0.300	mm	
Factor for Unevenly Distributed Tensile Stress	k	1.000		
Basic tensile strength of concrete for Table 7.2	$f_{ct,0}$	2.90	N/mm^2	
⊞ Axial Tensile Strength of Concrete	$f_{ct,eff}$	2.90	N/mm^2	
⊟ Area of the Tension Zone	A_{ct}	1000.00	cm^2	
Width of the Element	b_w	1.000	m	
Depth of Structural Member	h	0.200	m	
⊞ Bottom surface (+z)				
⊟ Top surface (-z)				
⊟ Minimum Reinforcement into Direction 1	$a_{s,min,-z,1}$	5.02	cm^2/m	
⊟ Factor of Stress Distribution Prior to Initial Crack Format	k_c	0.400		
⊞ Governing Design Forces				
Factor for Unevenly Distributed Tensile Stress	k	1.000		
Axial Tensile Strength of Concrete	$f_{ct,eff}$	2.90	N/mm^2	
⊞ Area of the Tension Zone	$A_{ct,-z,\varphi 1}$	1000.00	cm^2	
⊟ Allowable stress in the reinforcement acc. to Table 7.2	perm $\sigma_{s,-z,\Phi 1}$	231.11	N/mm^2	
⊟ Limit Diameter of Reinforcement	$d_{s,-z,\Phi 1}^*$	1.80	cm	
⊟ First Calculated Value of Limit Diameter	$d_{s,calc1}^*$	1.80	cm	
Existing Diameter of Reinforcement	$d_{s,min,-z,\varphi 1}$	1.20	cm	
Effective Depth	$d_{-z,\Phi 1}$	0.170	m	
⊞ Minimum Reinforcement into Direction 2	$a_{s,min,-z,2}$	5.83	cm^2/m	

Figure 2.98 Limit diameter for reinforcement direction φ_1

Analogously, the limit bar diameter $d^*_{s,-z,\varphi 2}$ is obtained for the reinforcement direction φ_2:

$$d^*_{s,-z,\phi_2} = 12 \cdot \frac{2.9}{2.9} \cdot \frac{2 \cdot (200 - 158)}{0.4 \cdot 100} = 25.20 \text{ mm}$$

⊞ Internal Forces of Linear Statics				
⊞ Check if internal forces cause cracks				
⊞ Check Existing Longitudinal Reinforcement				
⊞ Design Internal Forces in Serviceability Limit State				
⊟ Determination of minimum reinforcement				
⊞ Calculation Parameter for All Directions				
⊞ Bottom surface (+z)				
⊟ Top surface (-z)				
⊞ Minimum Reinforcement into Direction 1	$a_{s,min,-z,1}$	5.02	cm^2/m	
⊟ Minimum Reinforcement into Direction 2	$a_{s,min,-z,2}$	5.83	cm^2/m	
⊟ Factor of Stress Distribution Prior to Initial Crack Format	k_c	0.400		
⊞ Governing Design Forces				
Factor for Unevenly Distributed Tensile Stress	k	1.000		
Axial Tensile Strength of Concrete	$f_{ct,eff}$	2.90	N/mm^2	
⊞ Area of the Tension Zone	$A_{ct,-z,\varphi 2}$	1000.00	cm^2	
⊟ Allowable stress in the reinforcement acc. to Table 7.2	perm $\sigma_{s,-z,\Phi 2}$	198.86	N/mm^2	
⊟ Limit Diameter of Reinforcement	$d_{s,-z,\Phi 2}^*$	2.52	cm	
⊟ First Calculated Value of Limit Diameter	$d_{s,calc1}^*$	2.52	cm	
Existing Diameter of Reinforcement	$d_{s,min,-z,\varphi 2}$	1.20	cm	
Effective Depth	$d_{-z,\Phi 2}$	0.158	m	

Figure 2.99 Limit diameter for reinforcement direction φ_2

In window 1.3 *Surfaces*, the allowable crack width $w_{k,max}$ is given as 0.3 mm (see Figure 2.97). With the maximum bar diameters $d^*_{s,-z,\varphi 1}$ = 18.00 mm and $d^*_{s,-z,\varphi 2}$ = 25.20 mm, we can interpolate the allowable stress σ_s from EN 1992-1-1, Table 7.2N (see Figure 2.96).

$$\sigma_{s,-z,\phi_1} = 240 + \frac{280 - 240}{16 - 25} \cdot (18.00 - 16) = 231.11 \text{ N/mm}^2$$

$$\sigma_{s,-z,\phi_2} = 200 + \frac{200 - 160}{25 - 32} \cdot (25.20 - 25) = 198.86 \text{ N/mm}^2$$

These allowable steel stresses are also shown in Figure 2.98 and Figure 2.99.

The steel stress in the direction φ_2 is governing.

The area of the concrete tension zone in the cross-section is determined as follows:

$$A_{ct} = b \cdot \frac{h}{2} = 100 \cdot \frac{20}{2} = 1\,000\ \text{cm}^2$$

Thus, according to Equation 2.71, the following minimum reinforcement for reinforcement direction 2 is obtained:

$$a_{s,min,\phi_2} = \frac{0.4 \cdot 1.0 \cdot 2.9 \cdot 1\,000}{198.86} = 5.83\ \text{cm}^2/m$$

For this reinforcement direction, the applied reinforcement is greater than the minimum reinforcement. The following check criterion is thus obtained:

$$\frac{a_{s,min,-z,2}}{a_{s,exist,-z,2}} = \frac{5.83}{11.31} = 0.516$$

⊞ Design Internal Forces in Serviceability Limit State			
⊞ Determination of minimum reinforcement			
⊞ Existing Longitudinal Reinforcement			
⊟ Check			
Minimum Reinforcement at the Top (-z) Surface in Direction 2	$a_{s,min,-z,2}$	5.83	cm^2/m
Existing Reinforcement at the Top (-z) Surface in Direction 2	$a_{s,exist,-z,2}$	11.31	cm^2/m
Criterion of Check	Criterion	0.516	

Figure 2.100 Check criterion for minimum reinforcement

2.6.4.10 Check of rebar diameter

The rebars' limit diameter max d_s is checked according to EN 1992-1-1, Equation (7.6N) (see Equation 2.72).

At the top side of the plate, the program determines the maximum bar diameter $d^*_{s,-z,\varphi 1}$ of the first reinforcement direction depending on the stress available in this direction. In the check for the limitation of the steel stress, this stress was calculated as $\sigma^*_{s,-z,\varphi 1} = 208.18\ N/mm^2$. Together with the selected crack width $w_k = 0.3$ mm, the following limit diameter $d^*_{s,-z,\varphi 1}$ is obtained in Table 7.2N by interpolation:

$$d^*_{s,-z,\phi_1} = 25 + \frac{25 - 16}{200 - 240} \cdot (208.18 - 200) = 23.16\ \text{mm}$$

⊞ Internal Forces of Linear Statics			
⊞ Check if internal forces cause cracks			
⊞ Check Existing Longitudinal Reinforcement			
⊞ Design Internal Forces in Serviceability Limit State			
⊟ Determination of the maximum steelbar diameter			
⊟ Calculation Parameter for All Directions			
Maximum allowable crack width at the bottom (+z) surface	$w_{k,-z,limit}$	0.300	mm
Maximum allowable crack width at the top (-z) surface ac	$w_{k,-z,limit}$	0.300	mm
⊟ Axial Tensile Strength of Concrete	$f_{ct,eff}$	2.90	N/mm^2
Mean Axial Tensile Strength	f_{ctm}	2.90	N/mm^2
Depth of Structural Member	h	0.200	m
Reference tensile strength acc. to Table 7.2	$f_{ct,0}$	3.00	N/mm^2
Width of the Element	b_w	100.00	cm
⊞ Bottom surface (+z)			
⊟ Top surface (-z)			
⊟ Maximum Steelbar Diameter into Reinforcement Direction	$d_{s,max,-z,\varphi 1}$	1.54	cm
⊟ Limit Diameter of Reinforcement	$d_{s,-z,\Phi 1}^*$	2.32	cm
Steel Stress into Reinforcement Direction 1	$\sigma_{s,-z,\Phi 1}$	208.18	N/mm^2
⊟ First Calculated Value of Maximum Steelbar Diameter	$d_{s,max,calc1}$	1.54	cm
Effective Depth	$d_{-z,\Phi 1}$	0.170	m
Factor of Stress Distribution Prior to Initial Crack Form	k_c	0.400	
Depth of Tension Zone	h_{cr}	0.100	m
Axial Tensile Strength of Concrete	$f_{ct,eff}$	2.90	N/mm^2

Figure 2.101 Limit diameter in reinforcement direction 1

The limit diameter $d^*_{s,\text{-}z,\varphi 2}$ for reinforcement direction 2 is analogously determined from the tension stress $\sigma^*_{s,\text{-}z,\varphi 2} = 167.09\ \text{N/mm}^2$ and the crack width $w_k = 0.3$ mm:

$$d^*_{s,-z,\phi_2} = 32 + \frac{32 - 25}{160 - 200} \cdot (167.09 - 160) = 30.76\ \text{mm}$$

⊞ Internal Forces of Linear Statics			
⊞ Check if internal forces cause cracks			
⊞ Check Existing Longitudinal Reinforcement			
⊞ Design Internal Forces in Serviceability Limit State			
⊟ Determination of the maximum steelbar diameter			
⊞ Calculation Parameter for All Directions			
⊞ Bottom surface (+z)			
⊟ Top surface (-z)			
⊞ Maximum Steelbar Diameter into Reinforcement Direction 1	$d_{s,max,\text{-}z,\varphi 1}$	1.54	cm
⊟ Maximum Steelbar Diameter into Reinforcement Direction 2	$d_{s,max,\text{-}z,\varphi 2}$	1.46	cm
⊟ Limit Diameter of Reinforcement	$d_{s,\text{-}z,\Phi 2}^*$	3.08	cm
Steel Stress into Reinforcement Direction 2	$\sigma_{s,\text{-}z,\Phi 2}$	167.09	N/mm²
⊟ First Calculated Value of Maximum Steelbar Diameter	$d_{s,max,calc1}$	1.46	cm
Existing Longitudinal Reinforcement	$a_{s,exist,\text{-}z,2}$	0.158	m
Effective Depth	$d_{\text{-}z,\Phi 2}$	0.400	
Factor of Stress Distribution Prior to Initial Crack Formation	k_c	0.100	m
Factor for Unevenly Distributed Tensile Stress	k	2.90	N/mm²

Figure 2.102 Limit diameter in reinforcement direction 2

With the limit diameters d^*_s for the two reinforcement directions and the respective steel stresses, the maximum bar diameters d_s are determined.

$$d_{s,max,-z,\phi_1} = 23.16 \cdot \frac{2.9}{2.9} \cdot \frac{0.4 \cdot 100}{2 \cdot (200 - 170)} = 15.44\ \text{mm}$$

$$d_{s,max,-z,\phi_2} = 30.76 \cdot \frac{2.9}{2.9} \cdot \frac{0.4 \cdot 100}{2 \cdot (200 - 158)} = 14.65\ \text{mm}$$

⊞ Internal Forces of Linear Statics			
⊞ Check if internal forces cause cracks			
⊞ Check Existing Longitudinal Reinforcement			
⊞ Design Internal Forces in Serviceability Limit State			
⊟ Determination of the maximum steelbar diameter			
⊞ Calculation Parameter for All Directions			
⊞ Bottom surface (+z)			
⊟ Top surface (-z)			
⊟ Maximum Steelbar Diameter into Reinforcement Direction 1	$d_{s,max,\text{-}z,\varphi 1}$	1.54	cm
⊞ Limit Diameter of Reinforcement	$d_{s,\text{-}z,\Phi 1}^*$	2.32	cm
⊞ First Calculated Value of Maximum Steelbar Diameter	$d_{s,max,calc1}$	1.54	cm
Axial Tensile Strength of Concrete	$f_{ct,eff}$	2.90	N/mm²
⊟ Maximum Steelbar Diameter into Reinforcement Direction 2	$d_{s,max,\text{-}z,\varphi 2}$	1.46	cm
⊞ Limit Diameter of Reinforcement	$d_{s,\text{-}z,\Phi 2}^*$	3.08	cm
⊞ First Calculated Value of Maximum Steelbar Diameter	$d_{s,max,calc1}$	1.46	cm
Axial Tensile Strength of Concrete	$f_{ct,eff}$	2.90	N/mm²

Figure 2.103 Maximum bar diameters

Rebar diameters $d_s = 12$ mm are respectively provided for both reinforcement directions. Thus, the check criterion for the governing reinforcement direction φ_1 is obtained as:

$$\frac{d_{s,\text{exist},-z,\phi_2}}{\max d_{s,-z,\phi_2}} = \frac{12.0}{14.65} = 0.819$$

⊞ Design Internal Forces in Serviceability Limit State			
⊞ Determination of the maximum steelbar diameter			
⊞ Existing Steelbar Diameter			
⊟ Check			
Existing steelbar diameter at the top of the surface in direction 2	$d_{s,exist,\text{-}z,\varphi 2}$	1.20	cm
Maximum Steelbar Diameter at the Top Surface into Direction 2	$d_{s,max,\text{-}z,\varphi 2}$	1.46	cm
Criterion of Check	Criterion	0.819	

Figure 2.104 Check criterion for bar diameter

2.6.4.11 Design of bar spacing

In the *Longitudinal Reinforcement* tab of window 1.4 *Reinforcement*, a bar spacing of a = 100 mm has been specified for both reinforcement directions by using the [Rebars] button.

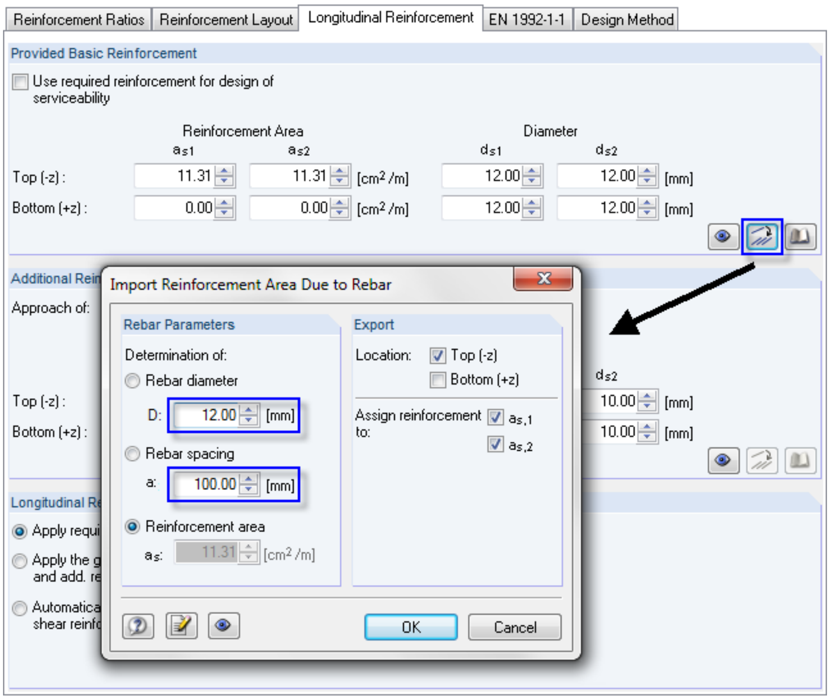


Figure 2.105 *Import Reinforcement Area Due to Rebar* dialog box

The maximum bar spacing max $s_{l,-z,\varphi 1}$ is determined by interpolation according to EN 1992-1-1, Table 7.3N for the existing tension stress $\sigma_{s,-z,\varphi 1}$ = 208.18 N/mm² and the crack width w_k = 0.3 mm.

Table 7.3N Maximum bar spacing for crack control[1]

Steel stress[2] [MPa]	Maximum bar spacing [mm]		
	w_k=0,4 mm	w_k=0,3 mm	w_k=0,2 mm
160	300	300	200
200	300	250	150
240	250	200	100
280	200	150	50
320	150	100	-
360	100	50	-

Figure 2.106 Maximum values of bar spacings according to EN 1992-1-1, Table 7.3N

$$\max s_{l,-z,\phi_1} = 250 + \frac{250 - 200}{200 - 240} \cdot (208.18 - 200) = 139.8\,\text{mm}$$

Analogously, the maximum bar spacing for the direction φ_2 is determined from the existing tension stress $\sigma_{s,-z,\varphi 2}$ = 167.09 N/mm² as max $s_{l,-z,\varphi 2}$ = 291.1 mm.

⊞ Internal Forces of Linear Statics			
⊞ Check if internal forces cause cracks			
⊞ Check Existing Longitudinal Reinforcement			
⊞ Design Internal Forces in Serviceability Limit State			
⊟ Determination of maximum steelbar distance			
⊟ Calculation Parameter for All Directions			
Maximum allowable crack width at the bottom (+z) surface acc	$w_{k,-z,limit}$	0.300	mm
Maximum allowable crack width at the top (-z) surface acc. to (	$w_{k,-z,limit}$	0.300	mm
⊞ Bottom surface (+z)			
⊟ Top surface (-z)			
⊟ Maximum Steelbar Distance in Direction 1	$s_{l,max,-z,\Phi 1}$	0.240	m
Steel Stress into Reinforcement Direction 1	$\sigma_{s,-z,\Phi 1}$	208.18	N/mm²
⊟ Maximum Steelbar Distance in Direction 2	$s_{l,max,-z,\Phi 2}$	0.291	m
Steel Stress into Reinforcement Direction 2	$\sigma_{s,-z,\Phi 2}$	167.09	N/mm²

Figure 2.107 Maximum bar spacings in both reinforcement directions

The existing bar spacing $s_{l,exist,-z,\varphi 1}$ = 100 mm available for the reinforcement direction φ_1 is smaller than the maximum allowable bar spacing max $s_{l,max,-z,\varphi 1}$ = 240 mm.

Therefore, the following check criterion applies for reinforcement direction φ_1:

$$\frac{s_{l,\ \mathrm{exist},\ -z,\phi_1}}{\max s_{l,\max,\ -z,\phi_1}} = \frac{0.100}{0.240} = 0.417$$

⊞ Design Internal Forces in Serviceability Limit State			
⊞ Determination of maximum steelbar distance			
⊞ Existing Steelbar Distance			
⊟ Check			
Existing steelbar spacing at the top (-z) surface in direction 1	$s_{l,exist,-z,\Phi 1}$	0.100	m
Maximum steelbar spacing at the top (-z) surface in direction 1	$s_{l,max,-z,\Phi 1}$	0.240	m
Criterion of Check	Criterion	0.417	

Figure 2.108 Check criterion for bar spacing

2.6.4.12 Check of crack width

The calculation value w_k of the crack width is determined according to Equation (7.8) of EN 1992-1-1, clause 7.3.4.

$$w_k = s_{r,\max} \cdot (\varepsilon_{sm} - \varepsilon_{cm})$$

Equation 2.73

where

$s_{r,max}$ maximum crack spacing in final crack state (see Equation 2.74 or Equation 2.75)

ε_{sm} mean strain of the reinforcement under governing action combination, including the effects of applied deformations and taking the concrete's effect of tension between the cracks into account (only the additional concrete tensile strain beyond the zero strain at the same level is considered)

ε_{cm} mean strain of concrete between cracks

Maximum crack spacing $s_{r,max}$

If the spacing of the rebars in the bonded reinforcement is not larger than 5 · (c + φ/2) in the tension zone, the maximum crack spacing for the final crack state may be determined according to EN 1992-1-1, Equation (7.11):

$$s_{r,max} = k_3 \cdot c + k_1 \cdot k_2 \cdot k_4 \cdot \frac{\phi}{\rho_{p,eff}}$$

Equation 2.74

If the spacing of the rebars in the bonded reinforcement exceeds 5 · (c + φ/2) in the tension zone or if no bonded reinforcement is available within the tension zone, the limit for the crack width may be determined with the following maximum crack spacing:

$$s_{r,max} = 1.3 \cdot (h - x)$$

Equation 2.75

The depth of the compression zone x in state II therefore has to be calculated for the check of the crack width. It is determined with the neutral axis depth ξ that is related to the depth of the structural element.

$$x = \xi \cdot h = \frac{0.5 + \alpha_e \cdot \frac{a_{s,exist}}{b \cdot h} \cdot \frac{d}{h}}{1.0 + \alpha_e \cdot \frac{a_{s,exist}}{b \cdot h}}$$

Equation 2.76

⊞ Internal Forces of Linear Statics				
⊞ Check if internal forces cause cracks				
⊞ Check Existing Longitudinal Reinforcement				
⊞ Design Internal Forces in Serviceability Limit State				
⊞ Check Steel Stress				
⊟ Check Final Crack Spacing				
⊞ Bottom surface (+z)				
⊟ Top surface (-z)				
⊟ Maximum Crack Spacing into Reinforcement Direction 1	$s_{r,max,-z,\Phi 1}$	0.177	m	
⊟ Limiting Spacing of Bonded Reinforcement	$s_{l,limit,-z,\Phi 1}$	0.150	m	
Concrete Cover	$c_{-z,\Phi 1}$	2.40	cm	
Existing Steelbar Diameter	$d_{s,exist,-z,\varphi 1}$	0.012	m	
⊟ Existing Steelbar Distance	$s_{l,exist,-z,\varphi 1}$	0.100	m	
Existing Reinforcement in Direction 1	$a_{s,max,-z,1}$	11.31	cm^2/m	
Existing Steelbar Diameter	$d_{s,max,-z,\varphi 1}$	0.012	m	
⊟ Existing bar spacing is not larger than limiting spacing => Formula (7.11)				
Concrete Cover	$c_{-z,\Phi 1}$	2.40	cm	
Existing Steelbar Diameter	$d_{s,max,-z,\varphi 1}$	0.012	m	
Coefficient for consideration of bond properties	k_1	0.800		
Coefficient for consideration of the strain distribution	k_2	0.500		
Parameter in National Annex	k_3	3.400		
Parameter in National Annex	k_4	0.425		
⊟ Effective Reinforcement Ratio	$\rho_{eff,-z,\Phi 1}$	0.021		
Existing Reinforcement in Direction 1	$a_{s,max,-z,1}$	11.31	cm^2/m	
⊟ Concrete area in which the reinforcement is effect	$a_{c,eff,-z,\Phi 1}$	527.05	cm^2	
Width of the Element	b_w	100.00	cm	
⊟ Depth of the area in which the reinforcement is	$h_{eff,-z,\Phi 1}$	0.053	m	
⊟ First calculated depth of the area in which th	$h_{eff,calc1}$	0.075	m	
Concrete Cover to Rebar Centroid	$d'_{-z,\Phi 1}$	3.00	cm	
⊟ Second calculated depth of the area in whic	$h_{eff,calc2}$	0.053	m	
Depth of Structural Member	h	0.200	m	
⊞ Maximum Crack Spacing into Reinforcement Direction 2	$s_{r,max,-z,\Phi 2}$	0.218	m	
⊟ Maximum Final Crack Spacing acc. (7.15)	$s_{r,max,-z,res}$	0.137	m	
Angle Between Reinforcement Direction and Crack Op	Φ_{-z}	40.254	°	

Figure 2.109 Maximum crack spacing in reinforcement direction 1

Furthermore, the maximum crack spacing is analyzed according to EN 1992-1-1, Equation (7.15):

$$s_{r,max} = \frac{1}{\dfrac{\cos\theta}{s_{r,max,x}} + \dfrac{\sin\theta}{s_{r,max,y}}}$$

Equation 2.77

where

θ	angle between reinforcement in x-direction and direction of principal tension stress
$s_{r,max,x}$ $s_{r,max,y}$	maximum crack spacing in x- or y-direction

This equation is important if the first method, *By assuming an identical deformation ratio of the longitudinal reinforcement* for determining the design internal forces in the serviceability limit state, has been selected in the *Settings for Analytical Method of Serviceability Limit State Design* dialog box (see Figure 2.89).

In the third method (*By taking into account the deformation ratio of the longitudinal reinforcement*), on the other hand, the direction of the compression strut is determined according to Baumann. The limit angle of 15° is ignored because the crack width in this area is not governing.

⊞ Internal Forces of Linear Statics			
⊞ Check if internal forces cause cracks			
⊞ Check Existing Longitudinal Reinforcement			
⊞ Design Internal Forces in Serviceability Limit State			
⊞ Check Steel Stress			
⊟ Check Final Crack Spacing			
⊟ Bottom surface (+z)			
Concrete does not crack on this side.			
⊟ Top surface (-z)			
⊞ Maximum Crack Spacing into Reinforcement Direction 1	$s_{r,max,-z,\Phi 1}$	0.177	m
⊞ Maximum Crack Spacing into Reinforcement Direction 2	$s_{r,max,-z,\Phi 2}$	0.218	m
⊟ Maximum Final Crack Spacing acc. (7.15)	$s_{r,max,-z,res}$	0.137	m
Angle Between Reinforcement Direction and Crack Opening	Φ_{-z}	40.254	°

Figure 2.110 Maximum crack spacings for both reinforcement directions

Difference in mean strain (ε_{sm} - ε_{cm})

For the calculation value of the crack width w_k according to Equation 2.73, we need to determine the factor (ε_{sm} - ε_{cm}) for each reinforcement direction and for the direction of the resulting strain.

The difference in the mean strain of concrete and reinforcing steel is determined according to [7], clause 7.3.4, Equation (7.9):

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \cdot \dfrac{f_{ct,\,eff}}{\rho_{eff}} \cdot (1 + \alpha_e \cdot \rho_{eff})}{E_s} \geq 0.6 \cdot \frac{\sigma_s}{E_s}$$

Equation 2.78

The maximum mean strain $(\varepsilon_{sm} - \varepsilon_{cm})_{-z,res}$ is obtained as the resulting mean strain of the individual reinforcement directions as 1.291 ‰.

⊞ Internal Forces of Linear Statics			
⊞ Check if internal forces cause cracks			
⊞ Check Existing Longitudinal Reinforcement			
⊞ Design Internal Forces in Serviceability Limit State			
⊞ Check Steel Stress			
⊞ Check Final Crack Spacing			
⊟ Determination of difference in the main strain			
⊟ Calculation Parameter for All Directions			
Axial Tensile Strength of Concrete	$f_{ct,eff}$	2.90	N/mm²
⊟ Ratio of Modulus of Elasticity	α_e	6.061	
Modulus of Elasticity of Reinforcement	E_s	200000.00	N/mm²
Mean Secant Modulus of Elasticity	E_{cm}	33000.00	N/mm²
Factor to Consider the Load Duration	k_t	0.400	
⊞ Bottom surface (+z)			
⊟ Top surface (-z)			
⊟ Difference in the main strain into the direction 1	$(\varepsilon_{sm} - \varepsilon_{cm})_{-z,\Phi 1}$	0.735	‰
⊞ First Calculated Difference in the Main Strain	$(\varepsilon_{sm} - \varepsilon_{cm})_{calc1}$	0.735	‰
Second Calculated Difference in the Main Strain	$(\varepsilon_{sm} - \varepsilon_{cm})_{calc2}$	0.625	‰
⊞ Steel Stress into Reinforcement Direction 1	$\sigma_{s,-z,\Phi 1}$	208.18	N/mm²
⊟ Difference in the main strain into the direction 2	$(\varepsilon_{sm} - \varepsilon_{cm})_{-z,\Phi 2}$	0.527	‰
⊞ First Calculated Difference in the Main Strain	$(\varepsilon_{sm} - \varepsilon_{cm})_{calc1}$	0.527	‰
Second Calculated Difference in the Main Strain	$(\varepsilon_{sm} - \varepsilon_{cm})_{calc2}$	0.501	‰
⊞ Steel Stress into Reinforcement Direction 2	$\sigma_{s,-z,\Phi 2}$	167.09	N/mm²
⊟ Resulting Difference in the Main Strain	$(\varepsilon_{sm} - \varepsilon_{cm})_{-z,res}$	1.291	‰
⊟ Differential Angle According to Baumann			
1st Differential Angle	$\alpha_{m,-z}$	30.000	°
2nd Differential Angle	$\beta_{m,-z}$	120.000	°
Concrete Strut Direction acc. to Baumann	$\gamma_{strut,Bau,exist,3}$	79.746	°

Figure 2.111 Difference in mean strain for both reinforcement directions

To simplify the expression, we introduce symbols for the sought mean strain ($\varepsilon_{sm} - \varepsilon_{cm}$): *s* for the side length in the reinforcement direction, *d* for the partial length of the compression struts, *l* for the perpendicular to the compression strut, and ε.

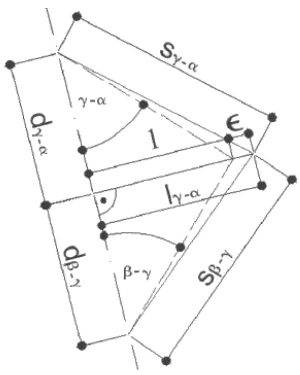


Figure 2.112 Mean strain ε

The partial length $d_{\gamma\text{-}\alpha}$ is determined as follows for a selected compression strut inclination:

$$d_{\gamma-\alpha} = \frac{1}{\tan(\gamma - \alpha)}$$

The length is unitless (the perpendicular to the compression strut was included without unit).

Then the length $s_{\gamma\text{-}\alpha}$ is determined.

$$s_{\gamma-\alpha} = \frac{1 + \varepsilon_\alpha}{\tan(\gamma - \alpha)}$$

If the reinforcement direction θ_1 forms the smallest differential angle with the principal moment m_1, we have to insert the previously determined difference in the mean strains $(\varepsilon_{sm} - \varepsilon_{cm})_{\theta 1}$ of concrete and reinforcing steel for ε_α:

$$s_{\gamma-\alpha} = \frac{1 + (\varepsilon_{sm} - \varepsilon_{cm})_{\theta_1}}{\tan(\gamma - \alpha)}$$

If the reinforcement direction θ_2 forms the smallest differential angle with the principal moment m_1, we have to insert the previously determined difference in the mean strains $(\varepsilon_{sm} - \varepsilon_{cm})_{\theta 2}$ of concrete and reinforcing steel for ε_α:

With the Pythagorean theorem, we can determine the value $l_{\gamma\text{-}\alpha}$ from the lengths $d_{\gamma\text{-}\alpha}$ and $s_{\gamma\text{-}\alpha}$:

$$l_{\gamma-\alpha} = \sqrt{s^2_{\gamma-\alpha} - d^2_{\gamma-\alpha}}$$

Since all formulas are based on an initial length of 1.0 units of length, the strain ε is determined as follows:

$$\varepsilon = l_{\gamma-\alpha} \cdot -1.0$$

This strain $\varepsilon = (\varepsilon_{sm} - \varepsilon_{cm})$ is checked again by means of the intermediate angle $(\beta - \gamma)$.

For the determination of the SLS design internal forces according to the *By assuming an identical deformation ratio of the longitudinal reinforcement* method, the strain ratio of the reinforcements can significantly deviate from the assumed geometric strain ratio. To correctly determine the resulting strain ratio, the program therefore uses the strain of the reinforcement that is closer to the main action.

Crack width w_k

The calculated value of the crack width w_k is determined according to Equation 2.73.

⊟ Determination of calculated value of crack width			
⊟ Bottom surface (+z)	Ratio	0.000	
Concrete does not crack on this side.			
⊟ Top surface (-z)	Ratio	0.591	
⊟ Calculated Value of Crack Width into Reinforcement Direction 1	$w_{k,-z,\varphi 1}$	0.130	mm
Maximum Crack Spacing into Reinforcement Direction 1	$s_{r,max,-z,\Phi 1}$	0.177	m
Difference in the mean strain into the direction 1	$(\varepsilon_{sm} - \varepsilon_{cm})_{-z,\Phi 1}$	0.735	‰
⊟ Calculated Value of Crack Width into Reinforcement Direction 2	$w_{k,-z,\varphi 2}$	0.115	mm
Maximum Crack Spacing into Reinforcement Direction 2	$s_{r,max,-z,\Phi 2}$	0.218	m
Difference in the mean strain into the direction 2	$(\varepsilon_{sm} - \varepsilon_{cm})_{-z,\Phi 2}$	0.527	‰
⊟ Crack width into the resulting difference in the mean strain	$w_{k,-z,res}$	0.177	mm
Maximum Final Crack Spacing acc. (7.15)	$s_{r,max,-z,res}$	0.137	m
Resulting Difference in the Mean Strain	$(\varepsilon_{sm} - \varepsilon_{cm})_{-z,res}$	1.291	‰

Figure 2.113 Calculated value of crack width

In window 1.3 *Surfaces*, we have specified the maximum allowable crack width $w_k = 0.3$ mm. The following criterion of check for the governing resulting direction is thus obtained:

⊞ Design Internal Forces in Serviceability Limit State			
⊞ Check Steel Stress			
⊞ Check Final Crack Spacing			
⊞ Determination of difference in the mean strain			
⊞ Determination of calculated value of crack width			
⊟ Check			
Crack width at the top (-z) surface in direction of the resulting strain	$w_{k-z,,res}$	0.177	mm
Maximum allowable crack width at the top (-z) surface acc. to user input	$w_{k,-z,limit}$	0.300	mm
Criterion of Check	Criterion	0.591	

Figure 2.114 Check criterion for crack width

2.6.5 Governing Loading

In RFEM, we can define the various loadings in individual load cases (LC). These load cases can be superimposed into load combinations (CO) and result combinations (RC). The differences between these types of combinations is described in the chapters 5.5 and 5.6 of the RFEM manual.

While load cases and load combinations respectively yield only one set of internal forces, up to 16 sets of internal forces can be created in a result combination, depending on the type of model:

- For the model types 2D - XZ (u_X / u_Z / φ_Y) and 2D - XY (u_X / u_Y / φ_Z) (wall), only the axial forces n_x, n_y, and n_{xy} are obtained in the surfaces.

Their combination yields six sets of internal forces, with one of these axial forces respectively showing its maximum or minimum value.

- For the model type 2D - XY (u_Z / φ_X / φ_Y) (plate), the maximum and minimum values of the moments m_x, m_y, and m_{xy} and the shear forces v_x and v_y are determined.

Ten sets of internal forces are thus obtained.

- The model type 3D contains all axial forces, moments, and shear forces mentioned above and therefore yields 16 sets of internal forces.

The analysis core for the serviceability limit state designs processes the internal forces of the selected load cases and load combinations one by one. The same is true for the sets of internal forces of a result combination. This shows that the design of a result combination is much more time-consuming.

In most checks for the individual reinforcement directions, the internal forces or sets of internal forces result in a loading. The program determines the greatest loading among all reinforcement directions. If the resistance is different for the individual reinforcement directions, the program searches for the reinforcement direction that yields the largest quotient from loading over resistance.

2.7 Deformation Analysis with RF-CONCRETE Deflect

For the deformation analysis, you need a license of the add-on module **RF-CONCRETE Deflect**.

2.7.1 Material and Geometry Assumptions

For the deformation analysis with RF-CONCRETE Deflect, a linear-elastic compression and tension behavior of the reinforcing steel is assumed. A linear-elastic compression behavior and a linear-elastic behavior is applied for concrete until the tension strength is reached. This is sufficiently accurate for the serviceability limit state. If the provided stress exceeds the tensile strength of concrete, damage develops according to EN 1992-1-1, clause 7.3.4.

The calculation uses a simple isotropic model of fracture mechanics that is defined independently in the two reinforcement directions. From an engineering point of view, the material stiffness matrix is calculated by interpolation between the uncracked (state I) and cracked state (state II) according to EN 1992-1-1, clause 7.4.3, Equation (7.18). Thus, the reinforced concrete is modeled as an orthotropic material. All laws of damage development can take the tension stiffening effect and simple long-term effects (shrinkage and creep) into account.

The calculation of the material stiffness matrices occurs for the model types *2D - XY* (u_Z / φ_X / φ_Y) and *3D*. For the model type *3D*, the eccentricities' influence of the ideal centroid (see below) is additionally considered in the stiffness matrix.

2.7.2 Design Internal Forces

As described above, the calculation of stiffnesses is based on linear-elastic assumptions. The internal forces are transformed in the orthogonal reinforcement directions φ as well as on both surfaces s (top and bottom). The determined internal forces — bending moments $m_{\varphi,s}$ and axial forces $n_{\varphi,s}$ (torsion moments are eliminated by a transformation in the reinforcement directions) — depend on the

(a) type of model,

(b) method of calculation,

(c) classification criterion.

2.7.3 Critical Surface

To determine the critical surface, each reinforcement direction φ is considered separately. The state of stress is analyzed on both surfaces s — bottom surface (in the direction of the local +z-axis) and top surface (in the direction of the local -z-axis). The side with the greatest tensile stress in the concrete is classified as governing. The internal forces on the critical side are designated as n_Φ and m_Φ.

The axial force $n_{\Phi,s}$ that is transformed in the reinforcement direction Φ has the same value for both surfaces s ($n_\Phi = n_{\Phi,top} = n_{\Phi,bottom}$). The axial forces are therefore not relevant for determining the critical side; only the moments are considered in finding the governing surface. The algebraic signs for the bending moments $n_{\Phi,s}$ are determined regarding whether the moments cause tension or compression on the respective surface s. Therefore, the critical surface is the side with the larger bending moment (i.e. the side that is more strongly subjected to tension).

For the calculation of the stiffness, only the internal forces n_Φ and m_Φ on the critical side are taken into account. Until now, the term "bottom surface" has referred to the local +z-axis; in the following, however, "bottom surface" refers to the critical side of the surface.

2.7.4 Cross-Section Properties

The cross-section properties are calculated for both reinforcement directions Φ and both cross-section states c (cracked/uncracked). A linear-elastic behavior of the concrete on the tension side is applied for state I (uncracked cross-section) and the concrete tensile strength is not considered for state II (cracked cross-section).

If no axial forces n_Φ act as is the case for the model type *2D - XY (u_Z / φ_X / φ_Y)*, for example, this part of the calculation is independent of the internal forces and a direct calculation of the cross-section properties is possible. In the remaining cases, the compression zone depth is calculated by means of an iterative method of calculation, the so-called "binary method". For numerical reasons, the program uses the minimum value for the reinforcement ratio $\rho_{min} = 10^{-4}$ in every iteration step, meaning that if there is no reinforcement, a virtual minimum reinforcement area is applied. This small value has no noticeable influence on the results (stiffnesses).

The calculated ideal cross-section properties (related to the concrete cross-section) in a reinforcement direction φ and the crack state c are

- the moment of inertia to the ideal center of gravity $I_{\Phi,c}$,
- the moment of inertia to the geometric center of the cross-section $I_{0.\Phi,c}$,
- the cross-section area $A_{\Phi,c}$,
- the eccentricity of the ideal center of gravity $e_{\Phi,c}$.

2.7.5 Considering Long-Term Effects

The influence of creep and shrinkage is what is regarded as a long-term effect. According to EN 1992-1-1, long-term effects are to be considered separately.

2.7.5.1 Creep

The creep effects are considered through a reduction of the modulus of elasticity E, with the effective creep coefficient φ_{eff} being applied according to EN 1992-1-1, Equation (7.20):

$$E_{cd,eff} = \frac{E_{cd}}{1 + \varphi_{eff}}$$

Equation 2.79

2.7.5.2 Shrinkage

In the deformation calculation according to EN 1992-1-1, there are two areas that are influenced by shrinkage effects.

Reduction of material stiffness

The material stiffness in each reinforcement direction Φ is reduced by the so-called coefficient of shrinkage influence $\kappa_{sh,\Phi,c}$. For the two crack states c (cracked/uncracked), the axial forces $n_{sh,\Phi,c}$ and bending moments $m_{sh,\Phi,c}$ can be calculated from the free shrinkage strain ε_{sh}:

$$n_{sh,\phi} = -\varepsilon_{sh} \cdot E_s \cdot (a_{s1} + a_{s2})$$

$$m_{sh,\phi} = n_{sh} \cdot e_{sh}$$

Equation 2.80

where

$n_{sh,\Phi}$	additional axial force from shrinkage in reinforcement direction φ
$m_{sh,\Phi}$	additional moment from shrinkage in the centroid of the ideal cross-section in reinforcement direction Φ
a_{s1}	bottom reinforcement surface
a_{s2}	top reinforcement surface
E_s	modulus of elasticity of reinforcing steel
ε_{sh}	shrinkage strain
e_{sh}	eccentricity of shrinkage forces (state I and state II) from the center of gravity of the ideal cross-section

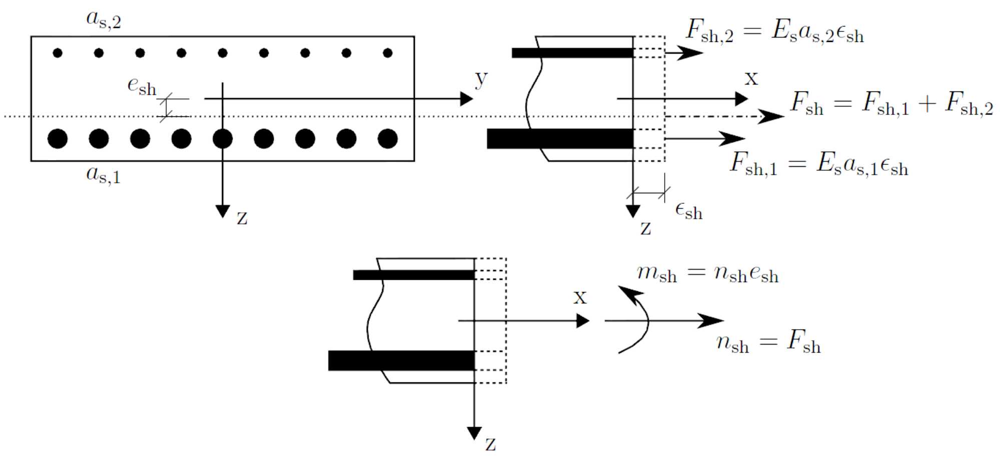


Figure 2.115 Internal forces $n_{sh,\Phi}$ and $m_{sh,\Phi}$

With these internal forces from shrinkage, the additional curvature $\kappa_{sh,\Phi,c}$ induced by shrinkage is calculated in the analyzed point — without influence of the surrounding model. Subsequently, the new coefficient of shrinkage influence $\kappa_{sh,\Phi,c}$ is calculated according to:

$$\kappa_{sh,\phi,c} = \frac{\kappa_{sh,\phi,c} + \kappa_{\phi}}{\kappa_{\phi}}$$

Equation 2.81

where

κ_Φ	curvature induced by external loading without shrinkage influence in reinforcement direction Φ
$\kappa_{sh,\Phi,c}$	curvature induced by shrinkage (and reinforcement arrangement) without influence of creep in reinforcement direction Φ

The coefficient $\kappa_{sh,\Phi,c}$ is limited to the interval $\kappa_{sh,\Phi,c} \in \langle 1, 100 \rangle$: Therefore, $\kappa_{sh,\Phi,c}$ may not reduce the stiffness by more than 100 times for numerical and physical reasons. Furthermore, the minimum value $\kappa_{sh,\Phi,c} = 1$ means that it is not possible to consider the influence of shrinkage if the influence of shrinkage has an orientation opposite to the loading-induced curvature κ_d.

The influence of shrinkage on the membrane stiffness is not considered.

Calculation of distribution coefficient

The second influence of shrinkage lies in the calculation of the distribution coefficient ζ according to EN 1992-1-1, clause 7.4.3, Equation (7.18). The following chapter describes the distribution coefficient in detail.

2.7.6 Distribution Coefficient

The calculation of the distribution coefficient ζ_d is shown for a reinforcement direction Φ. First, the maximum concrete tension stress $\sigma_{max,\Phi}$ is calculated under the assumption of linear-elastic material behavior:

$$\sigma_{max,\phi} = \frac{n_{\phi} + n_{sh,\phi}}{A_{\phi,I}} + \frac{m_{\phi} - n_{\phi} \cdot \left(x_{\phi,I} - \frac{h}{2}\right) + m_{sh,\phi,I}}{I_{\phi,I}} \cdot \left(h - x_{\phi,I}\right)$$

Equation 2.82

where

n_Φ	axial force from external loading in reinforcement direction Φ
$n_{sh,\Phi}$	additional axial force from shrinkage in reinforcement direction Φ
m_Φ	moment from external loading in reinforcement direction Φ
$m_{sh,\Phi,I}$	additional moment from shrinkage in reinforcement direction φ in state I
$x_{\Phi,I}$	depth of concrete compression zone in uncracked state in reinforcement direction Φ

h	depth of cross-section
$A_{\Phi,I}$	ideal cross-section area in state I in reinforcement direction Φ
$I_{\Phi,I}$	ideal moment of inertia in state I in reinforcement direction Φ

The influence of the shrinkage forces on the maximum tension stress $\sigma_{max,\Phi}$ is considered via the additional internal forces from shrinkage.

The calculation of the distribution coefficient ζ_Φ depends on whether the influence of tension stiffening is taken into account in the deformation calculation according to EN 1992-1-1.

Distribution coefficient ζ_Φ while taking tension stiffening into account

- for $\sigma_{max,\Phi} > f_{ctm}$:

$$\zeta_\phi = 1 - \beta \cdot \left(\frac{f_{ctm}}{\sigma_{max,\phi}} \right)^n$$

- for $\sigma_{max,\Phi} \leq f_{ctm}$:

$$\zeta_\phi = 0$$

Equation 2.83

where

β	parameter for the load duration
f_{tm}	mean tensile strength of concrete
n	2 for EN 1992-1-1

Distribution coefficient ζ_Φ without taking tension stiffening into account

- for $\sigma_{max,\Phi} > f_{ctm}$:

$$\zeta_\phi = 1$$

- for $\sigma_{max,\Phi} \leq f_{ctm}$:

$$\zeta_\phi = 0$$

Equation 2.84

2.7.7 Cross-Section Properties for Deformation Analysis

For the material stiffness matrix D for the deformation analysis, the program requires the cross-section properties dependant on the cracked state that are available in every reinforcement direction. These are in detail

- the moment of inertia to the ideal center of gravity I_Φ,
- the moment of inertia to the geometric center of the cross-section $I_{0,\Phi}$,
- the cross-section area A_Φ,
- the eccentricity of the ideal center e_Φ to the geometric center.

The mean strain ε_Φ and mean curvature K_Φ are interpolated from the cracked and uncracked state according to EN 1992-1-1, Equation (7.18):

$$\varepsilon_\phi = \zeta_\phi \cdot \varepsilon_{\phi,II} + (1 - \zeta_\phi) \cdot \varepsilon_{\phi,1}$$
$$\kappa_\phi = \zeta_\phi \cdot \kappa_{\phi,II} + (1 - \zeta_\phi) \cdot \kappa_{\phi,1}$$

Equation 2.85

The strains in the cracked state c (state I and II) are calculated according to the following equations:

$$\varepsilon_{\phi,c} = \frac{n_\phi}{E \cdot A_{\phi,c}}$$
$$\kappa_{\phi,c} = \kappa_{sh,\phi,c} \cdot \frac{m_\phi - n_\phi \cdot e_{\phi,c}}{E \cdot I_{\phi,c}}$$

Equation 2.86

The influence of shrinkage is therefore considered with the factor $k_{sh,\varphi,c}$.

When no axial forces n_Φ act, such as with the model type *2D - XY (u_Z / φ_X / φ_Y)*, only the ideal cross-section properties that relate to the ideal center of the cross-section are relevant:

$$A_\phi = \frac{A_{\phi,I} \cdot A_{\phi,II}}{\zeta_\phi \cdot A_{\phi,I} \cdot k_{sh,\phi,II} + (1 - \zeta_\phi) \cdot A_{\phi,II} \cdot k_{sh,\phi,I}}$$
$$I_\phi = \frac{I_{\phi,I} \cdot I_{\phi,II}}{\zeta_\phi \cdot I_{\phi,I} \cdot k_{sh,\phi,II} + (1 - \zeta_\phi) \cdot I_{\phi,II} \cdot k_{sh,\phi,I}}$$

Equation 2.87

If axial forces are available, the cross-section properties are related to the geometric center of the cross-section:

$$A_\phi = \frac{n_\phi}{A \cdot \varepsilon_\phi} \qquad \text{where } \varepsilon_\phi = \frac{m_\phi - \kappa_\phi \cdot E \cdot I_\phi}{n_\phi}$$
$$I_{\phi 0} = I_\phi + A_\phi \cdot e_\phi^2 \qquad \text{where } I_\phi \text{ as per Equation 2.87}$$

Equation 2.88

In the course of the calculation of the cross-section properties, the initial value of poisson's ratio ν_{init} is reduced according to the following equation:

$$\nu = \left(1 - \max_{\phi \in \{1,2\}} (\zeta_\phi)\right) \cdot \nu_{init}$$

Equation 2.89

2.7.8 Material Stiffness Matrix D

Bending stiffness — plates and shells

The bending stiffnesses in the reinforcement directions φ are determined as follows:

$$D_{d,d} = I_{0,d} \cdot \frac{E}{(1 - \nu^2)} \quad \text{where } d = \{1,2\}$$

$$D_{d,d} = I_{d} \cdot \frac{E}{(1 - \nu^2)} \quad \text{where } d = \{1,2\}$$

The non-diagonal component of the material stiffness matrix is calculated identically for plates and shells:

$$D_{1,2} = D_{2,1} = \nu \cdot \sqrt{(D_{1,1} \cdot D_{2,2})}$$

For shells, the differences in the bending stiffnesses due to the moments of inertia are compensated via the eccentricity components in the material stiffness matrix.

Torsional stiffness — plates and shells

The stiffness matrix elements for torsion are calculated as follows for plates and shells:

$$D_{3,3} = \frac{1 - \nu}{2} \cdot \sqrt{(D_{1,1,} \cdot D_{2,2})}$$

Shear stiffness — plates and shells

The stiffness matrix elements for shear are not reduced for the deformation analysis. They are calculated from the shear modulus G of the ideal cross-section and the cross-section height h. The following applies for shells and plates:

$$D_{3+d,3+d} = \frac{5}{6} \cdot G \cdot h \quad \text{where } d = \{1,2\}$$

Membrane stiffness — shells

The membrane stiffnesses in the reinforcement directions φ are determined as follows:

$$D_{5+d,5+d} = E \cdot \frac{A_d}{(1 - \nu^2)} \quad \text{where } d = \{1,2\}$$

The non-diagonal component of the material stiffness matrix is calculated from:

$$D_{6,7} = D_{7,6} = \nu \cdot \sqrt{(D_{6,6} \cdot D_{7,7})}$$

The shear stiffness component is:

$$D_{8,8} = G \cdot h$$

Eccentricity — shells

The stiffness matrix elements for the eccentricity of the centroid (ideal cross-section) in the reinforcement direction φ are calculated as follows:

$$D_{d,6} = D_{6,d} = D_{5+d,5+d} \cdot e_d \qquad \text{where } d = \{1,2\}$$

The non-diagonal component of the material stiffness matrix is determined from:

$$D_{1,7} = D_{7,1} = \frac{\nu}{2} \cdot \left(e_{\phi_1} + e_{\phi_2}\right) \cdot \sqrt{\left(D_{6,6} \cdot D_{7,7}\right)}$$

The eccentricity components for torsion are calculated as follows:

$$D_{3,8} = D_{8,3} = \frac{1}{2} \cdot G \cdot h \cdot \left(e_{\phi_1} + e_{\phi_2}\right)$$

2.7.9 Positive Definiteness of Material Stiffness Matrix

The positive definiteness of the material stiffness matrix D is tested by Sylvester's criterion, which has been modified regarding the null blocks. If the stiffness matrix D is not positive definite, the non-diagonal components of the material stiffness matrix are set to zero one after another. In extreme cases, only the positive components from the diagonal remain.

2.7.10 Example

The determination of the material stiffness matrix D described above is illustrated by means of a simple example. The model contains a single finite element and the surface is only reinforced on one side (top). For simplicity's sake, the manual calculation is carried out in the reinforcement direction φ_1.

2.7.10.1 Geometry

The model with the dimensions 1m x 1m and the thickness 0.20m is fixed at one side. The free side is subjected to the bending moment $m_x = -30$ kNm/m and the axial force $n_x = -100$ kN/m. The automatic self-weight is not taken into account.

The longitudinal reinforcement in φ_1 is 1 000 mm^2.

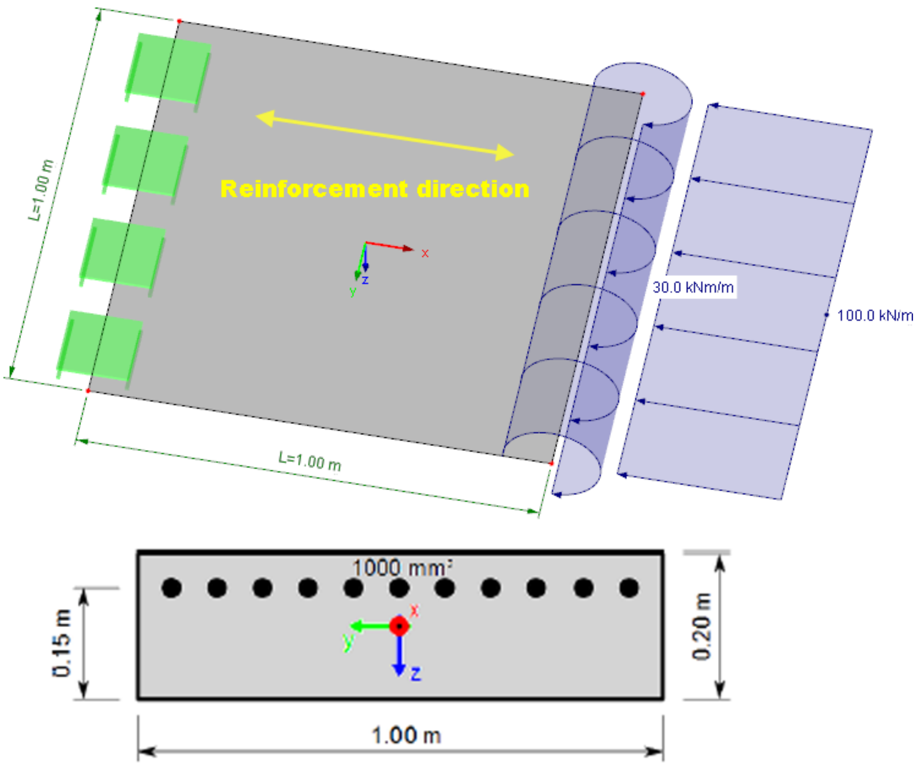


Figure 2.116 Model with loading and reinforcement

2.7.10.2 Materials

The following table shows the material properties.

⊞ Internal Forces of Linear Statics				
⊞ Principal Internal Forces				
⊞ Check if internal forces cause cracks				
⊞ Design Internal Forces				
⊞ Selection of Internal Design Forces				
⊞ Determination of Critical Surfaces				
⊟ Material Data for Stiffness Calculation				
⊟ Concrete				
Modulus of Elasticity	E_b	33000.0	MPa	
Shear Modulus	G_c	11800.0	MPa	
Poisson's Ratio	ν	0.20		
Mean Axial Tensile Strength	f_{ctm}	2.9	MPa	
Creep coefficient	φ	2.00		
Coefficient of Torsion	Φ	1.00		
⊟ Reinforcement				
Modulus of Elasticity	E_s	200000.0	MPa	

Figure 2.117 Material data for stiffness calculation

2.7.10.3 Selection of internal design forces

First, the internal forces are transformed in the first reinforcement direction φ_1. The bending moments have different values for the bottom (+z) and the top (-z) surface; the axial forces have the same algebraic signs after the transformation.

$$m_{\phi_1,+z} = -30\text{ kN}$$

$$m_{\phi_1,-z} = 30\text{ kN}$$

$$n_{\phi_1,+z} = n_{\phi_1,-z} = -100\text{ kN}$$

⊞ Internal Forces of Linear Statics				
⊞ Principal Internal Forces				
⊞ Check if internal forces cause cracks				
⊞ Design Internal Forces				
⊟ Selection of Internal Design Forces				
⊟ into Reinforcement Direction 1				
Design Axial Force	$n_{\Phi 1}$	-100.0	kN/m	
⊟ Design Moment	$m_{\Phi 1}$	30.0	kNm/m	
Design Moment at the Bottom Surface (+z)	$m_{+z,\Phi 1}$	-30.0	kNm/m	
Design Moment at the Top Surface (-z)	$m_{-z,\Phi 1}$	30.0	kNm/m	
⊟ into Reinforcement Direction 2				
Design Axial Force	$n_{\Phi 2}$	0.0	kN/m	
⊟ Design Moment	$m_{\Phi 2}$	1.1	kNm/m	
Design Moment at the Bottom Surface (+z)	$m_{+z,\Phi 2}$	-1.1	kNm/m	
Design Moment at the Top Surface (-z)	$m_{-z,\Phi 2}$	1.1	kNm/m	

Figure 2.118 Selection of internal design forces

2.7.10.4 Determining the critical surface

The top surface (-z) proves to be the critical surface. For further calculations, only the bending moment and the axial force of this surface are considered.

$$m_{\phi_1} = m_{\phi_1,-z} = 30\text{ kN}$$

$$n_{\phi_1} = n_{\phi_1,+z} = -100\text{ kN}$$

⊞ Internal Forces of Linear Statics				
⊞ Principal Internal Forces				
⊞ Check if internal forces cause cracks				
⊞ Selection of Internal Design Forces				
⊟ Determination of Critical Surfaces				
⊟ into Reinforcement Direction 1				
⊟ Critical Surface	topen (-z)			
Design Moment at the Bottom Surface (+z)	$m_{+z,\Phi 1}$	-30.0	kNm/m	
Design Moment at the Top Surface (-z)	$m_{-z,\Phi 1}$	30.0	kNm/m	
⊟ into Reinforcement Direction 2				
⊟ Critical Surface	oben (-z)			
Design Moment at the Bottom Surface (+z)	$m_{+z,\Phi 2}$	-1.1	kNm/m	
Design Moment at the Top Surface (-z)	$m_{-z,\Phi 2}$	1.1	kNm/m	

Figure 2.119 Determining the critical surface

2.7.10.5 Cross-section properties (cracked and uncracked state)

The cross-section properties depend on the governing side and the reinforcement direction φ_1. The minimum values are used for the reinforcement surfaces $a_{s2,\varphi1}$, $a_{s1,\varphi2}$, and $a_{s2,\varphi2}$.

The following cross-section properties for the uncracked and cracked state are to be calculated in order to be able to assemble the stiffness matrix of the material **D**.

Centroid

The distance of the centroid of the ideal cross-section from the concrete surface in compression is calculated directly for the uncracked state.

$$z_{I,\phi_1} = \frac{\frac{b \cdot h^2}{2} + \alpha\left(a_{s_1,\phi_1} \cdot d_{1,\phi_1} + a_{s_2,\phi_1} \cdot d_{2,\phi_1}\right)}{b \cdot h + \alpha \cdot \left(a_{s_1,\phi_1} + a_{s_2,\phi_1}\right)} =$$

$$= \frac{\frac{1000 \cdot 200^2}{2} + 6.061 \cdot (1000 \cdot 150 \cdot 15 \cdot 50)}{1000 \cdot 200 \cdot 6.061 \cdot (1000 + 15)} = 101.4 \text{ mm}$$

For the cracked state, the depth $\chi_{II,\varphi1}$ of the zone in compression must be calculated with the iterative method. Then the distance of the centroid of the ideal cross-section from the surface in compression is calculated for the cracked state.

Ideal cross-section area $A_{c,d}$

The effective cross-section area in the uncracked state without the influence of creep is:

$$A_{I,\phi_1} = b \cdot h + \alpha \cdot \left(a_{s_1,\phi_1} + a_{s_2,\phi_1}\right) = 1000 \cdot 200 + 6.061 \cdot (1000 + 15) = 2061.5 \text{ cm}^2$$

The effective cross-section area in the cracked state is determined with the influence of creep.

$$A_{II,\phi_1} = b \cdot \chi_{II,\phi_1} + \alpha \cdot \left(a_{s_1,\phi_1} + a_{s_2,\phi_1}\right) = 1000 \cdot 68.3 + 18.182 \cdot \left(1000 + 15\right) = 867.19 \text{ cm}^2$$

The coefficient α is the ratio of the moduli of elasticity of steel and concrete with or without the influence of creep.

Ideal moment of inertia to ideal centroid $I_{c,d}$

The effective moment of inertia to the ideal centroid in the uncracked state without the influence of creep is:

$$I_{I,\phi_1} = \frac{1}{12} \cdot b \cdot h^3 + b \cdot h \cdot \left(z_{I,\phi_1} - \frac{h}{2}\right)^2 + \alpha \cdot a_{s_1,\phi_1} \cdot \left(d_{1,\phi_1} - z_{I,\phi_1}\right)^2 + \alpha \cdot a_{s_2,\phi_1} \cdot \left(z_{I,\phi_1} - d_{2,\phi_1}\right)^2 =$$

$$= \frac{1}{12} \cdot 1000 \cdot 200^3 + 1000 \cdot 200 \cdot \left(101.4 - \frac{200}{2}\right)^2 + 6.061 \cdot 1000 \cdot (150 - 101.4)^2 + 6.061 \cdot 15 \cdot (101.4 - 50)^2 =$$

$$= 68\,161.30 \text{ cm}^4$$

The effective moment of inertia to the ideal centroid in the cracked state is determined with the influence of creep.

$$I_{II,\phi_1} = \frac{1}{12} \cdot b \cdot \chi^3_{II,\phi_1} + b \cdot \chi_{II,\phi_1} \cdot \left(z_{II,\phi_1} - \frac{\chi_{II,\phi_1}}{2}\right)^2 +$$
$$+ \alpha \cdot a_{s_1,\phi_1} \cdot \left(d_{1,\phi_1} - z_{II,\phi_1}\right)^2 + \alpha \cdot a_{s_2,\phi_1} \cdot \left(z_{II,\phi_1} - d_{2,\phi_1}\right)^2 =$$
$$= \frac{1}{12} \cdot 1000 \cdot 68.3^3 + 1000 \cdot 68.3 \cdot \left(58.5 - \frac{68.3}{2}\right)^2 +$$
$$+ 18.182 \cdot 1000 \cdot (150 - 58.5)^2 + 18.182 \cdot 15 \cdot (58.5 - 50)^2 =$$
$$= 21\,928.70 \text{ cm}^4$$

Ideal moment of inertia to the geometric center of the cross-section $I_{0,c,d}$

The ideal moment of inertia to the geometric center of the cross-section in the uncracked state without the influence of creep is:

$$I_{0,I,\phi_1} = \frac{1}{12} \cdot b \cdot h^3 + \alpha \cdot a_{s_1,\phi_1} \cdot \left(d_{1,\phi_1} - \frac{h}{2}\right)^2 + \alpha \cdot a_{s_2,\phi_1} \cdot \left(\frac{h}{2} - d_{2,\phi_1}\right)^2 =$$
$$= \frac{1}{12} \cdot 1000 \cdot 200^3 + 6.061 \cdot 200 \cdot \left(150 - \frac{200}{2}\right)^2 + 6.061 \cdot 15 \cdot \left(\frac{200}{2} - 50\right)^2 =$$
$$= 68\,204.50 \text{ cm}^4$$

The ideal moment of inertia to the geometric center of the cross-section in the cracked state is determined with the influence of creep.

$$I_{0,II,\phi_1} = \frac{1}{12} \cdot b \cdot \chi_{II,\phi_1}{}^3 + b \cdot \chi_{II,\phi_1} \cdot \left(\frac{h}{2} - \frac{\chi_{II,\phi_1}}{2}\right)^2 + \alpha \cdot a_{s_1,\phi_1} \cdot \left(d_{1,\phi_1} - \frac{h}{2}\right)^2 + \alpha \cdot a_{s_2,\phi_1} \cdot \left(\frac{h}{2} - d_{2,\phi_1}\right)^2 =$$
$$= \frac{1}{12} \cdot 1000 \cdot 68.3^3 + 1000 \cdot 68.3 \cdot \left(\frac{200}{2} - \frac{68.3}{2}\right)^2 + 18.182 \cdot 1000 \cdot \left(150 - \frac{200}{2}\right)^2 +$$
$$+ 18.182 \cdot 15 \cdot \left(\frac{200}{2} - 50\right)^2 =$$
$$= 36\,881.50 \text{ cm}^4$$

Eccentricity of centroid $e_{c,d}$

The eccentricity of the ideal centroid of the cross-section is determined as follows:

$$e_{c,\phi_1} = z_{c,\phi 1} - \frac{h}{2}$$

- uncracked state:

$$e_{\phi_1,I} = 101.4 - \frac{200}{2} = 1.4 \text{ mm}$$

- cracked state:

$$e_{\phi_1,II} = 58.5 - \frac{200}{2} = -41.5 \text{ mm}$$

⊞ Internal Forces of Linear Statics			
⊞ Principal Internal Forces			
⊞ Check if internal forces cause cracks			
⊞ Design Internal Forces			
⊞ Selection of Internal Design Forces			
⊞ Determination of Critical Surfaces			
⊞ Material Data for Stiffness Calculation			
⊟ Cross-Sectional Properties (Uncracked and Cracked State)			
⊟ into Reinforcement Direction 1			
⊟ Geometry			
Cross-Sectional Height	h	200.0	mm
Effective Height	$d_{1,\Phi 1}$	150.0	mm
Reinforcement area	$a_{s,1,\Phi 1}$	10.00	cm^2
Effective Height	$d_{2,\Phi 1}$	50.0	mm
Reinforcement area	$a_{s,2,\Phi 1}$	0.15	cm^2
⊟ Uncracked State (State I)			
Depth of the Concrete Compression Zone	$x_{I,\Phi 1}$	101.4	mm
Ideal Cross-Sectional Area	$A_{I,\Phi 1}$	2061.52	cm^2
Ideal Moment of Inertia to Ideal Center of Gravity	$I_{I,\Phi 1}$	68161.30	cm^4
Ideal Moment of Inertia to Geometrical Center of Cross-Section	$I_{0,I,\Phi 1}$	68204.50	cm^4
Eccentricity of Center of Gravity (Positive Value in Direction of Critical	$e_{I,\Phi 1}$	1.4	mm
⊟ Cracked State (State II)			
Depth of the Concrete Compression Zone	$x_{II,\Phi 1}$	68.3	mm
Ideal Cross-Sectional Area	$A_{II,\Phi 1}$	867.19	cm^2
Ideal Moment of Inertia to Ideal Center of Gravity	$I_{II,\Phi 1}$	21928.70	cm^4
Ideal Moment of Inertia to Geometrical Center of Cross-Section	$I_{0,II,\Phi 1}$	36881.50	cm^4
Eccentricity of Center of Gravity (Positive Value in Direction of Critical	$e_{II,\Phi 1}$	-41.5	mm

Figure 2.120 Cross-sectional properties in reinforcement direction 1

⊟ Cross-Sectional Properties (Uncracked and Cracked State)			
⊞ into Reinforcement Direction 1			
⊟ into Reinforcement Direction 2			
⊟ Geometry			
Cross-Sectional Height	h	200.0	mm
Effective Height	$d_{1,\Phi 2}$	139.0	mm
Reinforcement area	$a_{s,1,\Phi 2}$	0.14	cm^2
Effective Height	$d_{2,\Phi 2}$	60.0	mm
Reinforcement area	$a_{s,2,\Phi 2}$	0.14	cm^2
⊟ Uncracked State (State I)			
Depth of the Concrete Compression Zone	$x_{I,\Phi 2}$	100.0	mm
Ideal Cross-Sectional Area	$A_{I,\Phi 2}$	2001.68	cm^2
Ideal Moment of Inertia to Ideal Center of Gravity	$I_{I,\Phi 2}$	66692.90	cm^4
Ideal Moment of Inertia to Geometrical Center of Cross-Section	$I_{0,I,\Phi 2}$	66692.90	cm^4
Eccentricity of Center of Gravity (Positive Value in Direction of Critical	$e_{I,\Phi 2}$	0.0	mm
⊟ Cracked State (State II)			
Depth of the Concrete Compression Zone	$x_{II,\Phi 2}$	9.5	mm
Ideal Cross-Sectional Area	$A_{II,\Phi 2}$	100.42	cm^2
Ideal Moment of Inertia to Ideal Center of Gravity	$I_{II,\Phi 2}$	516.86	cm^4
Ideal Moment of Inertia to Geometrical Center of Cross-Section	$I_{0,II,\Phi 2}$	8734.84	cm^4
Eccentricity of Center of Gravity (Positive Value in Direction of Critical	$e_{II,\Phi 2}$	-90.5	mm

Figure 2.121 Cross-sectional properties in reinforcement direction 2

2.7.10.6 Shrinkage influence

The influence of shrinkage is directly introduced in the calculation with the defined value of the free shrinkage ε_{sh}. Thus, the influence of structural restraints or redistributions of the shrinkage forces is not taken into account.

In our example, the shrinkage strain is applied with the following value:

$$\varepsilon_{sh} = -0.5 \cdot 10^{-3}$$

The free shrinkage strain causes additional forces in the cross-section:

$$n_{sh,\phi_1} = -E_s \cdot \varepsilon_{sh} \cdot \left(a_{s_1,\phi_1} + a_{s_2,\phi_1}\right) = -200 \cdot 10^9 \cdot \left(-0.5 + 10^{-3}\right) \cdot \left(1000 + 15\right) \cdot 10^{-6} =$$
$$= 101.5\,\mathrm{kN/m}$$

The forces act for both crack states c (cracked and uncracked) with the eccentricity to the centroid of the ideal cross-section:

$$e_{sh,c,\phi_1} = \frac{a_{s_1,\phi_1} \cdot d_{1,\phi_1} + a_{s_2,\phi_1} \cdot d_{2,\phi_1}}{a_{s_1,\phi_1} + a_{s_2,\phi_1}} - z_{c,\phi_1}$$

- uncracked state:

$$e_{sh,c,\phi_1} = \frac{1000 \cdot 150 + 15 \cdot 50}{1000 + 15} - 101.4 = 47.1\,\mathrm{mm}$$

- cracked state:

$$e_{sh,c,\phi_1} = \frac{1000 \cdot 150 + 15 \cdot 50}{1000 + 15} - 58.5 = 90.0\,\mathrm{mm}$$

The bending moment caused by the axial force $n_{sh,\varphi 1}$ for both crack states c is:

$$m_{sh,c,\phi_1} = n_{sh,\phi_1} \cdot e_{sh,c,\phi_1}$$

- uncracked state:

$$m_{sh,I,\phi_1} = 101.5 \cdot 10^3 \cdot 0.047 = 4.8\,\mathrm{kNm/m}$$

- cracked state:

$$m_{sh,II,\phi_1} = 101.5 \cdot 10^3 \cdot 0.090 = 9.1\,\mathrm{kNm/m}$$

When determining the coefficient $k_{sh,c,d}$ for both crack states c, we have to distinguish:

- for $m_{\varphi 1} \neq 0$:

$$k_{sh,c,\phi_1} = \frac{m_{sh,c,\phi_1} + m_{\phi_1} - n_{\phi_1} \cdot e_{c,\phi_1}}{m_{\phi_1} - n_{\phi_1} \cdot e_{c,\phi_1}}$$

- for $m_{\varphi 1} = 0$:

$$k_{sh,c,\phi_1} = 1 \quad \text{where } k_{sh,c,\phi_1} \in \{1,100\}$$

In this example: $m_{\varphi 1} \neq 0$

- uncracked state:

$$k_{sh,I,\phi_1} = \frac{4.771 \cdot 10^3 + 30 \cdot 10^3 - (-100 \cdot 10^3) \cdot 1.4 \cdot 10^3}{30 \cdot 10^3 - (-100 \cdot 10^3) \cdot 1.4 \cdot 10^3} = 1.159$$

- cracked state:

$$k_{sh,II,\phi_1} = \frac{9.135 \cdot 10^3 + 30 \cdot 10^3 - (-100 \cdot 10^3) \cdot (-41.5 \cdot 10^3)}{30 \cdot 10^3 - (-100 \cdot 10^3) \cdot (-41.5 \cdot 10^3)} = 1.354$$

⊟ Shrinkage Influence				
⊟ into Reinforcement Direction 1				
⊟ Uncracked State (State I)				
Free Shrinkage	ε_{sh}	-0.001		
Axial Force Induced by Shrinkage	$n_{sh,I,\Phi1}$	101.5	kN/m	
Moment Induced by Shrinkage	$m_{sh,I,\Phi1}$	4.8	kNm/m	
Coefficient of Shrinkage Influence	$k_{sh,I,\Phi1}$	1.159		
⊟ Cracked State (State II)				
Free Shrinkage	ε_{sh}	-0.001		
Axial Force Induced by Shrinkage	$n_{sh,II,\Phi1}$	101.5	kN/m	
Moment Induced by Shrinkage	$m_{sh,II,\Phi1}$	9.1	kNm/m	
Coefficient of Shrinkage Influence	$k_{sh,II,\Phi1}$	1.354		
⊟ into Reinforcement Direction 2				
⊟ Uncracked State (State I)				
Free Shrinkage	ε_{sh}	-0.001		
Axial Force Induced by Shrinkage	$n_{sh,I,\Phi2}$	2.8	kN/m	
Moment Induced by Shrinkage	$m_{sh,I,\Phi2}$	0.0	kNm/m	
Coefficient of Shrinkage Influence	$k_{sh,I,\Phi2}$	1.000		
⊟ Cracked State (State II)				
Free Shrinkage	ε_{sh}	-0.001		
Axial Force Induced by Shrinkage	$n_{sh,II,\Phi2}$	2.8	kN/m	
Moment Induced by Shrinkage	$m_{sh,II,\Phi2}$	0.3	kNm/m	
Coefficient of Shrinkage Influence	$k_{sh,II,\Phi2}$	1.238		

Figure 2.122 Shrinkage influence

2.7.10.7 Calculation of distribution coefficient

The maximum stress in the uncracked state is:

$$\sigma_{max,\phi_1} = \frac{n_{\phi_1} + n_{sh,\phi_1}}{A_{\phi_1,I}} + \frac{m_{\phi_1} - n_{\phi_1}\left(\chi_{I,\phi_1} - \frac{h}{2}\right) + m_{sh,I,\phi_1}}{I_{\phi_1,I}} \cdot \left(h - \chi_{I,\phi_1}\right) =$$

$$= \frac{-100 \cdot 10^3 + 101.5 \cdot 10^3}{0.206} + \frac{30 \cdot 10^3 - \left(-100 \cdot 10^3\right)\left(0.101 - \frac{0.2}{2}\right) + 4.778 \cdot 10^3}{6.816 \cdot 10^{-4}} \cdot (0.2 - 0.1)$$

$$= 5.1 \text{ Mpa}$$

We assume a long-term loading:

$$\beta_{\varphi_1} = 0.5$$

Taking *Tension Stiffening* into account, the distribution coefficient is calculated according to the following equation:

- for $\sigma_{max,\varphi1} > f_{ctm}$:

$$\zeta_d = 1 - \beta_{\phi_1} \cdot \left(\frac{f_{ctm}}{\sigma_{max,\phi_1}}\right)^2$$

- for $\sigma_{max,\varphi1} \leq f_{ctm}$:

$$\zeta_{sh,c,\phi_1} = 0$$

In the example, the maximum tension stress in the concrete is larger than the concrete tensile strength.

$$\sigma_{max,\phi_1} > f_{ctm}$$

$$5.1 > 2.9$$

Thus, the distribution coefficient is:

$$\zeta_{\phi_1} = 1 - \beta_{\phi_1} \cdot \left(\frac{f_{ctm}}{\sigma_{max,\phi_1}}\right)^2 = 1 - 0.5 \cdot \left(\frac{2.9}{5.1}\right)^2 = 0.835$$

⊞ Cross-Sectional Properties (Uncracked and Cracked State)				
⊞ Shrinkage Influence				
⊟ Damage Parameter Calculation				
⊟ into Reinforcement Direction 1				
⊟ Damage Parameter	$\zeta_{\Phi 1}$	0.835		
Tension Stiffening		Yes		
Maximum Tensile Stress in Concrete	$\sigma_{c,I,\Phi 1}$	5.1	MPa	
Mean Axial Tensile Strength	f_{ctm}	2.9	MPa	
Coefficient of Duration of Load	$\beta_{\Phi 1}$	0.500		
⊟ into Reinforcement Direction 2				
⊟ Damage Parameter	$\zeta_{\Phi 2}$	0.000		
Tension Stiffening		Yes		
Maximum Tensile Stress in Concrete	$\sigma_{c,I,\Phi 2}$	0.2	MPa	
Mean Axial Tensile Strength	f_{ctm}	2.9	MPa	
Coefficient of Duration of Load	$\beta_{\Phi 2}$	0.500		
⊞ Final Cross-Sectional Properties				

Figure 2.123 Calculation of distribution coefficient (damage parameter)

2.7.10.8 Final cross-section properties

The curvature for both crack states c (cracked/uncracked) is calculated as follows:

$$\kappa_{c,\phi_1} = k_{sh,c,\phi_1} \cdot \frac{m_{\phi_1} - n_{\phi_1} \cdot e_{c,\phi_1}}{E \cdot I_{c,\phi_1}}$$

- uncracked state:

$$\kappa_{I,\phi_1} = 1.158 \cdot \frac{30 \cdot 10^3 - (-100 \cdot 10^3) \cdot 1.4 \cdot 10^{-3}}{11 \cdot 10^9 \cdot 6.816 \cdot 10^{-4}} = 4.655 \cdot 10^{-3}$$

- cracked state:

$$\kappa_{II,\phi_1} = 1.353 \cdot \frac{30 \cdot 10^3 - (-100 \cdot 10^3) \cdot (-41.5 \cdot 10^{-3})}{11 \cdot 10^9 \cdot 2.193^{-4}} = 14.499 \cdot 10^{-3}$$

The strain for both crack states is determined as follows:

$$\varepsilon_{c,\phi_1} = \frac{n_{\phi_1}}{E \cdot A_{c,\phi_1}}$$

- uncracked state:

$$\varepsilon_{I,\phi_1} = \frac{-100 \cdot 10^3}{11 \cdot 10^9 \cdot 0.206} = -4.413 \cdot 10^{-5}$$

- cracked state:

$$\varepsilon_{II,\phi_1} = \frac{-100 \cdot 10^3}{11 \cdot 10^9 \cdot 0.087} = -10.449 \cdot 10^{-5}$$

Thus, it is possible to determine the mean strain.

$$\varepsilon_{\phi_1} = \zeta_{\phi_1} \cdot \varepsilon_{II,\phi_1} + \left(1 - \zeta_{\phi_1}\right) \cdot \varepsilon_{I,\phi_1} =$$
$$= 0.835 \cdot (-10.449 \cdot 10^{-5}) + (1 - 0.835) \cdot (-4.413 \cdot 10^{-5}) = -9.459 \cdot 10^{-5}$$

The mean curvature is determined as follows:

$$\kappa_{\phi_1} = \zeta_{\phi_1} \cdot \kappa_{II,\phi_1} + \left(1 - \zeta_{\phi_1}\right) \cdot \kappa_{I,\phi_1} =$$
$$= 0.835 \cdot (14.449 \cdot 10^{-3}) + (1 - 0.835) \cdot 4.655 \cdot 10^3 = 12.885 \cdot 10^{-3}$$

With the mean curvature and the longitudinal strain, you can calculate the final cross-section properties while taking account of shrinkage, creep, and *tension stiffening*.

Ideal cross-section area

$$A_{\phi_1} = \frac{n_{\phi_1}}{E \cdot \varepsilon_{\phi_1}} = \frac{-100 \cdot 10^3}{11 \cdot 10^9 \cdot (-9.459 \cdot 10^{-5})} = 958.59 \text{ cm}^2$$

Ideal moment of inertia to the ideal center of the cross-section

$$I_{\phi_1} = \frac{I_{I,\phi_1} \cdot I_{II,\phi_1}}{\zeta_{\phi_1} \cdot I_{I,\phi_1} \cdot k_{dh,II,\phi_1} + \left(1 - \zeta_{\phi_1}\right) \cdot I_{II,\phi_1} \cdot k_{dh,I,\phi_1}} =$$

$$= \frac{6.816 \cdot 10^{-4} \cdot 2.193 \cdot 10^{-4}}{0.836 \cdot 6.816 \cdot 10^{-4} \cdot 1.353 + \left(1 - 0.836\right) \cdot 2.193 \cdot 10^{-4} \cdot 1.158} = 18\,391.50 \text{ cm}^4$$

Eccentricity of centroid

$$e_{\phi_1} = \frac{m_{\phi_1} - \kappa_{\phi_1} \cdot E \cdot I_{\phi_1}}{n_{\phi_1}} = \frac{30 \cdot 10^3 - 12.855 \cdot 10^{-3} \cdot 11 \cdot 10^9 \cdot 1.839 \cdot 10^{-4}}{-100 \cdot 10^3} = -39 \text{ mm}$$

Ideal moment of inertia to the geometric center of the cross-section

$$I_{0,\phi_1} = I_{\phi_1} + A_{\phi_1} \cdot e_{\phi_1}{}^2 = 1.839 \cdot 10^{-4} + 0.096 \cdot (-0.0393)^2 = 33\,207 \text{ cm}^4$$

Poisson's ratio is determined as follows:

$$\nu = \left(1 - \max_{d \in \{1,2\}} (\zeta_d)\right) \cdot \nu_{init} = (1 - max\,(0.0836)) \cdot 0.2 = 0.0328$$

⊞ Internal Forces of Linear Statics			
⊞ Principal Internal Forces			
⊞ Check if internal forces cause cracks			
⊞ Design Internal Forces			
⊞ Selection of Internal Design Forces			
⊞ Determination of Critical Surfaces			
⊞ Material Data for Stiffness Calculation			
⊞ Cross-Sectional Properties (Uncracked and Cracked State)			
⊞ Shrinkage Influence			
⊞ Damage Parameter Calculation			
⊟ Final Cross-Sectional Properties			
⊟ into Reinforcement Direction 1			
Ideal Cross-Sectional Area	$A_{\Phi 1}$	958.59	cm^2
Ideal Moment of Inertia to Ideal Center of Gravity	$I_{\Phi 1}$	18391.50	cm^4
Ideal Moment of Inertia to Geometrical Center of Cross-Section	$I_{0,\Phi 1}$	33207.10	cm^4
Eccentricity of Center of Gravity (Positive Value in Direction of Critical	$e_{\Phi 1}$	39.3	mm
⊟ into Reinforcement Direction 2			
Ideal Cross-Sectional Area	$A_{\Phi 2}$	2001.68	cm^2
Ideal Moment of Inertia to Ideal Center of Gravity	$I_{\Phi 2}$	66692.90	cm^4
Ideal Moment of Inertia to Geometrical Center of Cross-Section	$I_{0,\Phi 2}$	66692.90	cm^4
Eccentricity of Center of Gravity (Positive Value in Direction of Critical	$e_{\Phi 2}$	0.0	mm

Figure 2.124 Final cross-section properties

2.7.10.9 Stiffness matrix of the material

Bending stiffness

$$D_{11} = \frac{I_{0,\phi_1} \cdot E}{1 - \nu^2} = \frac{3.322 \cdot 10^{-4} \cdot 11 \cdot 10^9}{1 - 0.0328^2} = 3656.74 \text{ kNm}$$

$$D_{12} = D_{21} = 0.0328 \cdot \sqrt{(3.656 \cdot 10^6 \cdot 7.344 \cdot 10^6)} = 170.58 \text{ kNm}$$

Torsional stiffness

$$D_{33} = \frac{1 - \nu}{2} \cdot \sqrt{D_{11} \cdot D_{22}} = \frac{1 - 0.0328}{2} \cdot \sqrt{(3.656 \cdot 10^6 \cdot 7.344 \cdot 10^6)} = 2505.84 \text{ kNm}$$

Shear stiffness

$$D_{44} = D_{55} = \frac{5}{6} \cdot G \cdot h = \frac{5}{6} \cdot 11.8 \cdot 10^9 \cdot 0.2 = 1\,966\,670 \text{ kN/m}$$

Membrane stiffness

$$D_{66} = \frac{E \cdot A_{\phi_1}}{1 - \nu^2} = \frac{11 \cdot 10^9 \cdot 0.096}{1 - 0.0328^2} = 1\,055\,590 \text{ kN/m}$$

$$D_{67} = D_{76} \cdot \nu \cdot \sqrt{(D_{66} \cdot D_{77})} = 0.0328 \cdot \sqrt{(1\,055.59 \cdot 10^6 \cdot 2505.84 \cdot 10^6)} = 50\,210.6 \text{ kN/m}$$

$$D_{88} = G \cdot h = 11.8 \cdot 10^9 \cdot 0.2 = 2\,360\,000 \text{ kN/m}$$

Eccentricity

$$D_{16} = D_{61} = D_{61} \cdot e_{\phi_1} = 1\,055.590 \cdot 10^9 \cdot 0.0393 = 41\,499.2 \text{ kNm/m}$$

$$D_{27} = D_{72} = D_{77} \cdot e_{\phi_1} = 0$$

$$D_{17} = D_{71} = \frac{\nu}{2} \cdot \left(e_{\phi_1} + e_{\phi_2}\right) \cdot \sqrt{D_{66} \cdot D_{77}} =$$

$$= \frac{0.0328}{2} \cdot (0.393 + 0) \cdot \sqrt{1\,055.59 \cdot 10^6 \cdot 2505.84 \cdot 10^6} = 987.0 \text{ kNm/m}$$

$$D_{38} = D_{83} = \frac{1}{2} \cdot G \cdot h \cdot \left(e_{\phi_1} + e_{\phi_2}\right) = \frac{1}{2} \cdot 11 \cdot 10^9 \cdot 0.2 \cdot (0.393 + 0) = 46\,390.2 \text{ kNm/m}$$

⊞ Cross-Sectional Properties (Uncracked and Cracked State)			
⊞ Shrinkage Influence			
⊞ Damage Parameter Calculation			
⊞ Final Cross-Sectional Properties			
⊟ Material Stiffness Matrix			
⊟ Bending Stiffness			
into Reinforcement Direction 1	D_{11}	3656.74	kNm
into Reinforcement Direction 2	D_{22}	7344.18	kNm
transversal influence	D_{12}	170.58	kNm
torsion	D_{33}	2505.84	kNm
⊟ Shear Stiffness			
into Reinforcement Direction 1	D_{44}	1966670.0	kN/m
into Reinforcement Direction 2	D_{55}	1966670.0	kN/m
⊟ Membrane Stiffness			
into Reinforcement Direction 1	D_{66}	1055590.0	kN/m
into Reinforcement Direction 2	D_{77}	2204240.0	kN/m
transversal influence	D_{67}	50210.6	kN/m
torsion	D_{88}	2360000.0	kN/m
⊟ Eccentricity			
into Reinforcement Direction 1	D_{16}	41499.2	kNm/m
into Reinforcement Direction 2	D_{27}	0.0	kNm/m
transversal influence	D_{17}	987.0	kNm/m
torsion	D_{38}	46390.2	kNm/m

Figure 2.125 Stiffness matrix of the material

2.8 Nonlinear Method

2.8.1 General

The serviceability limit state design (SLS) is generally divided into the three following groups:

- Stress limitation (EN 1992-1-1, clause 7.2)
- Crack control (EN 1992-1-1, clause 7.3)
- Deflection control (EN 1992-1-1, clause 7.4)

The design of reinforced concrete structures is generally based on linear structural analyses: To determine the reinforcement including the serviceability limit state design, the internal forces are determined linearly; afterwards the cross-section analysis is performed. However, this procedure considers the cracking that is typical for reinforced concrete with the corresponding nonlinear material rules of reinforced concrete only at the cross-section level.

By including the nonlinear behavior of reinforced concrete in the determination of internal forces, you get realistic states of stresses and therefore distributions of internal forces that, in statically indeterminate systems, differ significantly from the linearly determined internal forces due to stiffness redistributions. For the serviceability limit state design, this means that the nonlinear material behavior of the reinforced concrete must be taken into account to realistically calculate deformations, stresses, and crack widths.

If the crack formation is not taken into account in the deformation calculation, the occurring deformations are underestimated. By considering creeping and shrinkage, the deformation may be 3 to 8 times larger, depending on the stress and boundary conditions. The **RF-CONCRETE NL** add-on module allows for a realistic calculation of the deformations, crack widths, and stresses of reinforced concrete surfaces by considering the nonlinear material behavior when determining internal forces.

2.8.2 Equations and Methods of Approximation

2.8.2.1 Theoretical approaches

"Nonlinear calculation" refers to the determination of internal forces and deformations while considering the nonlinear behavior of internal forces and deformations (physical).

Planar structures can be described as two-dimensional structures with the following state quantities: surface loads, deformations, internal forces, and strains in the surface centroids. However, since material properties that vary over the surface depth must be considered for the nonlinear reinforced concrete model, it becomes necessary to extend the 2D model by additionally taking the depth of the cross-section into account. The cross-section of the reinforced concrete is divided into a certain number of steel and concrete layers (see Figure 2.126).

Based on the strains in the surface centroids and under the assumption of the Bernoulli hypothesis, the strains leading to the stresses after the corresponding steel or concrete material law has been applied are obtained for each layer. Then the resulting stresses per layer can be integrated into the internal forces of the gross cross-section.

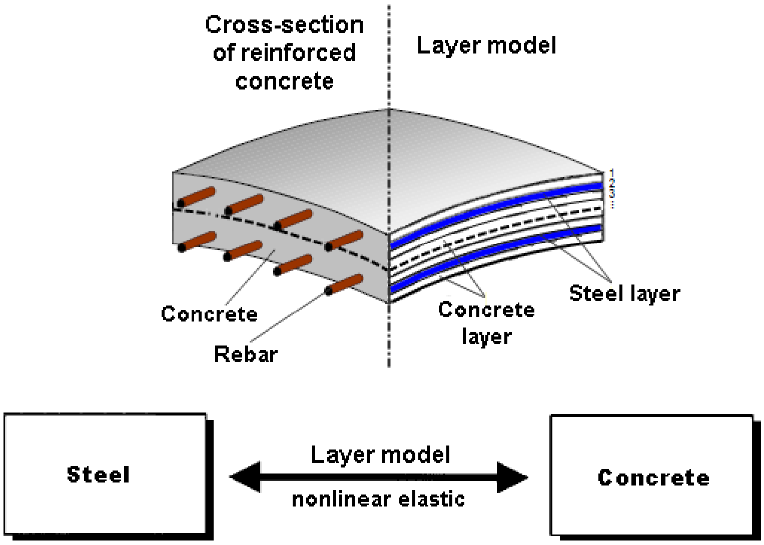


Figure 2.126 Layer model for reinforced concrete surfaces

If the concrete's tension strength is reached in a point of the structure, a discontinuity arises in the form of a crack. Strictly speaking, this discontinuity would require an adjustment of the discretization (*remeshing*) so that every crack is included in the calculation in its actual position and size. In case of several cracks, this method would result in a high numerical effort because every crack would increase the number of elements. Therefore, occurring cracks are "smeared" within an element and the stiffness-reducing influences of the cracks are taken into account in the calculation by adjusting the material rule.

If the first principal stress in a concrete layer reaches the concrete tensile strength, a crack is formed perpendicular to the first principal stress direction. This principal direction may change if the load changes. Here we can assume that a formed crack does not change its position and orientation (the so-called *fixed crack model*) or that the crack always runs orthogonally to the variable principal directions (*rotating crack model*). RF-CONCRETE NL uses the *rotating crack model*.

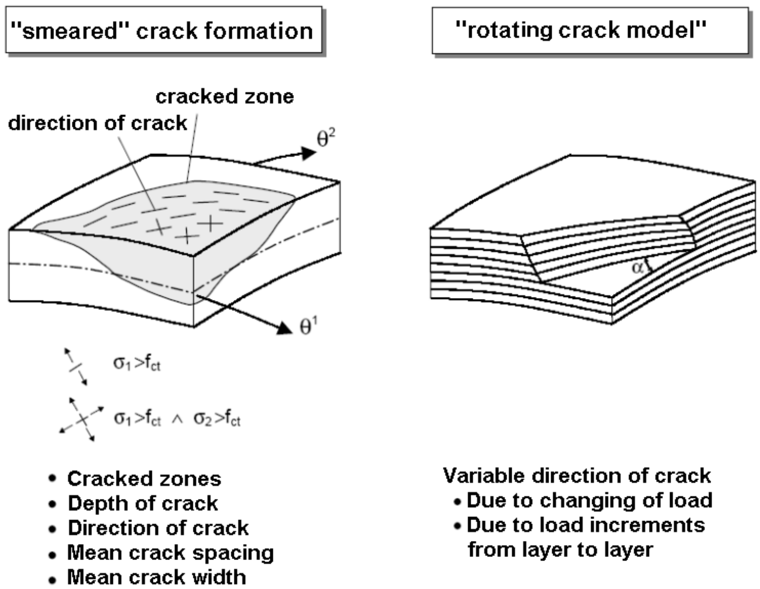


Figure 2.127 Crack models in reinforced concrete surface elements

2.8.2.2 Flowchart

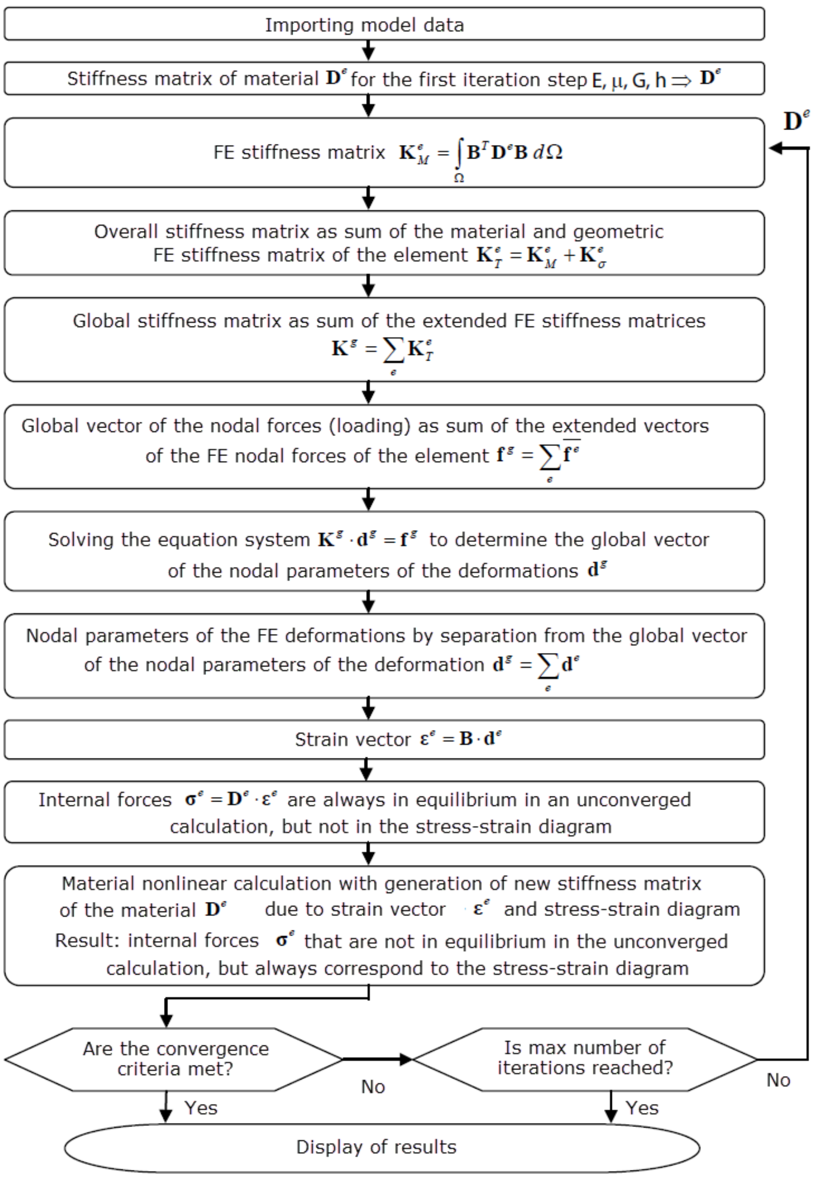


Figure 2.128 Flowchart

where

D^e	stiffness matrix of the material, constitutive matrix
B	matrix due to geometry and basic type of the FE function $\varepsilon^e = B \cdot d^e$
K_M^e	material FE stiffness matrix
K_σ^e	geometric FE stiffness matrix

K_T^e	FE overall stiffness matrix
K^g	global stiffness matrix of the entire model
f^e	FE nodal forces vector
f^g	global nodal forces vector (loading on entire model), vector of the right sides
d^e	nodal parameter vector of FE deformation
d^g	global nodal parameter vector of deformation, vector of the unknown
ε^e	strain vector
σ^e	vector of internal forces

2.8.2.3 Methods for solving nonlinear equations

The application of the FE method for solving nonlinear differential equations results in algebraic equations that can be expressed in the following form:

$$K(d) \cdot d = f$$

Equation 2.90

where

K	stiffness matrix of the model
d	vector of the unknown (usually of nodal parameters of the deformation)
f	vector of the right sides (usually of nodal forces)

The matrix K is the function of d and therefore cannot be evaluated without knowing the vector of the system roots d. Since this nonlinear system cannot be solved directly, iteration methods that are aimed at progressively increasing the precision of the solution are used.

RF-CONCRETE NL uses the iteration method according to Picard. This method is also known as the *Direct Iteration Method* or *Secant Modulus Method*.

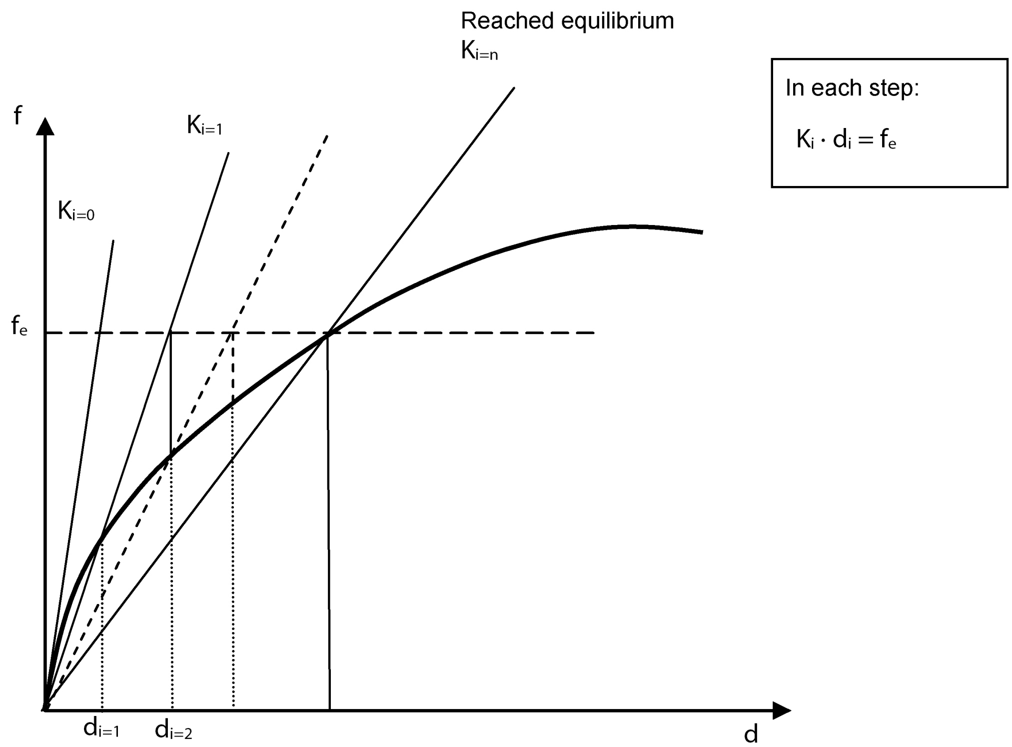


Figure 2.129 Direct iteration method

2.8.2.4 Convergence criteria

In solving the nonlinear equations, two convergence criteria are considered. The iteration step is deemed to be completed once a convergence criterion is fulfilled. The first convergence criterion observes how the diagonal components of the material's stiffness matrix change. A convergence is reached when the stiffness matrix of the material has stabilized for all finite elements.

$$D_{tot}^{i-1} = \begin{bmatrix} D_j^{i-1} & & \\ & \cdots & \\ & & D_n^{i-1} \end{bmatrix} \quad D_{tot}^{i} = \begin{bmatrix} D_j^{i} & & \\ & \cdots & \\ & & D_n^{i} \end{bmatrix} \quad \frac{\sum_{j=1}^{n} \left| D_j^i - D_j^{i-1} \right|}{\sum_{j=1}^{n} D_j^{i-1}} \le \varepsilon$$

Equation 2.91

where

D_{tot}^{i-1}	stiffness matrix of the material from the previous iteration step
D_{tot}^{i}	stiffness matrix of the material in the current iteration step
ε	desired precision (for RFEM precision 1, the following applies: ε = 0.05 %)

The second convergence criterion observes how the size of the maximum deformation changes. At the same time, the program checks whether the place of the maximum deformation within the structure has changed. Since the deformation normally converges faster than the stiffness matrix, the deformation criterion is only activated after 50 iteration steps (for RFEM precision 1).

$$\frac{d_{max}^{i} - d_{max}^{i-1}}{d_{max}^{i-1}} < \varepsilon \qquad \text{where } N_{max}^{i} = N_{max}^{i-1}$$

Equation 2.92

where

D_{max}^{i-1}	maximum nodal displacement from previous iteration step
D_{max}^{i}	maximum nodal displacement of structure in current iteration step
ε	desired precision (for RFEM precision 1, the following applies: ε = 0.05 %)
N_{max}^{i-1}	number of node with maximum displacement from previous iteration step
N_{max}^{i}	number of node with maximum displacement from current iteration step

The precision of the convergence criterion for the nonlinear calculation and the iteration step after which the deformation criterion is additionally considered are controlled in the *Global Calculation Parameters* tab of the *Calculation Parameters* dialog box in RFEM. This dialog box can also be accessed in window 1.1 *General Data* of RF-CONCRETE:

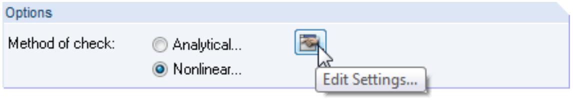


Figure 2.130 [Edit Settings] button in the *Serviceability Limit State* tab of window 1.1 *General Data*

The *Settings for Nonlinear Calculation* dialog box appears (see Figure 2.131).

You can then click the [Details] button to open the *Calculation Parameters* dialog box of RFEM.

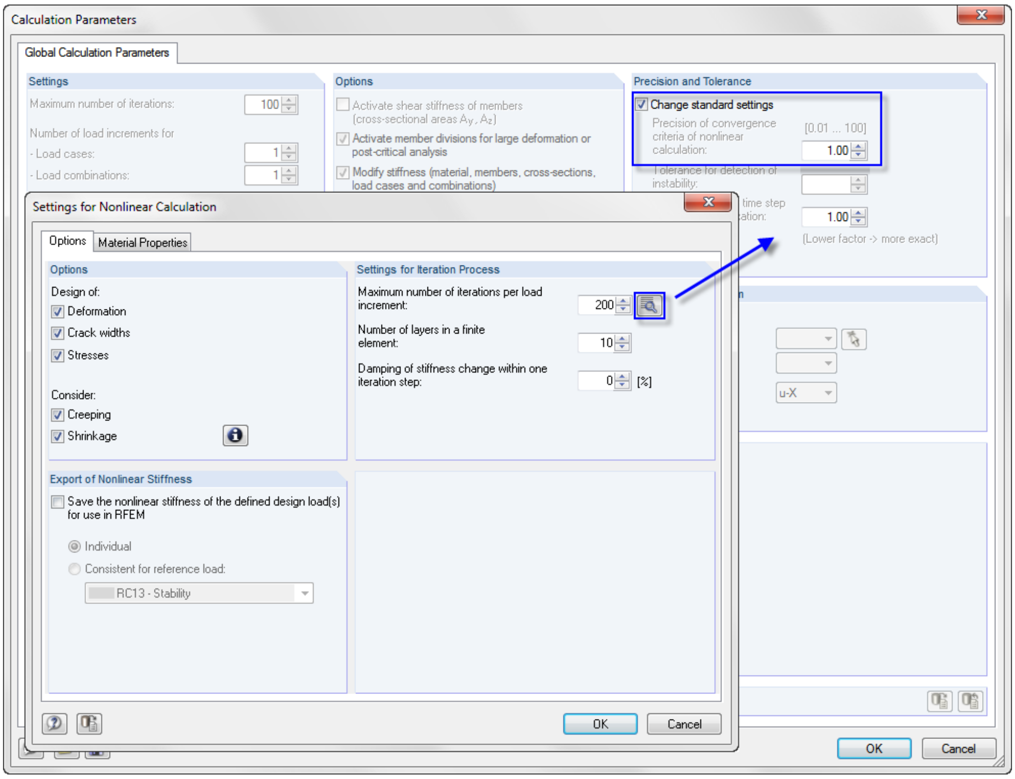


Figure 2.131 *Settings for Nonlinear Calculation* dialog box with access to convergence criterion

The value for the *Precision of convergence criteria* ("RFEM precision") indicated in the RFEM dialog box influences the break-off limit ε for the physically nonlinear calculation and the iteration step n_i from which the deformation criterion is additionally considered:

$$\varepsilon = \text{"RFEM precision"} \cdot 0.05\%$$

$$n_i = \frac{1}{\text{"RFEM precision"}} \cdot 50$$

The default value of the RFEM precision is 1. Thus, the precision of the convergence criterion for the physical nonlinearity is ε = 0.05 % and the additional consideration of the deformation criterion starts after the 50th iteration step. For a higher precision, the value of the RFEM precision must be reduced. Thus, ε becomes smaller and the deformation criterion is considered at a later time.

2.8.3 Material Properties

2.8.3.1 Concrete in compression area

For the serviceability limit state design, the program calculates using the mean strengths of the materials. In the compressed area, it is possible to choose between a parabolic and a parabolic-rectangular distribution of the stress-strain diagram.

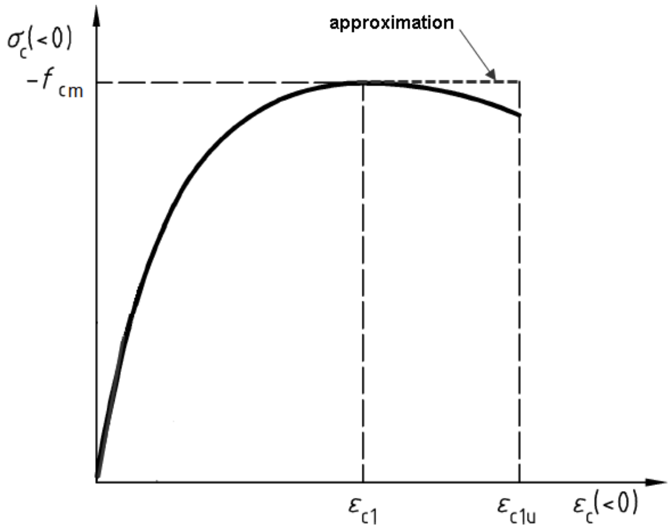


Figure 2.132 Stress-strain diagram for concrete in compression area

The settings are specified in the *Material Properties* tab of the *Settings for Nonlinear Calculation* dialog box (see Figure 3.12). This dialog box can be accessed in the *Serviceability Limit State* tab of window 1.1 *General Data* by clicking the button shown on the left (see Figure 2.130).

2.8.3.2 Concrete in tension area

There are several options available for the stress-strain diagram of the concrete in the tension area.

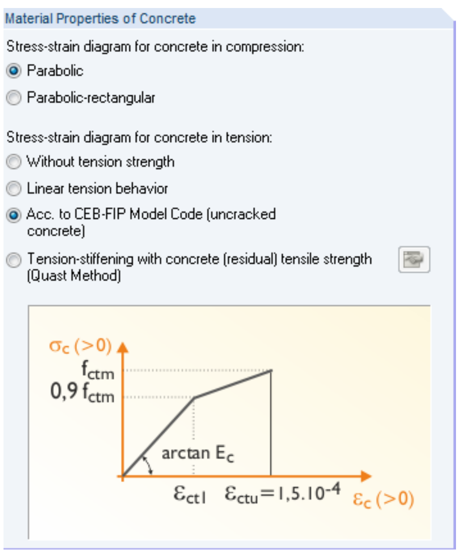


Figure 2.133 Stress-strain diagrams for tension area of concrete

The concrete tensile strength can, inter alia, be considered as per CEB-FIP Model Code 90:1993. Until the tensile strength of concrete f_{ctm} is reached, the program assumes the distribution for reaching the crack strain of 0.15 ‰ shown in Figure 2.134.

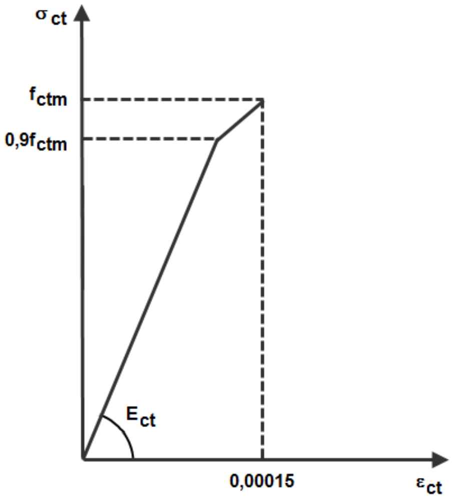


Figure 2.134 Stress-strain relation for concrete in the tension area as per CEB-FIP Model Code 90:1993

Alternatively, you can use the concrete's stiffening effect in the tension zone (*tension stiffening*). This procedure is described in the following chapter.

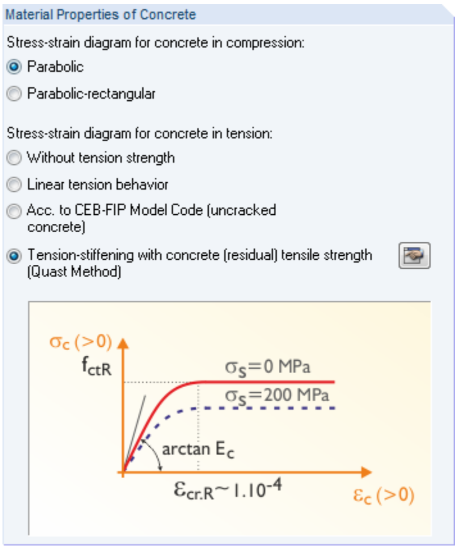


Figure 2.135 Stress-strain diagram for *Tension-stiffening*

2.8.3.3 Stiffening effect of concrete in tension area

In cracked parts of the reinforced concrete, the tensile forces in the crack are resisted by the reinforcement alone. Between two cracks, however, tension stresses are transferred to the concrete through the (displaceable) bond. Thus, the concrete contributes to the resistance of internal tension forces, which leads to an increased stiffness of the component. This effect refers to the stiffening contribution of the concrete in tension between the cracks and is also called *tension stiffening*.

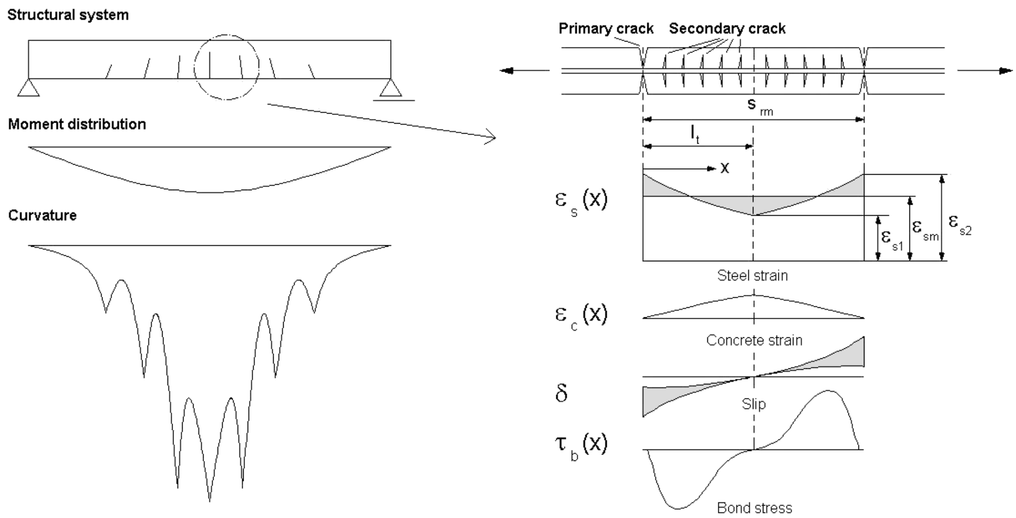


Figure 2.136 Stress and strain behavior between two primary cracks

The increase of the structural component stiffness due to tension stiffening can be considered in two ways:

- A residual, constant tension stress, which remains after the crack formation, can be involved in the concrete's stress-strain diagram.

The residual tension stress is clearly smaller than the tensile strength of the concrete. Alternatively, it is also possible to introduce modified stress-strain relations for the tension zone that consider the contribution of the concrete in tension between the cracks in the form of a decreasing branch in the graph after the tensile strength is reached.

- Another approach is to modify the "pure" stress-strain diagram of the reinforcing steel.

In this case, a reduced steel strain ε_{sm} is applied in the relevant cross-section, with the strain resulting from ε_{s2} and a reduction term due to the tension stiffening.

To consider *tension stiffening*, RF-CONCRETE NL uses the approach of modeling the concrete tensile strength according to Quast [4]. This model is based on a defined stress-strain relation of the concrete in the tension area (parabola-rectangle diagram). The basic assumptions of Quast's approach can be summarized as follows:

- full contribution of the concrete to tension until reaching the crack strain ε_{cr} or the calculational concrete tensile strength $f_{ct,R}$
- reduced stiffening contribution of the concrete in the tension zone according to the existing concrete strain
- no application of *tension stiffening* after the governing rebar starts yielding

To sum up, this means that the tensile strength $f_{ct,R}$ used for the calculation is **not** a fixed value but relates to the existing strain in the governing steel (tension) fiber. The maximum tensile strength $f_{ct,R}$ decreases linearly to zero, starting at the defined crack strain ε_{cr} until reaching the yield strain of the reinforcing steel in the governing steel fiber. This is achieved by means of the stress-strain relation in the tension area of the concrete shown in Figure 2.137 (parabola-rectangle diagram) and the determination of a reduction factor VMB (German: **V**ersteifende **M**itwirkung des **B**etons).

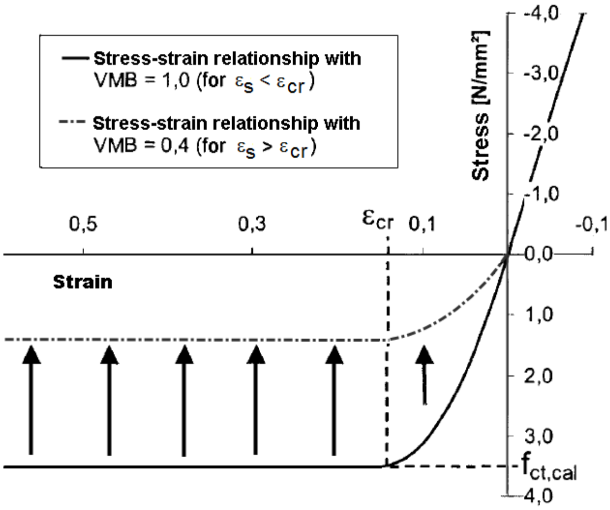


Figure 2.137 Stress-strain relation of concrete in the tension area with reduction factor VMB = 0.4

The stress-strain relation in the tension zone can be described with the following equations:

$$\sigma_c = \text{VMB} \cdot \left[f_{ct,R} \cdot \left(1 - \left(1 - \frac{\varepsilon}{\varepsilon_{cr}} \right)^{n_{PR}} \right) \right] \quad \text{for } 0 < \varepsilon < \varepsilon_{cr}$$

$$\sigma_c = \text{VMB} \cdot f_{ct,R} \quad \text{for } \varepsilon > \varepsilon_{cr} \text{ (constant distribution)}$$

Equation 2.93

The curvature of the parabola in the first section can be controlled by the exponent n_{PR}. The exponent should be adjusted in such a way that the transition from the compression zone to the tension zone is preferably achieved with the same modulus of elasticity.

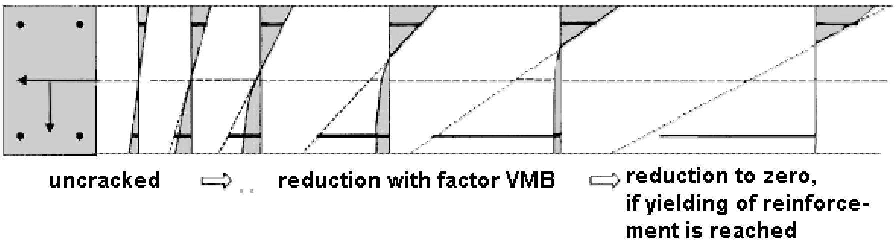


Figure 2.138 Stress conditions for increasing effect of *tension stiffening*

To determine the reduction factor VMB, the strain at the most tensioned steel fiber is used. The position of the reference point is shown in Figure 2.139.

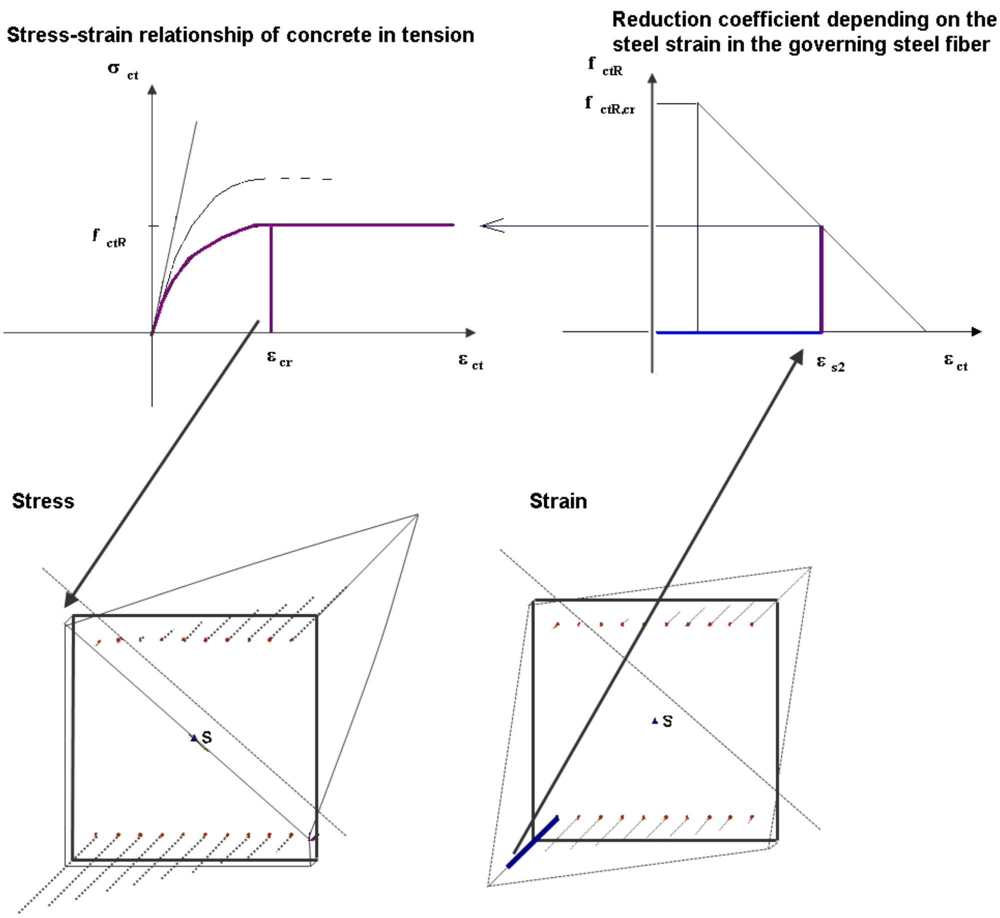


Figure 2.139 Determining the residual tensile strength for tension stiffening according to Quast [4].

The reduction parameter VMB decreases with increasing steel strain. In the diagram for the factor VMB (see Figure 2.140), it is evident that the factor VMB is reduced to zero exactly at the point when the yielding of the reinforcement starts.

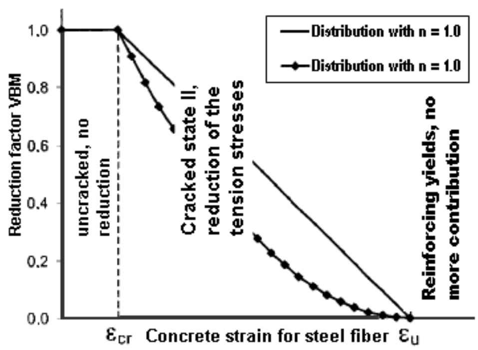


Figure 2.140 Reduction factor VMB

The distribution for the reduction factor VMB in state II ($\varepsilon > \varepsilon_{cr}$) can be controlled by means of the exponent n_{VMB}. According to Pfeiffer [5], the values n_{VMB} = 1 (linear) to n_{VMB} = 2 (parabola) are experiential values for structural elements subjected to bending. In his model, Quast [6] uses the exponent n_{VMB} = 1 (linear), thus achieving good concordance when recalculating column tests. According to Pfeiffer [5], it is possible to describe pure tension tests with acceptable concordance by using n_{VMB} = 2.

The assumption of a parabola-rectangle diagram for the cracked concrete tension zone can be regarded as a calculation aid. At first glance, there are great differences compared to the experimentally determined stress-strain diagrams on the tension side of the pure concrete.

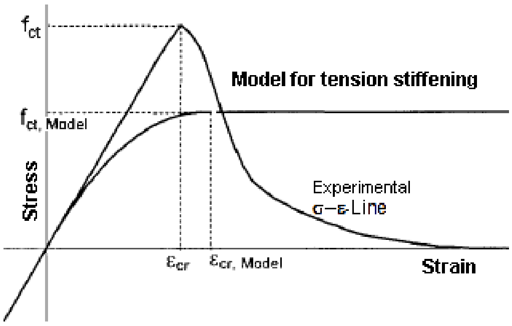


Figure 2.141 Comparison of model and laboratory test

The given stresses in the reinforced concrete cross-section in bending show that the parabola-rectangle diagram is indeed better suited to describe the mean of the strains and stresses.

In a bending beam, a concrete body forms between two cracks. It acts as a sort of wall into which tension forces are gradually reintroduced by the reinforcement. This results in a very irregular distribution of stress and strain. On average, however, we can create a plane of strain with a parabola-rectangle distribution with which it is possible to consider the mean curvature.

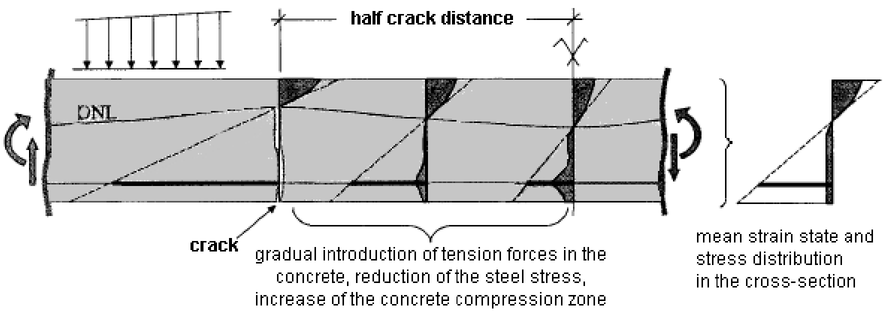


Figure 2.142 Existing state of stress when subjected to bending

Quast suggests the following calculation value for the tension strength $f_{ct,R}$ and the crack strain $\varepsilon_{cr,R}$ for his model.

$$f_{ct,R} = \left| \frac{1}{20} \cdot f_{cm} \right| \qquad \varepsilon_{cr,R} = \left| \frac{1}{20} \cdot \varepsilon_{c1} \right|$$

Equation 2.94

The calculational value for the tensile strength $f_{ct,R}$ is thus smaller than specified by the Eurocode. This is due to the description of the stress-strain relation and the determination of the reduction parameter VMB, in which the assumed tension stress and the resulting tension force are only slowly reduced after exceeding the tension strain. For a strain of $2 \cdot \varepsilon_{cr}$, there is also an acting tension stress of about $0.95 \cdot f_{ct,R}$. Thus, in case of bending, the reduction of the stiffness can be predicted well. In case of pure tension, the values for $f_{ct,R}$ mentioned above are too low. According to Pfeiffer [5], the values from EC 2 should be applied for the calculation value of the tensile strength.

The values for $f_{ct,R} = 1/20 \cdot f_{cm}$ recommended by Quast [6] can be reached by applying 60 % of the tensile strengths given in EC 2. On the one hand, the cracking of the cross-section is predicted too early when applying $f_{ct,R} = 0.6 \cdot f_{ctm}$. On the other hand, this already takes into account a reduction of the tensile strength under permanent load (about 70 %) or a temporarily higher load (e.g. the short-term application of the rare action combination) that results in a damaged tension zone.

The individual calculation values for the concrete's tension zone can be described as follows:

$$f_{ct,R} = 0.60 \cdot f_{ct,\text{standard}}$$ calculational tensile strength

$$\nu = \left| \frac{f_{cm}}{f_{ct,\text{standard}}} \right|$$ ratio, auxiliary factor

$$\varepsilon_{cr,R} = \left| \frac{\varepsilon_{c1}}{\nu} \right|$$ calculational crack strain

$$n_{PR} = 1.1 \cdot E_{cm} \cdot \frac{\varepsilon_{c1}}{f_{cm}}$$ exponent for general parabola (see Equation 2.93)

Equation 2.95

2.8.3.4 Reinforcing steel

For the serviceability limit state design, the program calculates with the mean strengths of the materials. The mean reinforcing steel strengths were published by the JCSS in the *Probabilistic Model Code*. The code specifies the mean value of the reinforcing steel's yield strength with $f_{ym} = 1.1 \cdot f_{yk}$.

RF-CONCRETE NL uses a bilinear distribution for the stress-strain relation of the reinforcing steel.

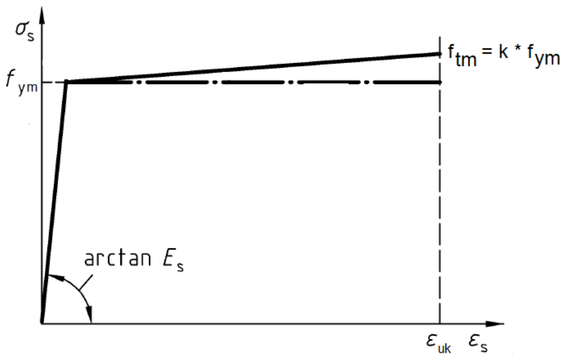


Figure 2.143 Stress-strain relation of reinforcing steel

The user can choose whether the plastic branch of the graph is horizontal or increases up to f_{tm}. The settings are specified in the *Settings for Nonlinear Calculation* dialog box (see Figure 3.12). To open this dialog box, click the button shown on the left that is available in the *Serviceability Limit State* tab of window 1.1 *General Data*.

2.8.4 Creep and Shrinkage

2.8.4.1 Considering creep

Creep describes the time-dependent deformation of the concrete with loading within a particular period of time. The essential influence values are similar to those of shrinkage (see chapter 2.8.4.2). Additionally, the so-called "creep-producing stress" has a considerable effect on the creep deformations.

Attention must be paid to the load duration, the time of load application, as well as the extent of the loading. Creep is taken into account by the creep coefficient $\varphi(t,t_0)$ at the point of time t.

In RF-CONCRETE Surfaces, the specifications for determining the creep coefficient are set in window 1.3 *Surfaces*. In it, you can specify the concrete's age at the considered point of time and at the beginning of loading, the relative air humidity, as well as the type of cement. Based on these specifications, the program determines the creep coefficient φ.

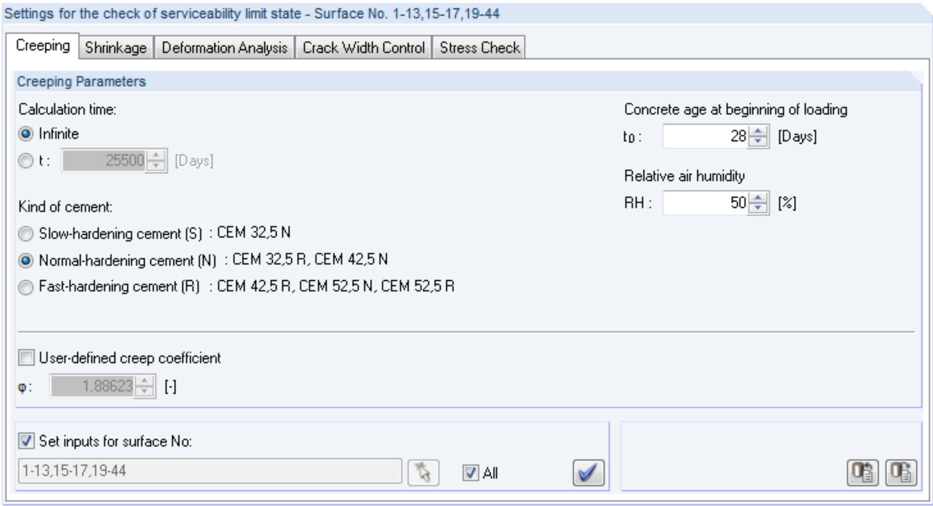


Figure 2.144 Window 1.3 *Surfaces, Creeping* tab

We now will briefly look at the determination of the creep coefficient φ according to EN 1992-1-1, clause 3.1.4. Using the following equations requires the creep-producing stress σ_c of the acting permanent load to not exceed the following value.

$$\sigma_c \leq 0.45 \cdot f_{ckj}$$

Equation 2.96

where

f_{ckj}	cylinder compressive strength of the concrete at the point of time when the creep-producing stress is applied

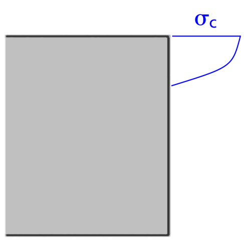


Figure 2.145 Creep-producing stress σ_c

Under the assumption of a linear creep behavior ($\sigma_c < 0.45 \cdot f_{ckj}$), the concrete's creep can be determined through a reduction of the concrete's modulus of elasticity.

$$E_{c,eff} = \frac{E_{cm}}{1.0 + \varphi(t, t_0)}$$

Equation 2.97

where

E_{cm}	mean modulus of elasticity according to EN 1992-1-1, Table 3.1
$\varphi(t,t_0)$	creep coefficient
t	age of concrete at relevant point of time in days
t_0	age of concrete when load application starts in days

According to EN 1992-1-1, clause 3.1.4, the creep coefficient $\varphi(t,t_0)$ at the analyzed point of time t can be calculated as follows.

$$\phi(t, t_0) = \phi_{RH} \cdot \beta(f_{cm}) \cdot \beta(t_0) \cdot \beta(t, t_0)$$

Equation 2.98

where

$$\varphi_{RH} = \left[1 + \frac{1 - \frac{RH}{100}}{0.10 \cdot \sqrt[3]{h_0}} \cdot \alpha_1\right] \cdot \alpha_2$$

RH	relative humidity [%]
$h_0 = \frac{2 \cdot A_c}{u}$	effective component thickness [mm] (for surfaces: $h_0 = h$) A_c cross-section area u cross-section perimeter
$\alpha_1 = \left(\frac{35}{f_{cm}}\right)^{0.7}$	adjustment factor f_{cm} mean value of cylinder compressive strength
$\alpha_2 = \left(\frac{35}{f_{cm}}\right)^{0.2}$	adjustment factor
$\beta(f_{cm}) = \frac{16.8}{\sqrt{f_{cm}}}$	coefficient for considering the concrete compressive strength

$$\beta(t_0) = \frac{1}{0.1 + t_{0,eff}^{0.2}}$$

coefficient for considering the age of concrete

$$t_{0,eff} = t_0 \left[1 + \frac{9}{2 + t_0^{1.2}}\right]^{\alpha} \geq 0.5 \cdot d$$

$$\beta(t,t_0) = \left[\frac{t - t_0}{\beta_H + t - t_0}\right]^{0.3}$$

coefficient for considering the load duration

t	age of concrete at relevant point of time in days
t_0	age of concrete when load application starts in days

$$\beta_H = 1.5 \cdot \left[1 + (0.012 \cdot RH)^{18}\right]^{\alpha} \cdot h_0 + 250 \cdot \alpha_3 \leq 1500 \cdot \alpha_3$$

RH	relative humidity [%]
h_0	effective component thickness [mm]

$$\alpha_3 = \left(\frac{35}{f_{cm}}\right)^{0.5}$$

adjustment factor

The influence of the type of cement on the concrete's creep coefficient can be taken into account by modifying the load application age t_0 with the following equation:

$$t_0 = t_{0,T} \cdot \left(1 + \frac{9}{2 + (t_{0,T})^{1.2}}\right)^{\alpha} \geq 0.5$$

Equation 2.99

where

$t_0 = t_T$	effective age of concrete when load application starts while taking the influence of temperature into account
α	exponent depending on type of cement:

-1 : slow-hardening cements (S) (32,5)
0 : normal- or rapid-hardening cements (N) (32,5 R; 42,5)
1 : rapid-hardening, high-strength cements (R) (42,5 R; 52,5)

Considering creep in the calculation

If the strains at the point of time t = 0 as well as at a later point of time t are known, it is possible to determine the creep coefficient φ for a calculational consideration in the model.

$$\varphi_t = \frac{\varepsilon_t}{\varepsilon_{t=0}} - 1$$

Equation 2.100

This equation is rearranged to the strain at the point of time t. Thus, we get the following relation, which is valid for uniform stresses (cf. Equation 2.96):

$$\varepsilon_t = \varepsilon_{t=0} \cdot (\varphi_t + 1)$$

Equation 2.101

For stresses higher than approximately 0.4 · f_{ck}, the strains increase disproportionately, resulting in the loss of the linearly assumed reference.

The calculation in RF-CONCRETE NL uses a common solution that is reasonable for construction purposes. The stress-strain diagram of the concrete is distorted by the factor (1 + φ).

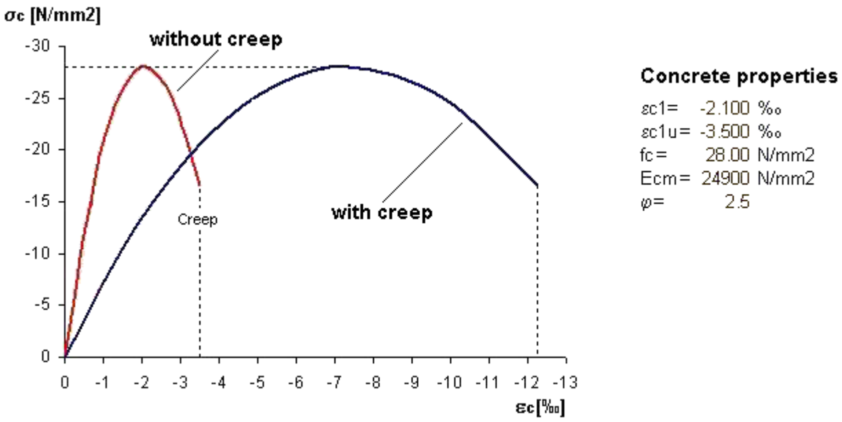


Figure 2.146 Distortion of the stress-strain relation for determining the creep effect

When taking creep into account, uniform creep-producing stresses are assumed during the period of load application, as can be seen in Figure 2.146. Because of neglected stress redistributions, the deformation is slightly overestimated due to this assumption. The stress reduction without a change in strain (relaxation) is only taken into account to a limited degree in this model. If we assume a linear elastic behavior, a proportionality could be presumed and the horizontal distortion would also reflect the relaxation at a ratio of (1 + φ). This correlation, however, is lost for the nonlinear stress-strain relationship.

Thus, it becomes clear that this procedure must be understood as an approximation. Therefore, a reduction of the stresses due to relaxation as well as nonlinear creep cannot or can only be approximately represented.

2.8.4.2 Taking shrinkage into account

Shrinkage describes a time-dependent change of the volume without the effect of external loads or temperature. This manual will not go into details regarding shrinkage problems and their individual types (drying shrinkage, autogenous shrinkage, plastic shrinkage, and carbonation shrinkage).

Significant influence values of shrinkage are relative humidity, effective thickness of structural components, aggregate, concrete strength, water-cement ratio, temperature, as well as the type and duration of curing. The shrinkage-determining value is the total shrinkage strain ε_{cs} at the considered point of time t.

According to EN 1992-1-1, clause 3.1.4, the total shrinkage strain ε_{cs} is composed of the components for drying shrinkage ε_{cd} and autogenous shrinkage ε_{ca}:

$$\varepsilon_{cs} = \varepsilon_{cd} + \varepsilon_{ca}$$

Equation 2.102 [7] Eq. (3.8)

The component from **drying shrinkage** ε_{cd} is determined as follows.

$$\varepsilon_{cd}(t) = \beta_{ds}(t, t_s) \cdot k_h \cdot \varepsilon_{cd,0}$$

Equation 2.103 [7] Eq. (3.9)

where

$$\beta_{ds}(t, t_s) = \frac{(t - t_s)}{(t - t_s) + 0.4 \cdot \sqrt{h_0^3}}$$

Equation 2.104 [7] Eq. (3.10)

t	age of concrete at relevant point of time in days
t_s	age of concrete when shrinkage starts in days
$h_0 = \frac{2 \cdot A_c}{u}$	effective component thickness [mm] (for surfaces: $h_0 = h$) A_c cross-section area u cross-section perimeter
k_h	coefficient according to [4] Table 3.3 depending on the effective cross-section thickness h_0
$\varepsilon_{cd,0}$	basic value according to [4] Table 3.2 or Annex B, Eq. (B.11):

$$\varepsilon_{cd,0} = 0.85 \cdot \left[(220 + 110 \cdot \alpha_{ds1}) \cdot exp\left(-\alpha_{ds2} \cdot \frac{f_{cm}}{f_{cmo}}\right)\right] \cdot 10^{-6} \cdot \beta_{RH}$$

α_{ds1}, α_{ds2}	factors for considering the type of cement (see Table 2.3)
f_{cm}	mean cylinder compressive strength of concrete in [N/mm²]
f_{cmo}	= 10 N/mm²

$$\beta_{RH} = 1.55 \cdot \left[1 - \left(\frac{RH}{RH_0}\right)^3\right]$$

RH	relative humidity of environment [%]
RH_0	100 %

Cement	Class	Property	α_{ds1}	α_{ds2}
32,5 N	S	slow-hardening	3	0.13
32,5 R; 42,5 R	N	normal-hardening	4	0.12
42,5 R; 52,5 N/R	R	rapid-hardening	6	0.11

Table 2.3 Factors α_{ds1} and α_{ds2} depending on the type of cement

The **autogenous shrinkage strain** ε_{ca} is determined as follows.

$$\varepsilon_{ca}(t) = \beta_{as}(t) \cdot \varepsilon_{ca}(\infty)$$ [7] Eq. (3.11)

where

$$\beta_{as}(t) = 1 - e^{-0.2\sqrt{t}}$$ [7] Eq. (3.12)

$$\varepsilon_{ca}(\infty) = 2.5 \cdot (f_{ck} - 10) \cdot 10^{-6}$$ [7] Eq. (3.13)

t in days

Taking shrinkage in RF-CONCRETE NL into account (while considering the reinforcement)

The data for the shrinkage strain is entered in window 1.3 *Surfaces*. In it, you can specify the age of concrete at the relevant point of time and at the beginning of shrinkage, the relative air humidity, and the type of cement. Based on these specifications, RF-CONCRETE NL determines the shrinkage strain ε_{cs}.

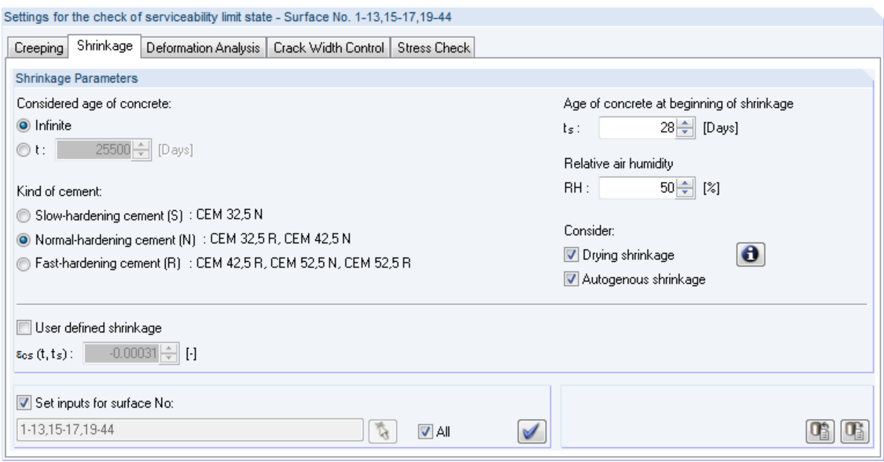


Figure 2.147 Window 1.3 *Surfaces*, *Shrinkage* tab

The shrinkage strain ε_{cs} (t,t_s) can also be specified manually, independent of standards.

The shrinkage strain is only applied to the concrete layers; the reinforcement layers remain unconsidered. Thus, there is a difference from the classical temperature loading, which also affects the reinforcement layers. Therefore, the model for shrinkage used in RF-CONCRETE NL considers the restraint of the shrinkage strain ε_{sh} that is exerted by the reinforcement or the cross-section curvature for an unsymmetrical reinforcement. The resulting loads from the shrinkage strain are automatically applied to the surfaces as virtual loads and calculated. Depending on the structural system, the shrinkage strain results in additional stresses (statically indeterminate system) or additional deformations (statically determinate system). For shrinkage, RF-CONCRETE NL therefore considers the influence of the structural boundary conditions in different ways.

The loads resulting from shrinkage are automatically assigned to the loading for serviceability defined in window 1.1 *General Data* and are therefore included in the nonlinear calculation.

The shrinkage depends on the correct distribution of the stiffness in the cross-section. Therefore, the consideration of *tension stiffening* (residual tensile strength of concrete according to Quast) as well as a small value for damping are recommended for the concrete's tension zone.

The 1D model shown in Figure 2.148 illustrates how shrinkage is considered in the program.

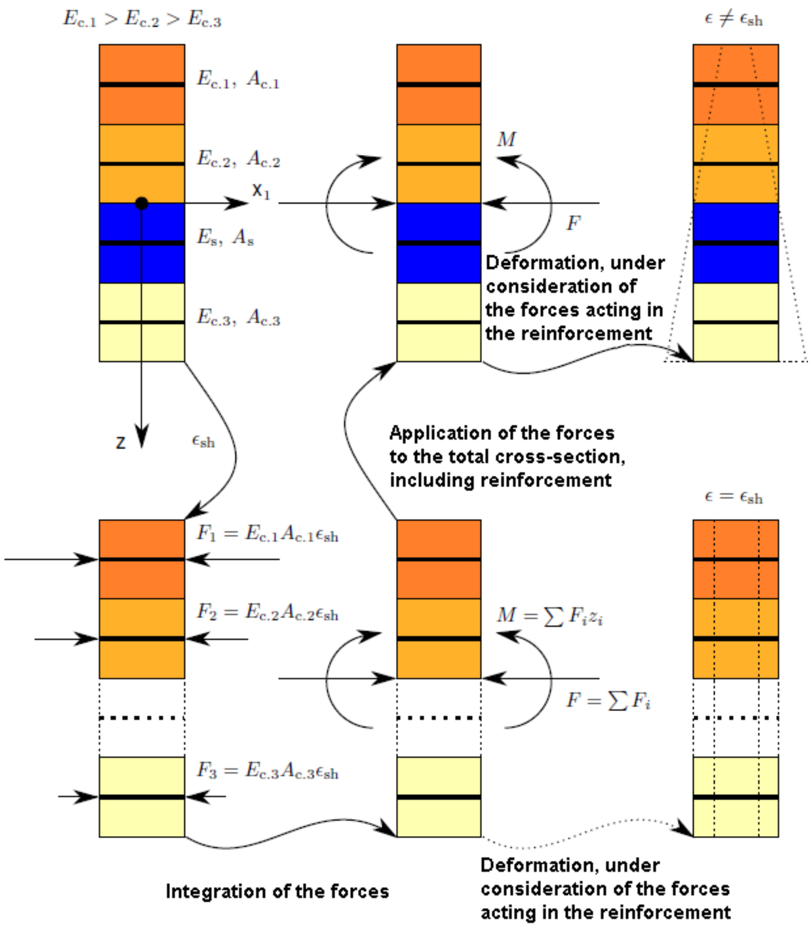


Figure 2.148 1D example for shrinkage

As a simplification, four layers are considered: The dark orange layers represent the concrete with little damage, the light orange layers the more heavily damaged concrete. The blue layer corresponds to the reinforcement. Each concrete layer is characterized by the actual modulus of elasticity $E_{c,i}$ and each cross-sectional area by $A_{c,i}$. The reinforcement is characterized by the actual modulus of elasticity E_s and the cross-sectional area A_s. Each layer is described by means of the coordinate z_i.

Considering shrinkage as an external load

Shrinkage strain can also be applied as an external load in RFEM: In the *New Surface Load* dialog box of RFEM, you can open the *Generate Surface Load Due to Shrinkage* dialog box by clicking the button shown on the left.

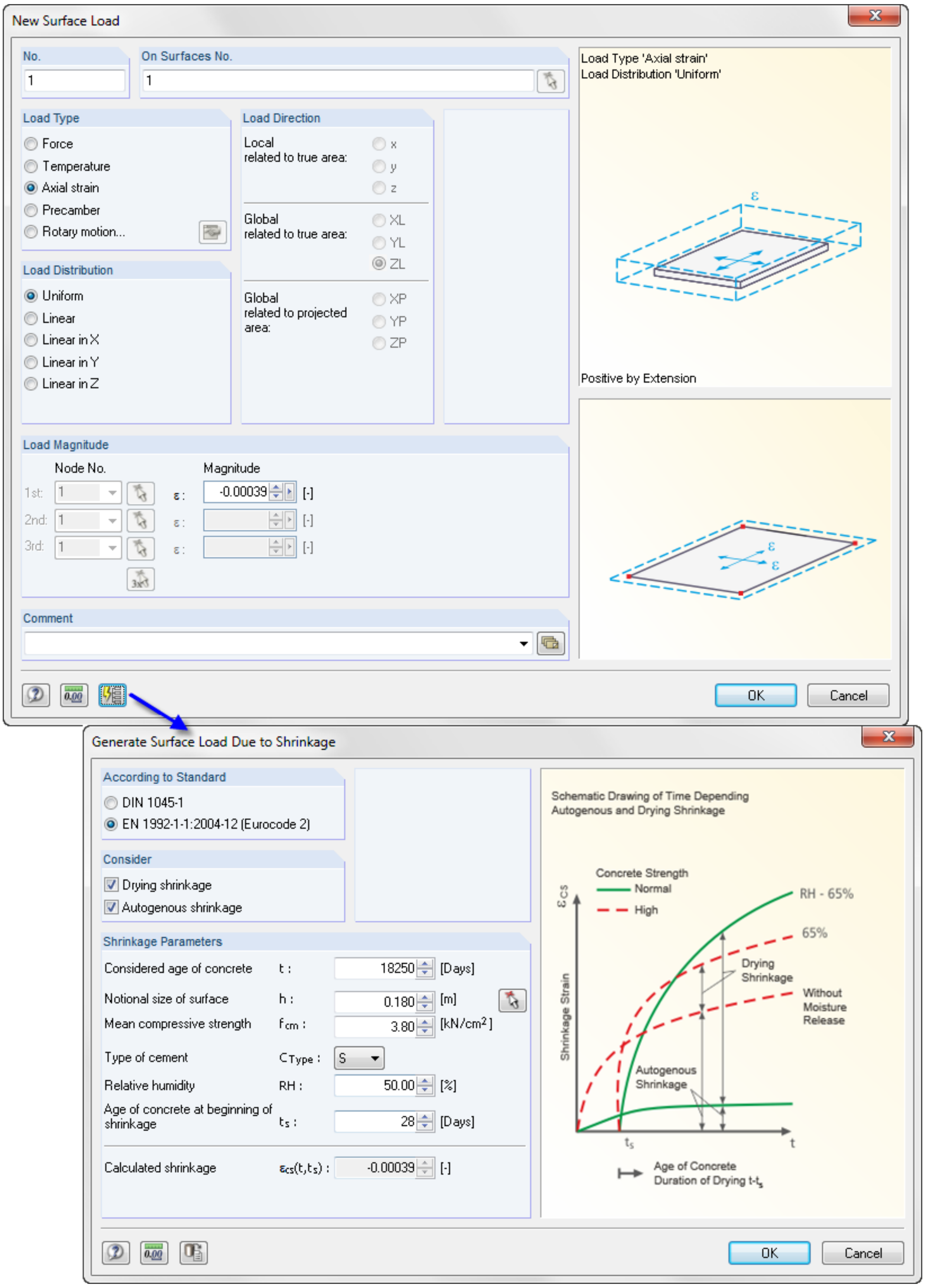


Figure 2.149 *Generate Surface Load Due to Shrinkage RFEM dialog box*

In this dialog box, you can enter the parameters for determining the shrinkage strain. To transfer the determined shrinkage value as a load magnitude into the initial dialog box, *New Surface Load*, click [OK]. The load type is automatically set to *Axial strain*. Please note that the shrinkage strain acts on the entire cross-section and that possible restraints or cross-section curvatures are not taken into account by the reinforcement.

3 Input Data

When you start the add-on module, a new window appears. A navigator that manages the available module windows is displayed on the left. The drop-down list above the navigator contains the design cases (see chapter 8.1).

The design-relevant data can be defined in several input windows. When you open RF-CONCRETE Surfaces for the first time, the following parameters are imported automatically:

- load cases, load combinations, and result combinations
- materials
- surfaces
- internal forces (in background, if calculated)

To open a window, click the corresponding entry in the navigator. Use the buttons shown on the left to set the previous or next window. You can also use the function keys [F2] (forwards) and [F3] (backwards) to go through the windows.

To save the entered data, click [OK]. RF-CONCRETE Surfaces closes and you return to the main program. To exit the add-on module without saving the data, click [Cancel].

3.1 General Data

In window 1.1 *General Data*, you can specify the design standard and the actions. The tabs manage the load cases, load combinations, and result combinations for the ultimate and serviceability limit state designs.

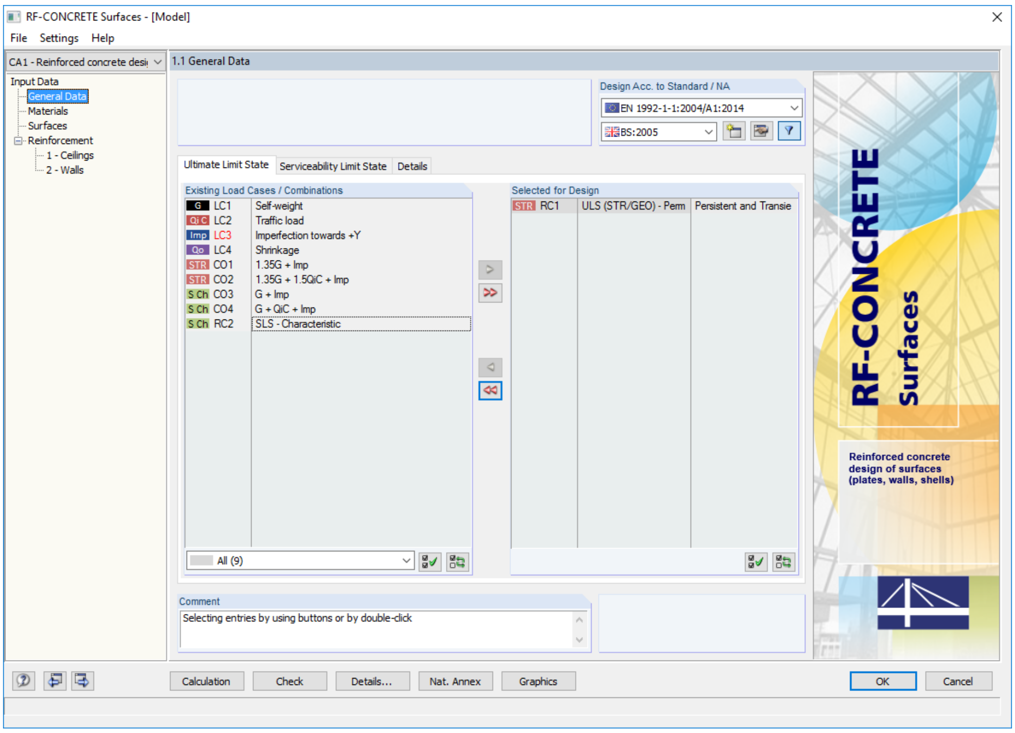


Figure 3.1 Window 1.1 *General Data*

Design Acc. to Standard / NA

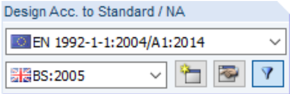
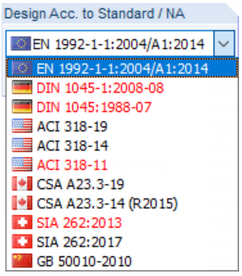


Figure 3.2 Standard and National Annex for reinforced concrete design

Standard



You can specify the standard according to which you want to perform the ultimate and serviceability limit state design. The following standards for reinforced concrete are available in the list:

EN 1992-1-1:2004/A1:2014	European Union
DIN 1045-1:2008-08	Germany
DIN 1045:1988-07	Germany
ACI 318-19	USA
ACI 318-14	USA
ACI 318-11	USA
CSA A23.3-19	Canada
CSA A23.3-14 (R2015)	Canada
SIA 262:2013	Switzerland
SIA 262:2017	Switzerland
GB 50010-2010	China

You can purchase each standard separately.

You can use the button to hide and display old standards in the list. They are marked in red.

National Annex

For the design according to the Eurocode (EN 1992-1-1:2004/A1:2014), you have to specify the National Annex whose parameters apply for the checks.

Figure 3.3 National Annexes for EN 1992-1-1

Nat. Annex...

Click the [Edit] button to view the preset parameters (see Figure 3.4). You can also access the *Parameters of National Annex* dialog box with the [Nat. Annex] button that is available in every input window.

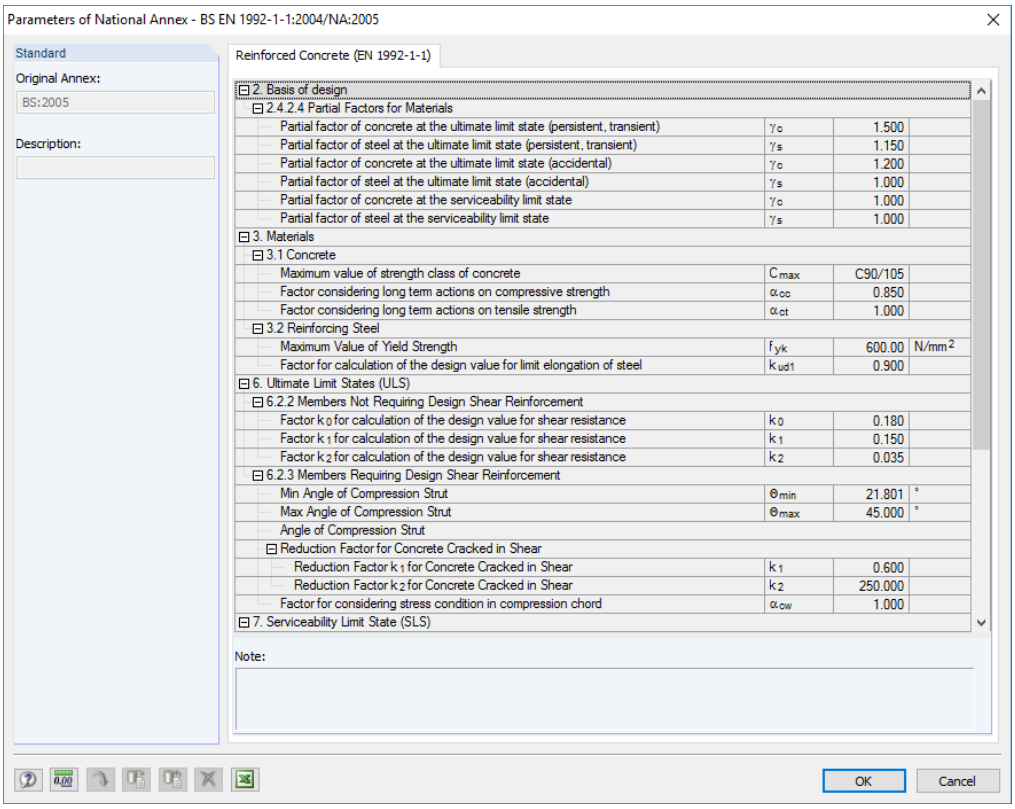


Figure 3.4 *Parameters of National Annex* dialog box

In this dialog box, you can find all design-relevant coefficients specified in the National Annexes. They are listed by the Eurocode's clause numbers.

If other specifications apply for partial safety factors, reduction factors, compression strut angles, etc., you can adjust the parameters. To do this, first click the [New] button to create a copy of the current National Annex. In this user-defined Annex, you can then change the parameters.

Comment

Figure 3.5 User-defined comment

In this text box, you can enter a user-defined note to describe the current design case, for example.

3.1.1 Ultimate Limit State

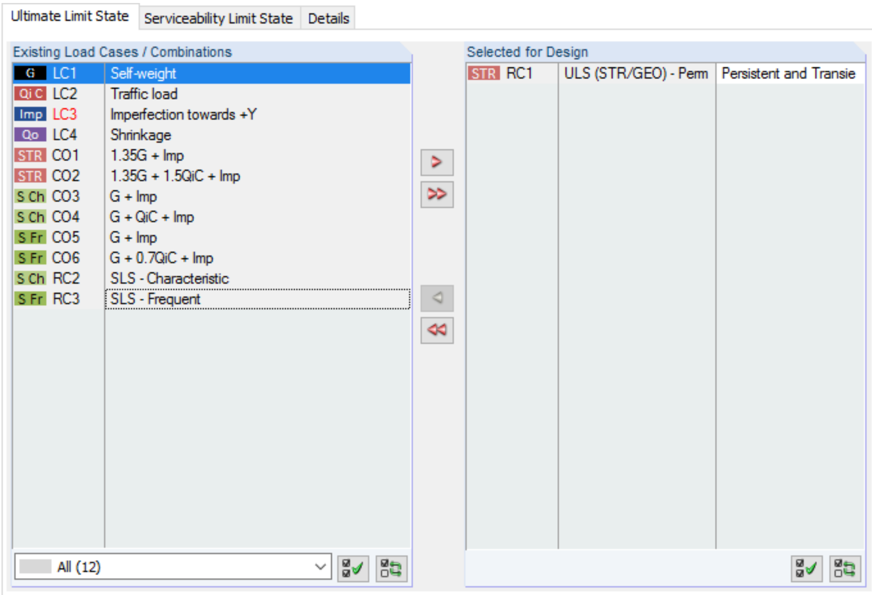


Figure 3.6 Window 1.1 *General Data, Ultimate Limit State* tab

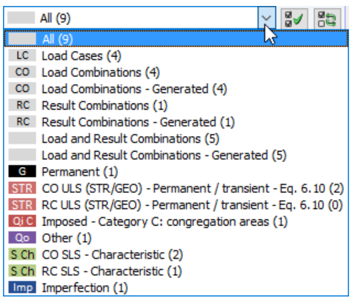
Existing Load Cases / Combinations

This column lists all load cases, load combinations, and result combinations that have been defined in RFEM.

Click [>] to transfer selected entries into the *Selected for Design* list on the right. You can also double-click the items to transfer them. To transfer the entire list to the right, click [>>] .

Selecting several load cases is possible by clicking them one by one while pressing the [Ctrl] key, as is common in Windows applications. This allows you to transfer several load cases at once.

If a load case is marked in red, like LC3 in Figure 3.6, it cannot be calculated: It indicates a load case without load data or a load case containing imperfections. When you transfer it, a corresponding warning appears.



Below the list, several filter options are available. They help you to assign the entries sorted by load cases, load combinations, or action categories. The buttons have the following functions:

[Select all button]	Selects all load cases in the list.
[Invert selection button]	Inverts the selection of load cases.

Table 3.1 Buttons in *Ultimate Limit State* tab

Selected for Design

The column on the right lists the load cases, load combinations, and result combinations selected for the design. To remove selected items from the list, click or double-click the entries. To empty the entire list, click .

You can assign the load cases, load combinations, and result combinations to the following design situations:

- *Persistent and Transient*
- *Accidental*

This classification controls the partial safety factors γ_c and γ_s according to EN 1992-1-1, Table 2.1 (see Figure 3.4 and Figure 3.44).

You can change the design situation by using the list that you can access by clicking at the end of the input field.

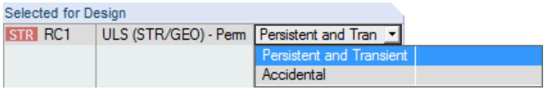


Figure 3.7 Assigning the design situation

Just like before, a multiple selection is possible by keeping the [Ctrl] key pressed, thus allowing you to change several items at once.

The analysis of an enveloping max/min result combination is faster than the analysis of all load cases and load combinations indiscriminately selected for the design. However, when analyzing a result combination, it is difficult to discern the influence that the included actions have (see also chapter 4.1).

3.1.2 Serviceability Limit State

The serviceability limit state design depends on the results of the ultimate limit state design. Therefore, it is not possible to perform the serviceability limit state design alone.

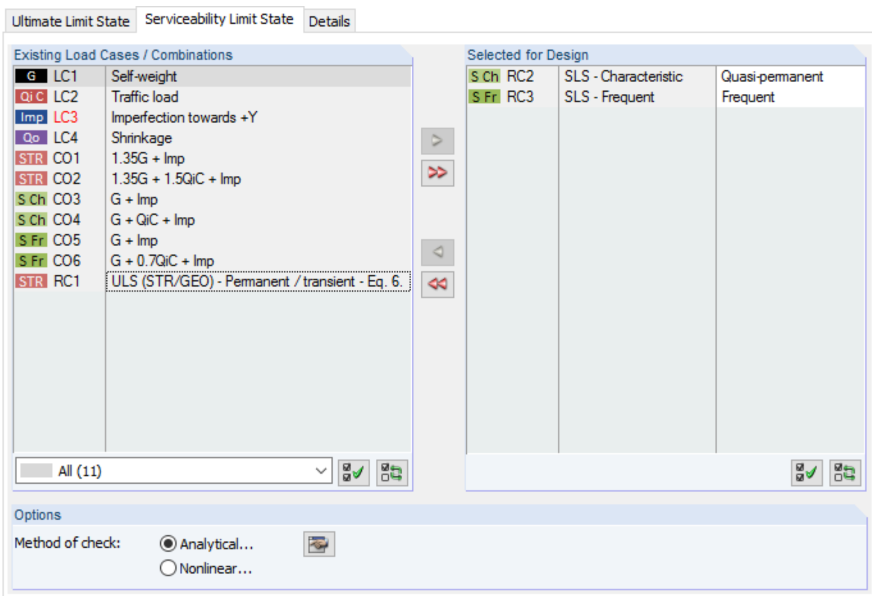


Figure 3.8 Window 1.1 *General Data, Serviceability Limit State* tab

Existing Load Cases / Combinations

This section lists all load cases, load combinations, and result combinations that have been defined in RFEM.

Normally, the actions and partial safety factors that are relevant for the serviceability limit state (SLS) design are different from the ones for the ultimate limit state. The corresponding combinations can be created in RFEM.

Selected for Design

Load cases, load combinations, and result combinations can be added or removed as described in chapter 3.1.1.

For EN 1992-1-1, you can assign different limit values for the deflection to the individual load cases, load combinations, and result combinations. The following design situations are available:

- *Characteristic with direct load*
- *Characteristic with imposed deformation*
- *Frequent*
- *Quasi-permanent*

You can change the design situation by using the list that can be accessed by clicking the button at the end of the input field.

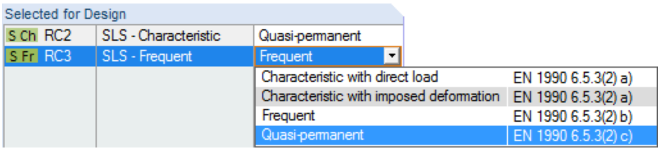


Figure 3.9 Assigning the design situation

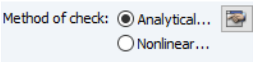
With the [Details] button, you can access settings for the individual design situations (see chapter 4.1.2).

Method of check

The two option buttons allow you to configure whether you want to perform the serviceability limit state designs according to the analytical or nonlinear method.

Both methods are described on the following pages.

3.1.2.1 Analytical method of check



The *Analytical* method is preset for the check. This method uses the equations given by the standards for reinforced concrete. This method is described in chapter 2.6 of the manual.

Click the button to open a dialog box for checking and, if necessary, adjusting the design parameters.

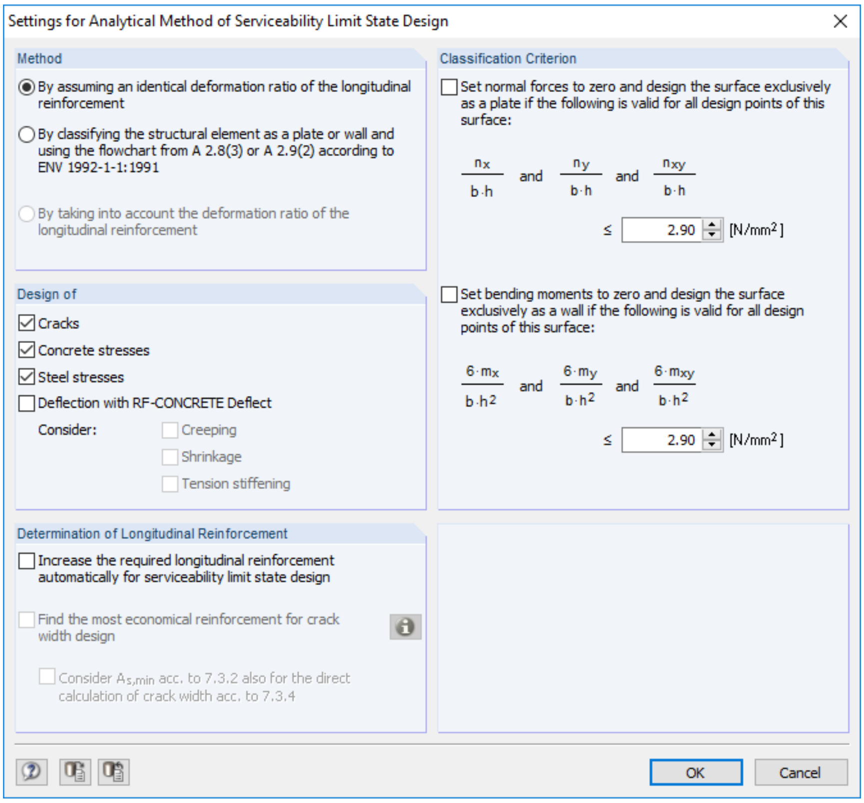


Figure 3.10 *Settings for Analytical Method of Serviceability Limit State Design* dialog box

Method

This dialog section allows you to control which strain ratio of the reinforcement directions is applied for the serviceability limit state design.

With *By assuming an identical deformation ratio of the longitudinal reinforcement*, the program assumes the same strain ratio of the provided reinforcement: All rebars are subjected to the same strain in the individual reinforcement directions. This approach is a fast and exact procedure. The selection of the compression strut inclination plays a significant role in this. This method is based on a purely geometrical division (see chapter 2.6). It is applicable if the provided reinforcement more or less corresponds to the required reinforcement.

The *By classifying the structural element as a plate or wall* option provides a simplified solution that you can use for a non-rotated, orthogonal reinforcement mesh: For each design point, the program checks if the tensile stresses from axial forces or bending moments do not exceed a certain stress. The limit value of the stress is defined in the *Classification Criterion* section. It is used to control whether the surface is designed as a plate (axial forces are set to zero) or a wall (moments are set to zero).

By neglecting small internal force components, it is possible to use the flowchart from ENV 1992-1-1, Annex A 2.8 or 2.9. The design internal forces correspond to the values shown in RFEM Table 4.16 (see RFEM manual, chapter 8.16).

Should the classification criterion for a design point of the surface not be fulfilled, an error message will appear during the calculation.

The *By taking into account the deformation ratio of the longitudinal reinforcement* option is only enabled for 2D model types (see Figure 2.1). This method considers the effective strain ratios due to the selected reinforcement and takes them into account for the serviceability limit state design.

Design of

In this dialog section, you can specify whether to analyze stresses and/or cracks in the design. You must select at least one of the check boxes.

If you select *Cracks*, it is possible to check the minimum reinforcements $a_{s,min}$, the limit diameters lim d_s, the maximum crack spacings max s_l, and the crack widths w_k. The settings for the individual checks can be specified in window 1.3 *Surfaces* (see chapter 3.3).

The analysis of the stresses can be differentiated with regards to the *Concrete stresses* σ_c and *Steel stresses* σ_s.

Furthermore, it is possible to calculate the *Deflection with RF-CONCRETE Deflect*, taking creeping, shrinkage, and tension stiffening into account (see chapter 2.7). To do so, you need a license of the add-on module **RF-CONCRETE Deflect**.

Determination of Longitudinal Reinforcement

The *Increase the required longitudinal reinforcement automatically* check box allows you to control if the longitudinal reinforcement is to be dimensioned to fulfill the serviceability limit state designs. If this check box is clear, only the specifications entered in the *Longitudinal Reinforcement* tab of window 1.4 (see chapter 3.4.3) are used: basic reinforcement, required reinforcement from ultimate limit state design, or basic reinforcement with provided additional reinforcement.

Dimensioning the reinforcement for the serviceability limit state design occurs by increasing the reinforcement iteratively. As the initial value for the iteration, the program uses the required ULS reinforcement to resist the given characteristic load. The dimensioning of the reinforcement has no result if the rebar spacing s_l of the applied reinforcement reaches the rebar diameter d_{sl}. In this case, the result windows indicate that the respective point cannot be designed.

In the design according to EN 1992-1-1, it is possible to *Find the most economical reinforcement for crack width design*. Click the [Info] button to display information about this option (see Figure 3.11). The *Information* dialog box describes when the check of crack width can be considered as being fulfilled. Moreover, clause 7.2 of EN 1992-1-1 describes the conditions under which the stresses should be limited.

This means that not **all** design ratios shown in window 3.1 have to be less than 1 in order for the serviceability limit state design to be fulfilled!

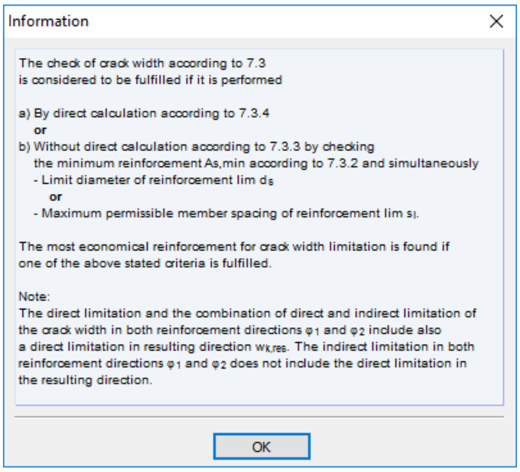


Figure 3.11 *Information* dialog box for determining the most economical reinforcement

Dimensioning the reinforcement with regards to the concrete and steel stress, the limit diameter, and the maximum bar spacing is done separately for each reinforcement direction. However, if the resulting crack width $w_{k,res}$ is governing to fulfill the crack width check, the reinforcement amount is increased equally for each direction.

Check criteria that do not have to be fulfilled for economical reasons are indicated by message 236) in the result windows of the serviceability check: "The check of the reinforcing layers does not need to be fulfilled for economical reasons". The check of crack width that is governing for the most economical reinforcement is marked by message 235): "The check restricts increase of reinforcement for economical reasons". This message applies to the designs for lim d_s, lim s_l, and w_k, but not for $a_{s,min}$.

If the *Find the most economical reinforcement for crack width design* check box is selected, you cannot specify a user-defined additional reinforcement for the SLS design in the *Longitudinal Reinforcement* tab of window 1.4 *Reinforcement*.

Classification Criterion

This dialog section (see Figure 3.10) is only available for 3D model types. The check boxes allow you to control if small *normal forces* and/or *bending moments* may be neglected in order to design surfaces as pure plates (upper box selected) or walls (lower box selected) in an idealized way. As the limit value, the mean value of the axial tensile strength f_{ctm} is respectively preset with 2.9 N/mm^2 of concrete C30/37: It is assumed that the tensile strength of concrete compensates for a crack formation due to minor tensile stresses, which is why they can be neglected.

If you have selected the classification as a plate or wall in the *Method* dialog section, you have to select at least one of the two check boxes.

3.1.2.2 Nonlinear method of check

In order to perform a design according to the *Nonlinear* method, a license of the **RF-CONCRETE NL** add-on module is required. This method is described in chapter 2.8. The program performs a physical and geometrical nonlinear calculation.

The nonlinear design method acts on the assumption of an interaction between model and loading, requiring a clear distribution of internal forces. Therefore, only load cases and load combinations can be analyzed, but result combinations (RC) cannot. In a result combination, two values are available for each FE node — maximum and minimum.

The internal forces for the nonlinear design are generally determined according to the second-order analysis.

Click the button to open a dialog box for checking and, if necessary, adjusting the design parameters. This dialog box is divided into the *Options* and *Material Properties* tabs.

Options

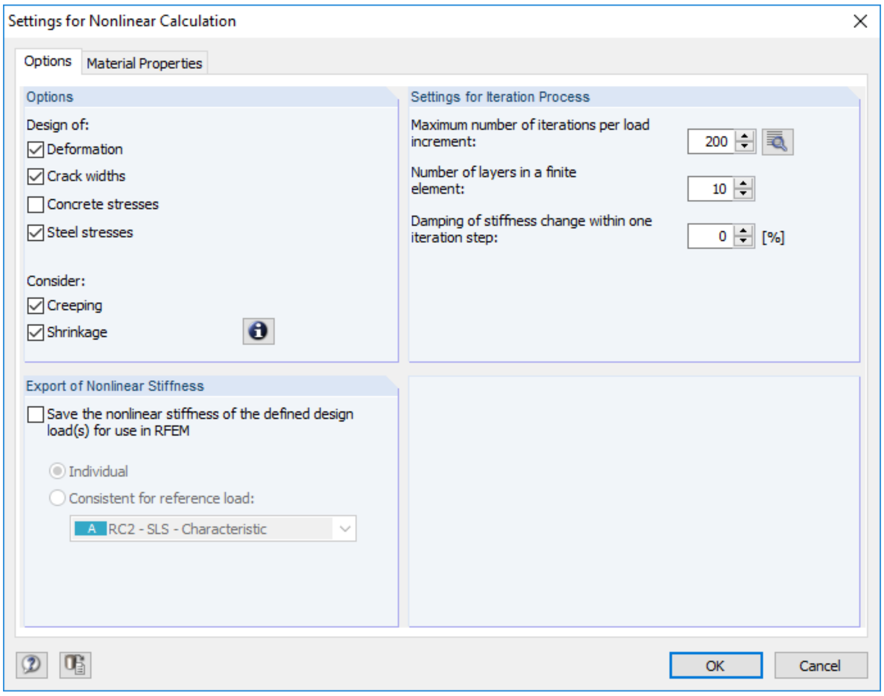


Figure 3.12 *Settings for Nonlinear Calculation* dialog box, *Options* tab

Options

This dialog section allows you to control which serviceability limit state designs you want to carry out: deformation, crack widths, as well as concrete and steel stresses. You must select at least one of the four check boxes.

You can also decide if the influence of creeping and shrinkage should be considered in the nonlinear calculation.

Detailed settings for the individual checks as well as for creep and shrinkage are specified in window 1.3 *Surfaces* (see chapter 3.3.2).

Export of Nonlinear Stiffness

The *Save the nonlinear stiffness* check box allows you to control if the nonlinearly determined stiffnesses are also available for a calculation in RFEM.

The stiffnesses can by exported *Individually* for each designed load case. In the *Load Cases and Combinations* dialog box of RFEM, you can then assign the according stiffness from RF-CONCRETE Surfaces to each of these load cases. RFEM allocates the load cases automatically. If you select *Consistent for reference load*, specify the governing load case in the drop-down list below. In RFEM, you can then assign the stiffness that results from these loads to all load cases that are defined.

The consideration of nonlinear stiffnesses in RFEM is described in chapter 7.3.1.3 of the RFEM manual.

Settings for Iteration Process

The settings in this dialog section influence the process of the nonlinear design method. You can find more information in chapter 2.8.2.4.

When modifying the precision of iterations, make sure that the *Maximum number of iterations* is higher than the point in the calculation process from which the deformation criterion is additionally taken into account. Click the [Details] button to open the *Calculation Parameters* dialog box of RFEM. In it, you can adjust the precision of the convergence criteria for the nonlinear calculation.

In the nonlinear calculation, the surface is divided into so-called *layers* (see chapter 2.8.2.1). The recommended number of layers is 10.

Furthermore, you can influence the convergence behavior with *Damping*: Damping controls the magnitude of the stiffness change in the subsequent calculation steps. For example, if you specify a damping of 50 %, the change of the stiffness between step 2 and 3 can at most be 50 % of the stiffness change between step 1 and step 2.

Material Properties

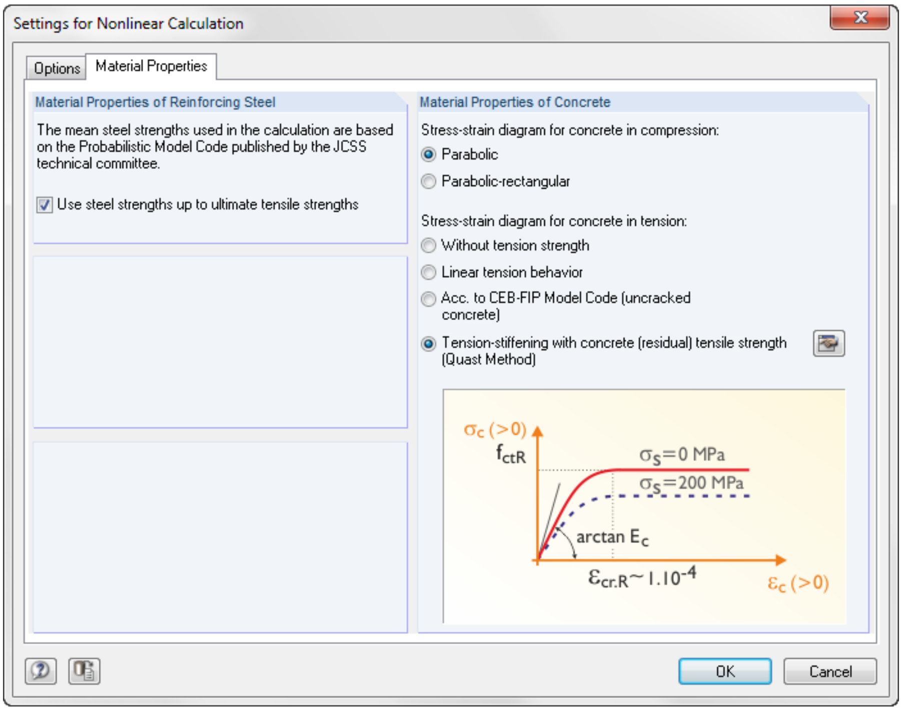


Figure 3.13 *Settings for Nonlinear Calculation* dialog box, *Material Properties* tab

Material Properties of Reinforcing Steel

The check box allows you to control if the calculation in the plastic zone of the reinforcing steel's stress-strain diagram is carried out with a rising or a horizontal graph (see chapter 2.8.3.4).

Material Properties of Concrete

In this dialog section, you can specify the stress-strain relations of the concrete *in compression* and *in tension*. A parabolic diagram for compression and tension stiffening for concrete tensile stresses is preset.

For *Tension stiffening* (consideration of the stiffening effect of concrete in the tension zone), you can specify the parameters used to apply the concrete tensile strength between the cracks in a separate dialog box. To open it, click the [Edit] button.

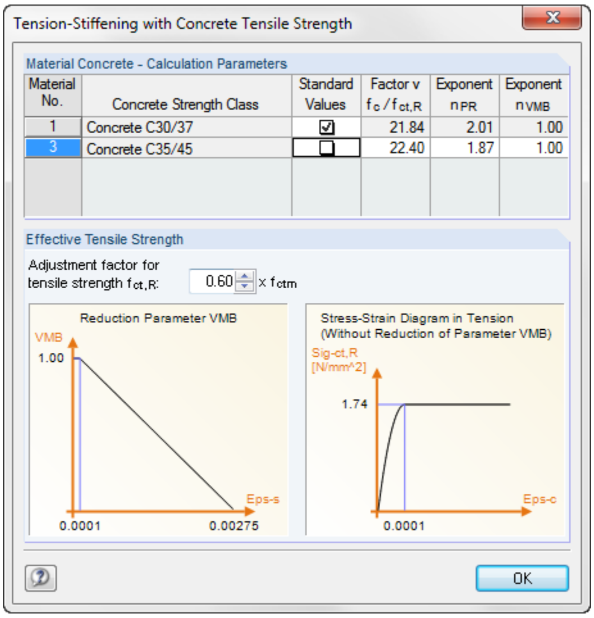


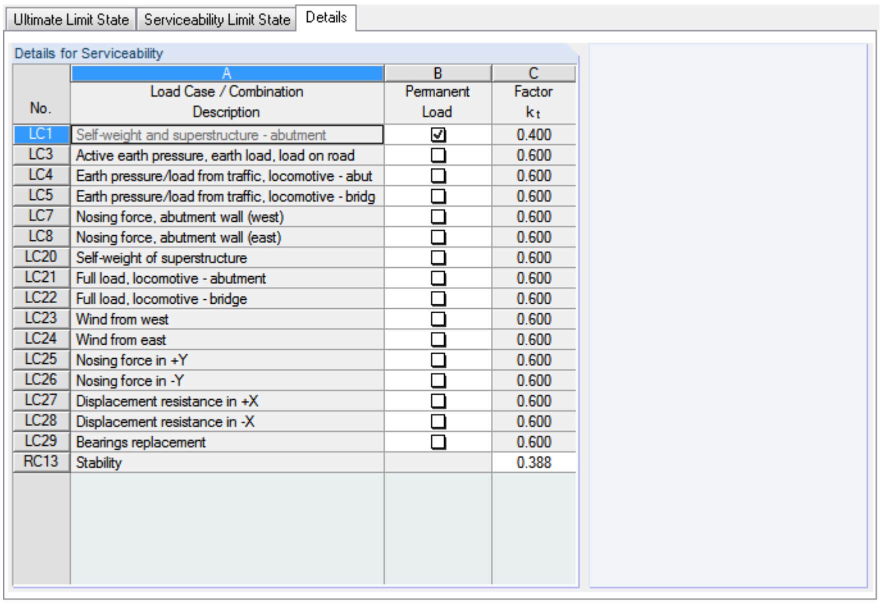
Figure 3.14 *Tension-Stiffening with Concrete Tensile Strength* dialog box

Modifications of the parameters are immediately displayed graphically in the diagrams.

The approach of *Tension Stiffening* is described in chapter 2.8.3.3.

3.1.3 Details

This tab appears when load cases have been selected for the serviceability limit state design and the standard EN 1992-1-1 is set.



No.	A Load Case / Combination Description	B Permanent Load	C Factor k_t
LC1	Self-weight and superstructure - abutment	☑	0.400
LC3	Active earth pressure, earth load, load on road	☐	0.600
LC4	Earth pressure/load from traffic, locomotive - abut	☐	0.600
LC5	Earth pressure/load from traffic, locomotive - bridg	☐	0.600
LC7	Nosing force, abutment wall (west)	☐	0.600
LC8	Nosing force, abutment wall (east)	☐	0.600
LC20	Self-weight of superstructure	☐	0.600
LC21	Full load, locomotive - abutment	☐	0.600
LC22	Full load, locomotive - bridge	☐	0.600
LC23	Wind from west	☐	0.600
LC24	Wind from east	☐	0.600
LC25	Nosing force in +Y	☐	0.600
LC26	Nosing force in -Y	☐	0.600
LC27	Displacement resistance in +X	☐	0.600
LC28	Displacement resistance in -X	☐	0.600
LC29	Bearings replacement	☐	0.600
RC13	Stability		0.388

Figure 3.15 Window 1.1 *General Data, Details* tab

In the crack width design, the program calculates the differences in the mean strains of concrete and reinforcing steel (see chapter 2.6.4.12). According to EN 1992-1-1, 7.3.4 (2), Eq. (7.9), the load duration factor k_t must be specified for this.

Load Case / Combination Description

This column lists all load cases, load combinations, and result combinations that have been selected for design in the *Serviceability Limit State* tab. For load and result combinations, the included load cases are shown as well.

Permanent Load

This column indicates the load cases that are to be applied as permanent loads. If an entry is marked (selected check box), the factor k_t is automatically set to 0.4 in the next column.

Factor k_t

The load duration factor k_t is used to consider the load duration. The factor k_t is 0.4 for long-term load actions and 0.6 for short-term actions.

For load and result combinations, the mean is formed from the k_t values of the load cases contained in the CO or RC.

$$k_t = \frac{\sum_{i=1}^{n} \gamma_i(\text{LC}) \cdot k_{t,i}(\text{LC})}{\sum_{i=1}^{n} \gamma_i(\text{LC})}$$

Equation 3.1

3.2 Materials

The window is divided into two parts. The upper section lists the concrete classes and steel grades relevant for the design. All materials of the concrete category used for surfaces in RFEM are preset. In the *Material Properties* section, the properties of the current material, i.e. the material whose table row is selected in the upper section, are displayed.

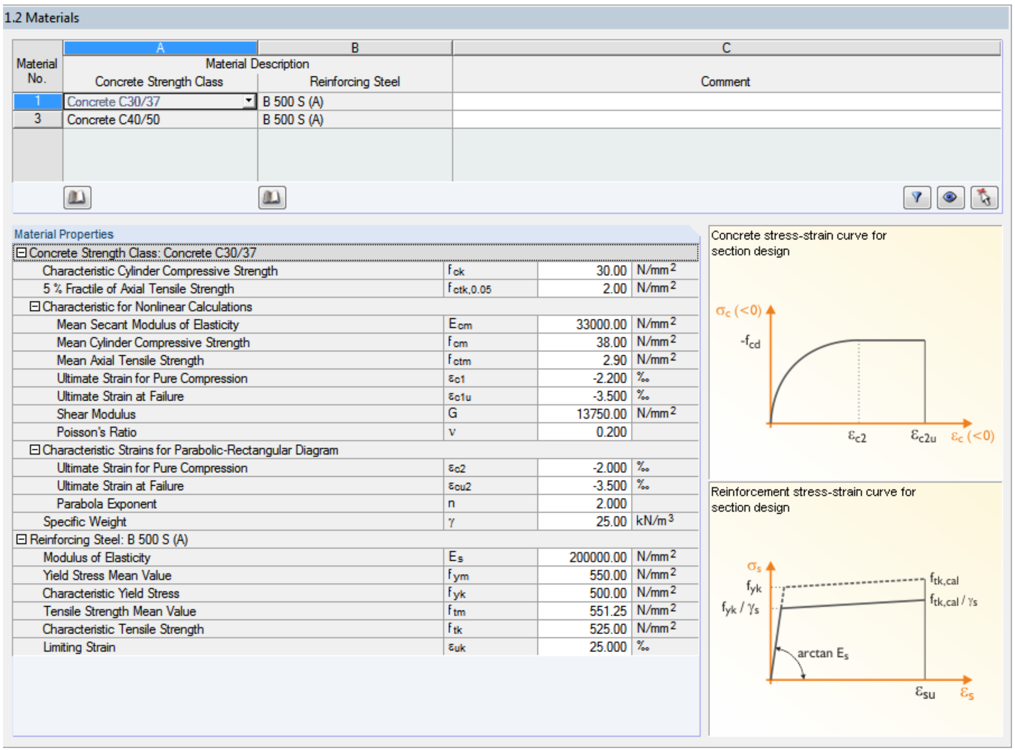


Figure 3.16 Window 1.2 *Materials*

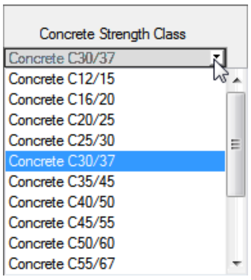
The table only lists materials that are used in the design. Invalid materials are highlighted in red, modified materials appear blue in color.

Chapter 4.3 of the RFEM manual describes the material properties used for determining the internal forces. The properties of the materials needed for the design are stored in the global material library as well. These values are preset for the *Concrete Strength Class* and the *Reinforcing Steel*.

To adjust the units and decimal places of the properties and stresses, select **Settings → Units and Decimal Places** in the menu (see chapter 8.2).

Material Description

Concrete Strength Class

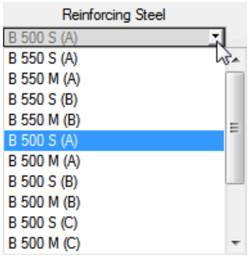


The concrete materials used in RFEM are preset; materials that are not relevant are hidden. The strength class can be modified at any time: click the material in column A, which activates the field. Then, click the button or press the [F7] key to open the list of the strength classes.

The list contains only strength classes that correspond to the design concept of the selected standard.

After the transfer, the design-relevant *Material Properties* are updated.

As a matter of principle, the material properties cannot be edited in RF-CONCRETE Surfaces.



Reinforcing Steel

In this column, the program presets a steel grade that corresponds to the design concept of the selected standard.

As with the concrete strength class, you can select a different reinforcing steel in the drop-down list: Click the material in column B to activate the field. Then, click the ⍗ button or press the [F7] key to open the list of the reinforcing steels.

As with the concrete strength classes, the list only contains steel grades that are relevant for the selected standard.

After the transfer, the *Material Properties* are updated.

Material Library

Many materials are stored in a database. To open the [Library], click the button shown on the left, which is available for the concrete strength classes and reinforcing steels at the end of the column.

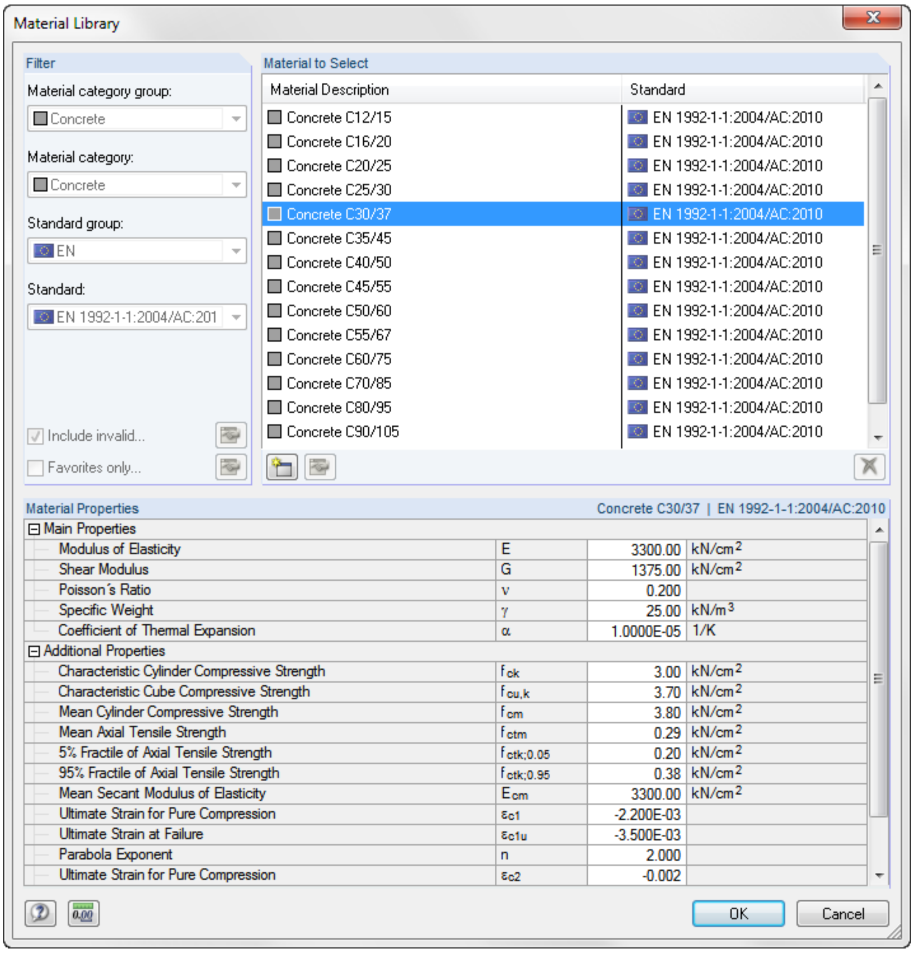
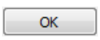


Figure 3.17 *Material Library* dialog box



In the *Filter* section, the standard-relevant materials are set as a preselection, thus excluding all other categories or standards. You can select the desired concrete strength class or steel grade from the *Material to Select* list; the properties can be reviewed in the lower section.

Click [OK] or press [↵] to transfer the selected material to window 1.2 of RF-CONCRETE Surfaces.

Chapter 4.3 of the RFEM manual describes how to filter, add, or reorganize materials.

3.3 Surfaces

This window manages the surfaces that are relevant for the design.

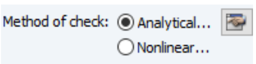
The makeup of the window depends on the settings in window 1.1 *General Data*: If you only design the ultimate limit state, the table merely lists the surfaces with their thicknesses. If you have selected load cases for the serviceability limit state design (see Figure 3.8), you can enter specific settings here. They differ depending on the selected method of check.

The buttons have the following functions:

Button	Function
	Only shows surfaces assigned to a reinforcement group in window 1.4 *Reinforcement* (see chapter 3.4)
	Allows you to go to the RFEM work window to change the view
	Allows you to select a surface in the RFEM work window

Table 3.2 Buttons in window 1.3 Surfaces

3.3.1 Analytical Method



The analytical method for the serviceability limit state design is described in detail in chapter 2.6.

If you use RF-CONCRETE Deflect, this window provides additional tabs and columns. They are described in chapter 3.3.2 *Nonlinear Method*.

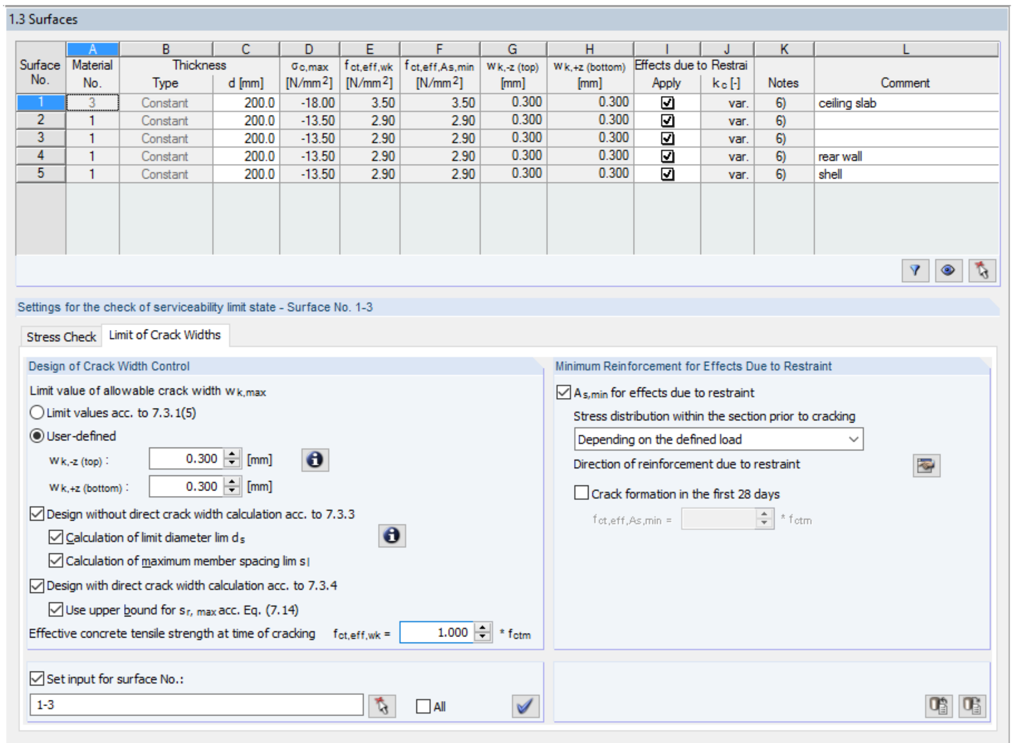


Figure 3.18 Window 1.3 *Surfaces* for analytical check method, *Limit of Crack Widths* tab

Material No.

The numbers of the materials are displayed according to window 1.2 *Materials* for each surface.

Thickness

Type

You can design constant and linearly variable thickness types. For orthotropic properties, the designs are limited to the ultimate limit state.

d

This column shows the surface thicknesses defined RFEM. The values can be changed for the design.

If the surface thicknesses are modified, the internal forces of RFEM, which result from the stiffnesses of the RFEM surface thicknesses, are used for the design. In a statically indeterminate system, the surface thicknesses modified in RF-CONCRETE Surfaces must also be adjusted in RFEM: In this way, the distribution of internal forces is correctly considered in the design.

The other column descriptions depend on the settings in the tabs below. These tabs can be controlled in the *Settings* dialog box (see Figure 3.10) where you can specify whether you want to design stresses and/or cracks.

The values in the columns are taken from the entries in the tabs below. These specifications apply to all surfaces by default. It is also possible to only assign the current settings to specific surfaces: Clear the *All* check box. Then, enter the numbers of the relevant surfaces or select them graphically in the RFEM work window with . With you assign the current settings to these surfaces. However, the assignment is **only** applicable for the active tab, for example *Stress Check*.

The following two parameters need to be defined in the *Stress Check* tab (see Figure 3.19).

$\sigma_{c,max}$

This column shows the value of the maximum (negative) concrete stress for limiting the concrete compressive stresses (see chapter 2.6.4.7). According to EN 1992-1-1, the following applies to

- quasi-permanent action combinations, if serviceability, ultimate limit state, or durability are considerably affected by creeping:

$$\sigma_c \leq 0.45 \cdot f_{ck} \qquad 7.2\ (3)$$

- rare (= characteristic) action combinations in exposure classes XD1 to XD3, XF1 to XF4, XS1 to XS3:

$$\sigma_c \leq 0.60 \cdot f_{ck} \qquad 7.2\ (2)$$

$\sigma_{s,max}$

This value represents the maximum reinforcing steel stress for limiting the reinforcement's tensile stresses (see chapter 2.6.4.8). According to EN 1992-1-1, the following applies to

- rare action combinations:

$$\sigma_s \leq 0.80 \cdot f_{yk} \qquad 7.2\ (5)$$

- pure effects due to restraint:

$$\sigma_s \leq 1.00 \cdot f_{yk} \qquad 7.2\ (5)$$

The remaining parameters have to be defined in the *Limit of Crack Widths* tab (see Figure 3.18).

$f_{ct,eff,wk}$

The value of the effective concrete tensile strength is required for the check of crack width according to EN 1992-1-1, clause 7.3.4 (see chapter 2.6.4.12).

$f_{ct,eff,As,min}$

This column manages the concrete's effective tensile strength that is to be applied for the determination of the minimum reinforcement to resist restraint according to EN 1992-1-1, clause 7.3.2 (see chapter 2.6.4.9). The concrete tensile strength depends on the time of the initial crack formation.

$w_{k,-z}$ (top) / $w_{k,+z}$ (bottom)

These parameters are the allowable crack widths at the top and bottom sides of the surfaces (see chapter 2.6.4.12).

Effects due to Restraint

If there are effects due to restraint, they must be considered when determining the minimum reinforcement for limiting the crack width (see chapter 2.6.4.9).

In the *Limit of Crack Widths* tab, you can use the [Edit] button to specify the minimum reinforcement to resist effects due to restraint (see Figure 2.97).

Apply

In column I or with the check box in the *Limitation of Crack Widths* tab, you can configure if there are effects due to restraint.

Type

In the *Limitation of Crack Widths* tab, you can specify whether there are internal or external effects due to restraint. This influences the factor k for considering nonlinearly distributed concrete tensile stresses (see Equation 2.71).

k_c

This factor takes account of the stress distribution in the tension zone (see Equation 2.71).

Notes

This column shows remarks in the form of footnotes described in detail in the status bar.

Comment

These input fields can be used to enter user-defined comments.

3.3.2 Nonlinear Method

Performing a design according to the *Nonlinear* method requires a license of the add-on module **RF-CONCRETE NL**. This method for the serviceability limit state design is described in detail in chapter 2.8.

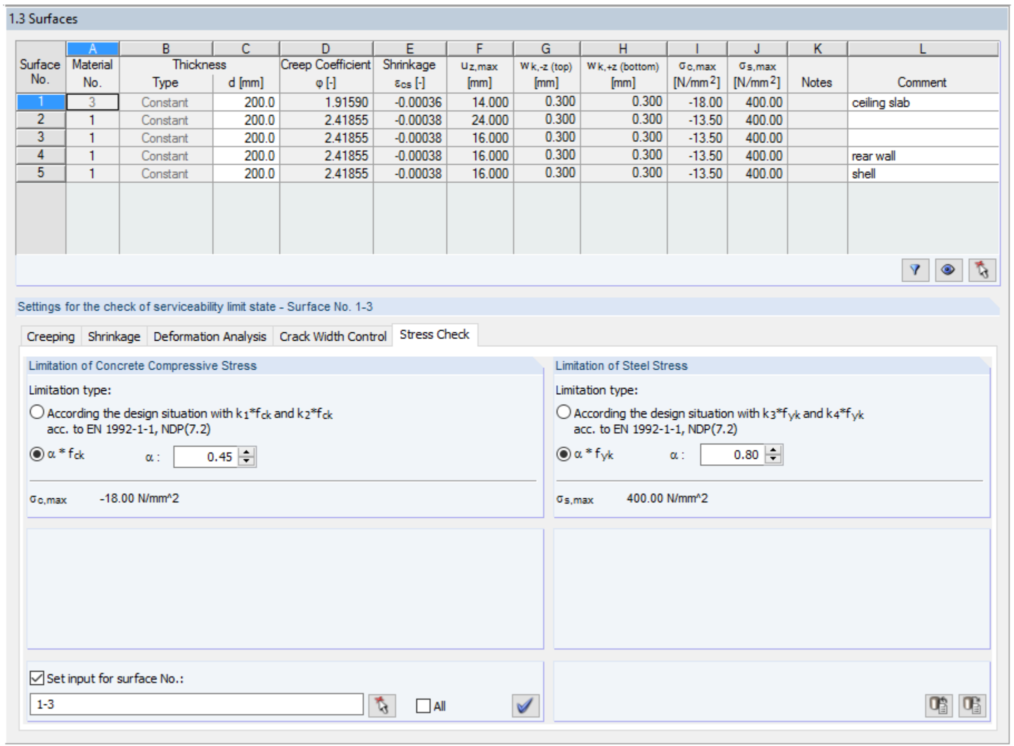


Figure 3.19 Window 1.3 *Surfaces* for nonlinear check method, *Stress Check* tab

The following columns are described in the previous chapter 3.3.1:

- Material
- Thickness
- $w_{k,-z(top)}$ / $w_{k,+z(bottom)}$
- $\sigma_{c,max}$
- $\sigma_{s,max}$

For orthotropic surfaces, no serviceability limit state design according to the nonlinear method is possible.

The values in the columns D through J are controlled in the tabs below. The settings specified in them are applied to all surfaces by default. It is also possible to only assign the current settings to specific surfaces: Clear the *All* check box. Then, enter the numbers of the relevant surfaces or use the select button to select them graphically. With the check button you can assign the current settings to these surfaces.

The assignment is only applicable for the active tab, for example *Stress Check*.

Creep Coefficient φ

The parameters for creep must be defined in the *Creeping* tab (see Figure 2.144). Based on these conditions, the program determines the creep coefficient φ. For the effective component thickness h_0, the program applies the surface thickness d.

Determining the creep coefficient is described in chapter 2.8.4.1.

Shrinkage ε_{cs}

This column shows the shrinkage strain. The relevant parameters are defined in the *Shrinkage* tab (see Figure 2.147). Based on these conditions, the program determines the shrinkage strain ε_{cs}. For the effective component thickness h_0, the program applies the surface thickness d.

Determining the shrinkage strain is described in chapter 2.8.4.2. If you do not wish to apply any shrinkage strain to a surface, set a user-defined shrinkage strain of zero in the *Shrinkage* tab and then assign it to the surface.

For pure plates that are defined as the model type 2D - XY ($u_Z/\varphi_X/\varphi_Y$), shrinkage cannot be considered: There are only degrees of freedom for bending.

$u_{z,max}$

This value represents the maximum allowable deformation that must be observed in the design of the serviceability limit state. The design criteria are defined in the *Deformation Analysis* tab.

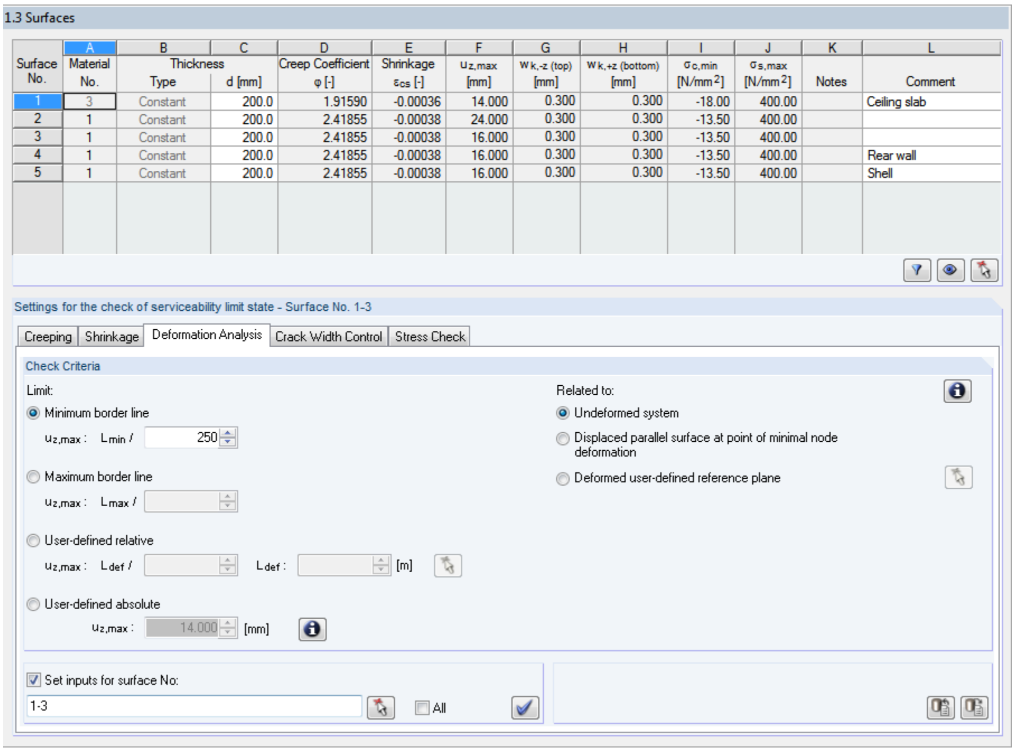


Figure 3.20 Window 1.3 *Surfaces*, *Deformation Analysis* tab

Limit

The serviceability for "common structures", for example according to EN 1992-1-1, clause 7.4, is ensured if the deflection in the quasi-permanent action combination does not exceed the following limit values.

Common case:

$$u_{z,max} = \frac{\ell_{eff}}{250}$$

Structural elements for which excessive deformations can result in subsequent damages:

$$u_{z,max} = \frac{\ell_{eff}}{500}$$

The *Minimum border line*, *Maximum border line*, and *User-defined relative* options determine which effective length l_{eff} is applied. For the two border line options, the program applies the smallest or largest border line of the respective surface.

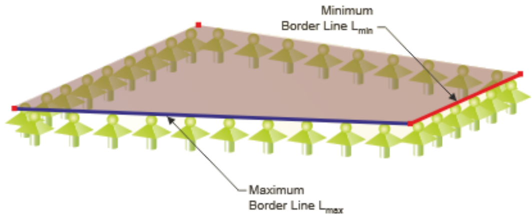


Figure 3.21 Maximum and minimum border line for determining $u_{z,max}$

When you choose the *User-defined relative* option, you can enter the length directly or select it graphically between any two points in the RFEM model with [button]. In addition, you must define a divisor by which the lengths are divided for all three options.

It is also possible to specify the allowable maximum deformation $u_{z,max}$ as *User-defined absolute*.

Related to

The deformation design criterion uses the deflection of a surface — the vertical deformation relative to the straight line connecting the points of support. The *Deformation Analysis* tab (see Figure 3.20) provides three options for calculating the local deformation $u_{z,local}$ applied in the design.

- *Undeformed system:* The deformation is related to the initial structure.
- *Displaced parallel surface:* This option is recommended for an elastic support of the surface. The deformation $u_{z,local}$ is related to a virtual reference surface that is displaced parallel to the undeformed system. The displacement vector of the reference surface is as long as the minimal nodal deformation within the surface.

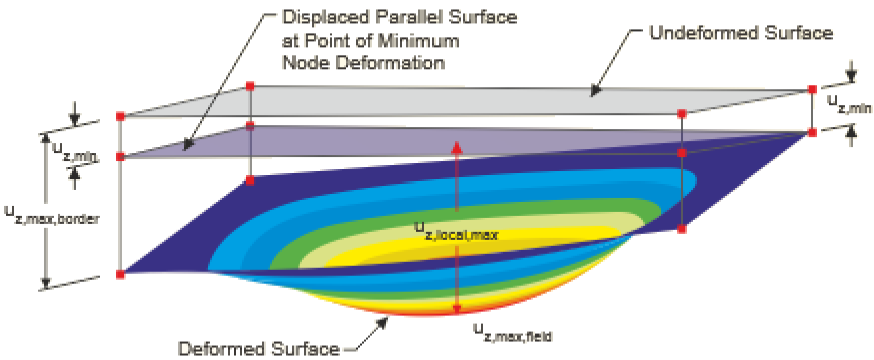


Figure 3.22 Displaced parallel surface (displacement vector: minimal nodal deformation $u_{z,min}$)

- *Deformed reference plane:* If the support deformations of a surface differ considerably from each other, you can define an inclined reference plane for the deformation $u_{z,local}$ that is to be designed. This plane must be defined by three points of the undeformed system. The program determines the deformation of the three definition points, places the reference plane in these displaced points, and then calculates the local deformation $u_{z,local}$.

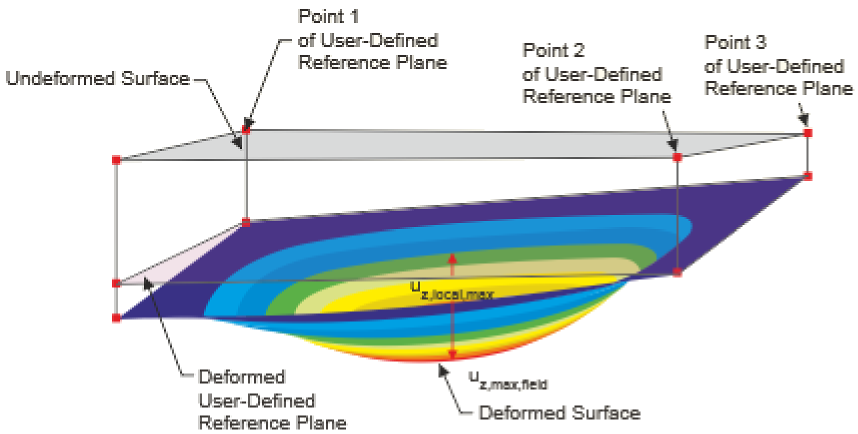


Figure 3.23 Displaced user-defined reference plane

3.4 Reinforcement

This window consists of five tabs where all settings for the reinforcement are specified. Since the surfaces to be designed often require different specifications, you can define so-called "reinforcement groups" in each RF-CONCRETE Surfaces case. Each reinforcement group manages the reinforcement parameters that apply to particular surfaces.

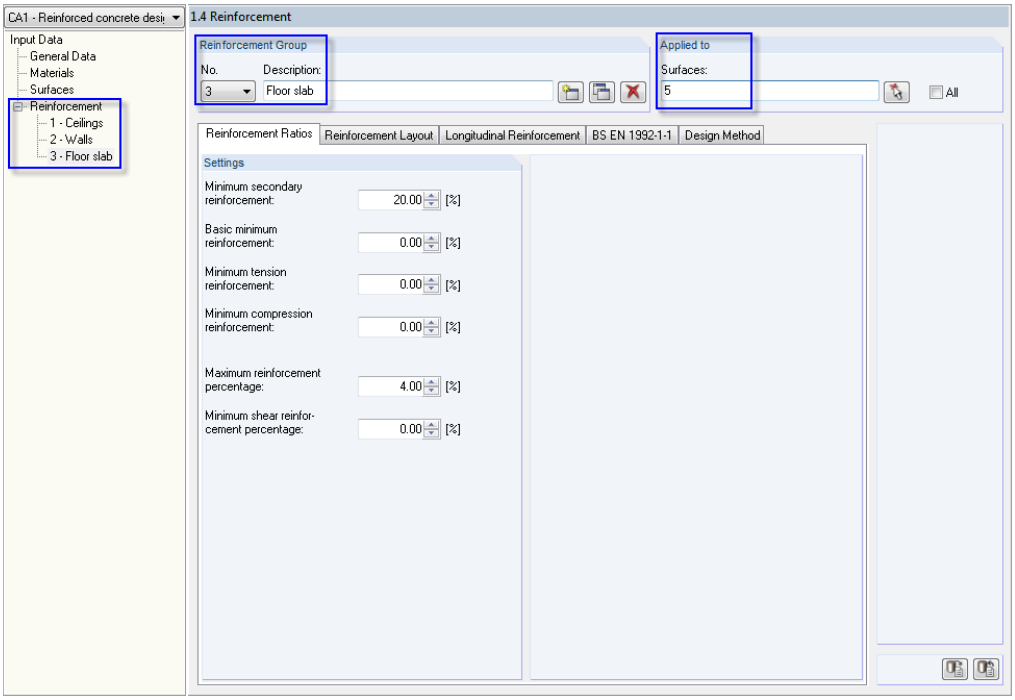
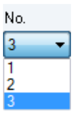


Figure 3.24 Window 1.4 *Reinforcement* with three reinforcement groups

Reinforcement Group

To create a new reinforcement group, click the button in the *Reinforcement Group* section. The number is automatically assigned. A user-defined *Description* helps you to overlook all reinforcement groups available in the design case.



To select the desired reinforcement group, use the *No.* list or click the entries in the navigator.

With the button, you can remove the selected reinforcement group from the design case without any further warning. Surfaces that were contained in such a reinforcement group are therefore not designed. To design them, they must be assigned to a new or existing reinforcement group.

Applied to Surfaces

This section allows you to specify the surfaces that the parameters of the current reinforcement group apply to. By default, *All* surfaces are preset. If the corresponding check box is selected, you cannot create another reinforcement group because a surface cannot be designed according to different rules (this is only possible in "design cases", see chapter 8.1). The *All* check box must therefore be cleared to use several reinforcement groups.

In the input field, you can enter the numbers of the surfaces that the reinforcement parameters in the tabs below apply to; you can also select them graphically in the RFEM work window with . Only surface numbers that have not yet been assigned to other reinforcement groups can be entered in the input field.

3.4.1 Reinforcement Ratios

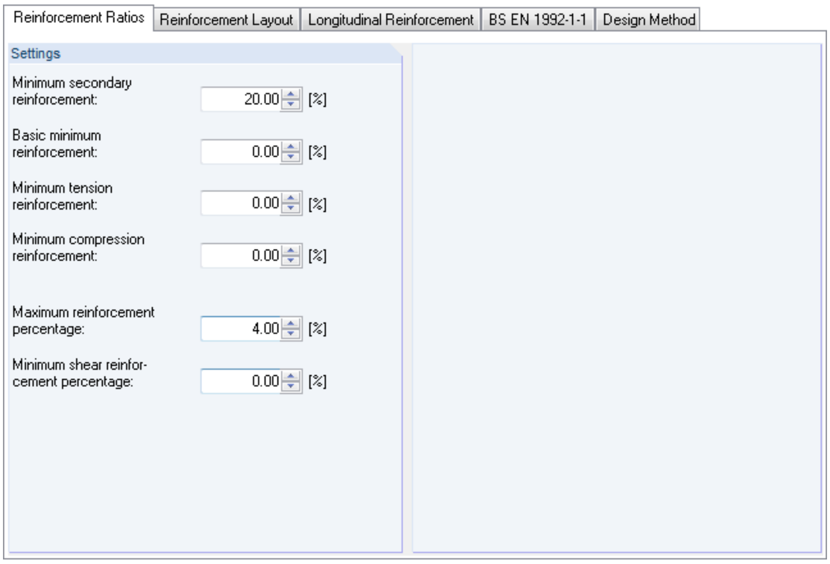


Figure 3.25 Window 1.4 *Reinforcement, Reinforcement Ratios* tab

In this tab, you can define the minimum and maximum reinforcements in percentages. The *Minimum secondary reinforcement* refers to the largest longitudinal reinforcement to be applied. All additional reinforcement ratios refer to the cross-sectional area of a surface stripe with a width of 1 meter.

Examples for minimum and maximum reinforcements can be found in chapter 2.3.7, chapter 2.4.5, and chapter 2.5.8.

3.4.2 Reinforcement Layout

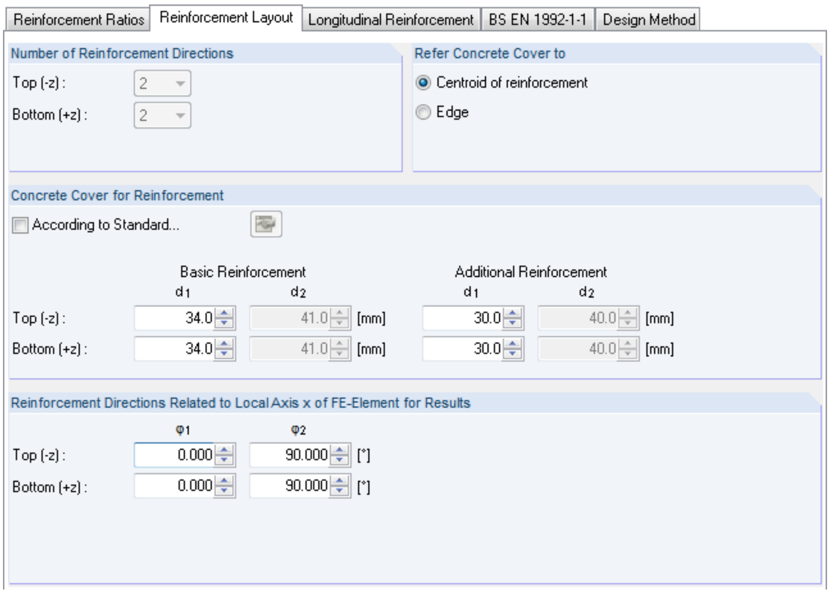


Figure 3.26 Window 1.4 *Reinforcement, Reinforcement Layout* tab

This tab manages the geometric specifications for the reinforcement.

Number of Reinforcement Directions

The reinforcement mesh can be defined with two or three reinforcement directions for each surface side.

For serviceability limit state designs, only a reinforcement mesh with two directions is allowed.

The definition of the "top" and "bottom" surface side is described below in the *Concrete Cover for Reinforcement* section.

Refer Concrete Cover to

You can refer the concrete covers that you can specify in the *Concrete Cover for Reinforcement* section to the reinforcement's *Centroid* or *Edge* distance.

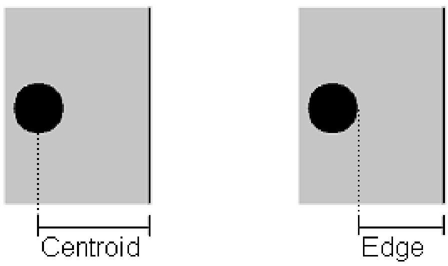


Figure 3.27 Reference of concrete cover

When you select the *Edge* option in a pure ultimate limit state design, you have to specify the *Bar diameter D*.

Concrete Cover for Reinforcement

You need to specify the concrete covers of the *Basic Reinforcement* and, if necessary, the *Additional Reinforcement* for both sides of the surface. The dimensions represent either the centroids d of the individual layers or the reinforcements' edge distances c_{nom} in the direction φ_1. The reinforcement directions must be defined in the dialog section below.

The "top" and the "bottom" surface side is defined as follows: The bottom surface is defined in the direction of the positive local surface axis z, the top surface accordingly in the direction of the negative local z-axis.

The RFEM graphic shows the xyz coordinate systems of the surfaces once you move the pointer across a surface. You can also use the shortcut menu of a surface (by right-clicking it) to display or hide the axes.

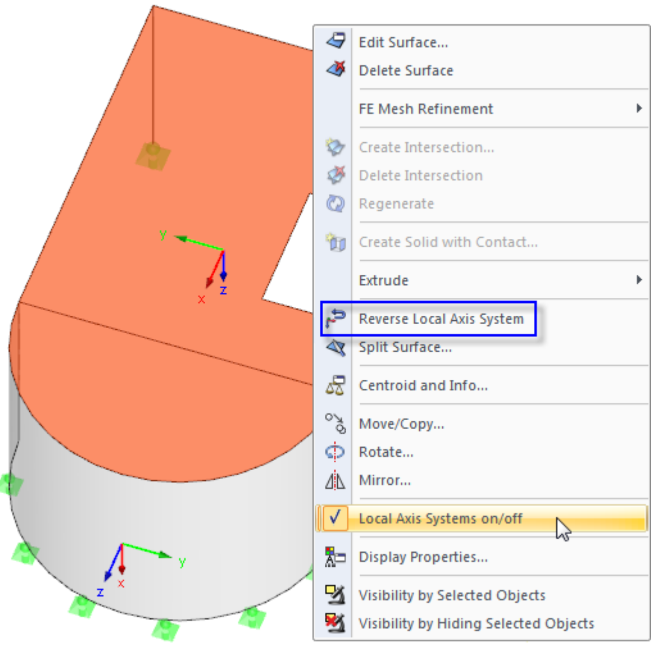
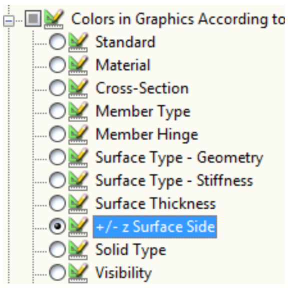


Figure 3.28 RFEM shortcut menu of a surface



To display the surface sides in different colors, select *Colors in Graphics According to* ⟶ *+/- z Surface Side* in the *Display* navigator (see figure on the left).

You can change the orientation of the local z-axis of a surface by using the *Reverse Local Axis System* option in the shortcut menu (see Figure 3.28). This way you can, for example, unify the orientation of walls in order to unambiguously assign the top and bottom reinforcement sides for vertical surfaces.

The model types "wall" – 2D - XZ ($u_X/u_Z/\varphi_Y$) or "diaphragm" – 2D - XY ($u_X/u_Y/\varphi_Z$) are models that are exclusively subjected to compression or tension in the component plane. In such a case, it is not possible to create different reinforcement meshes for each surface side so that the input possibilities are limited to uniform concrete covers on both sides.

If you select the *According to Standard* check box, the [Edit] button becomes available. Use this button to open the dialog box shown in Figure 3.29.

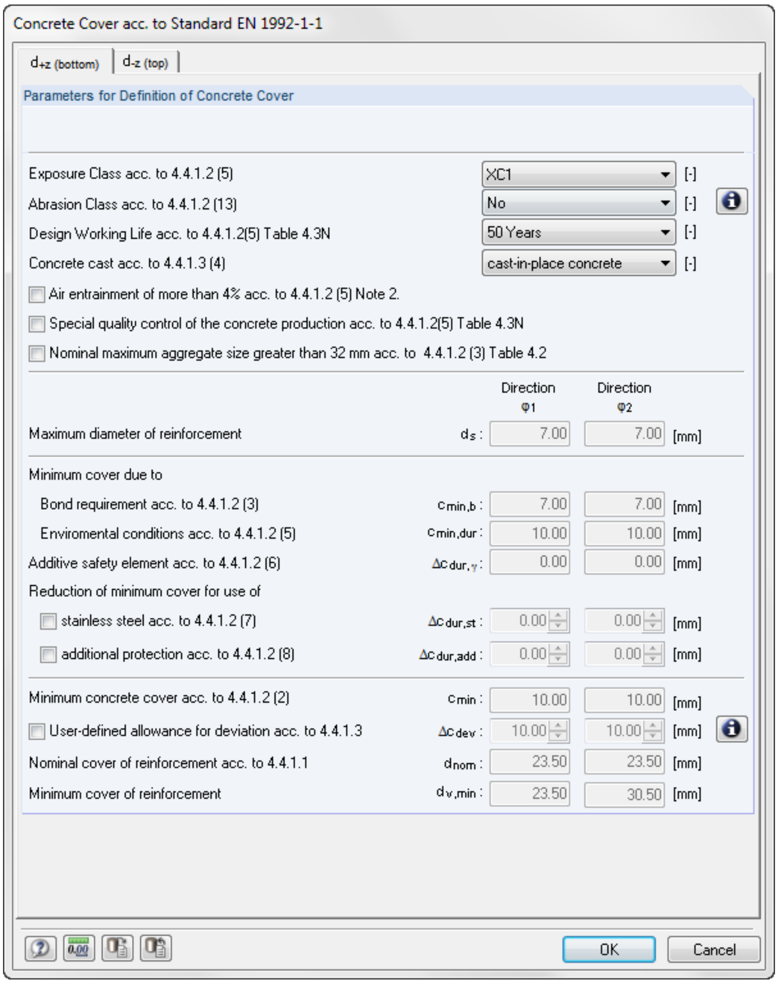
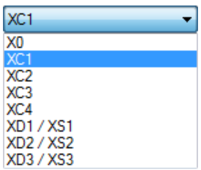


Figure 3.29 *Concrete Cover acc. to Standard* dialog box



Exposure classes

In the upper section, you can define the parameters (exposure class, abrasion class, etc.) according to the standard. Based on these parameters, RF-CONCRETE Surfaces determines the required concrete covers.

You can specify the parameters separately for each surface side in the two dialog tabs.

Reinforcement Directions Related to Local Axis x

The reinforcement directions φ are related to the local x-axes of the finite elements. In the *Edit Surface* dialog box of RFEM, you can check and, if necessary, adjust the axis system of the surfaces for the results output.

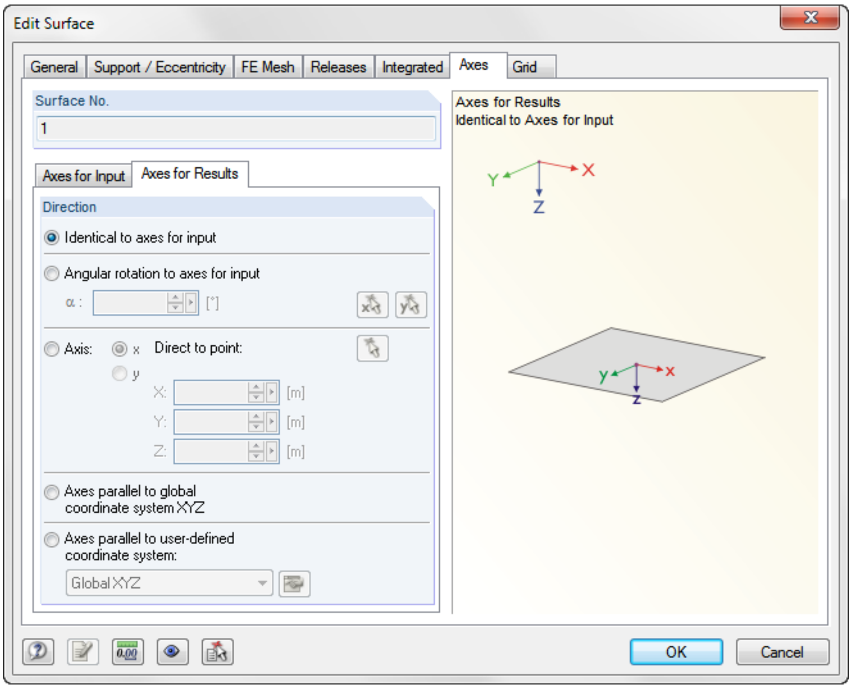
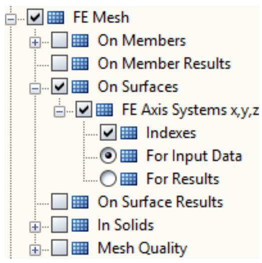


Figure 3.30 *Edit Surface* RFEM dialog box, *Axes* and *Axes for Results* tabs



For curved surfaces, it is recommended to check the axes of the finite elements graphically: In the *Display* navigator of RFEM, select *FE Mesh* ⟶ *On Surfaces* ⟶ *FE Axis Systems x, y, z,* ⟶ *Indexes*.

The reinforcement directions have to be specified for each layer by means of the angles φ. Only positive angles are allowed. They respectively represent the clockwise rotation of the reinforcement direction in relation to the corresponding x-axis.

For the model types "wall" – 2D - XZ ($u_X/u_Z/\varphi_Y$) or "diaphragm" – 2D - XY ($u_X/u_Y/\varphi_Z$), it is not possible to create different reinforcement meshes for each side of the surface. Thus, the input options are limited to uniform reinforcement directions on both surface sides.

3.4.3 Longitudinal Reinforcement

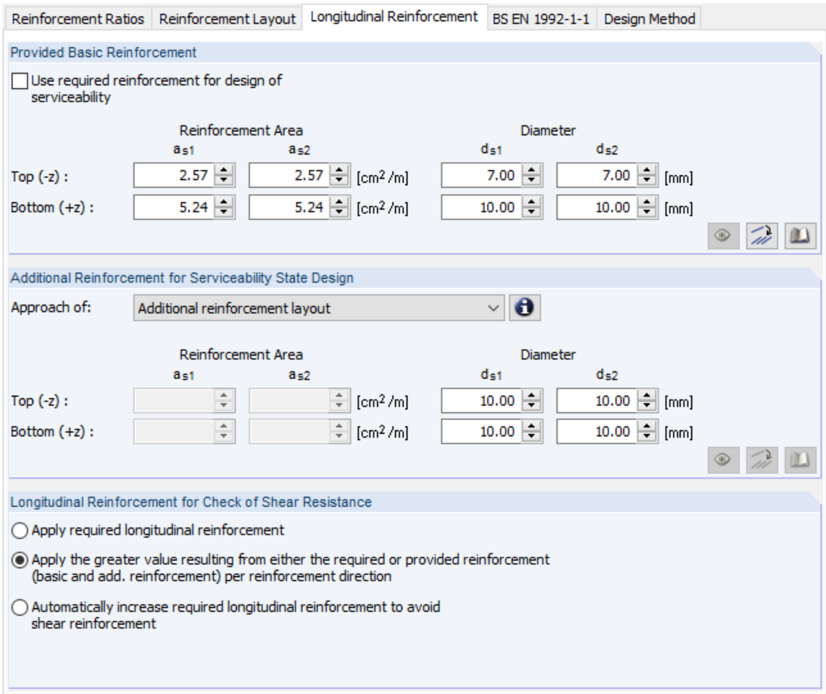


Figure 3.31 Window 1.4 *Reinforcement, Longitudinal Reinforcement* tab for serviceability limit state design

The sections in the tab depend on the designs selected in window 1.1 *General Data*: A pure ultimate limit state design does not require any specific reinforcement settings. You only need to decide which longitudinal reinforcement you want to use for the shear force check. For serviceability limit state designs, however, you must specify reinforcement areas.

For more information on the reinforcement specifications in the serviceability limit state design, see chapter 2.6.3.

Provided Basic Reinforcement

For each surface side and each reinforcement direction, you can define a basic reinforcement that applies to all surfaces of the reinforcement group. To do so, enter the *Reinforcement Area* and the *Diameter* that is relevant for the serviceability limit state design in the input fields.

If the user-defined basic reinforcement exceeds the required reinforcement, no additional reinforcement is needed. However, it is inefficient to apply large constant basic reinforcements to surfaces.

RF-CONCRETE Surfaces provides databases for rebars and mesh reinforcements that facilitate entering the reinforcement areas. To access these libraries, use the two buttons shown on the left.

Rebars

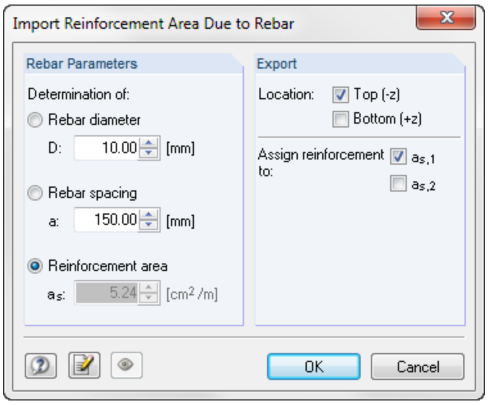


Figure 3.32 *Import Reinforcement Area Due to Rebar* dialog box

The three options in the *Rebar Parameters* section are interactive. Normally, the program calculates the reinforcement area from the rebar diameter and the rebar spacing.

The *Export* section allows you to control which input fields of the *Longitudinal Reinforcement* tab the determined reinforcement areas are imported into. The location and reinforcement direction can be specifically defined (or sweepingly, by selecting all check boxes).

Mesh reinforcements

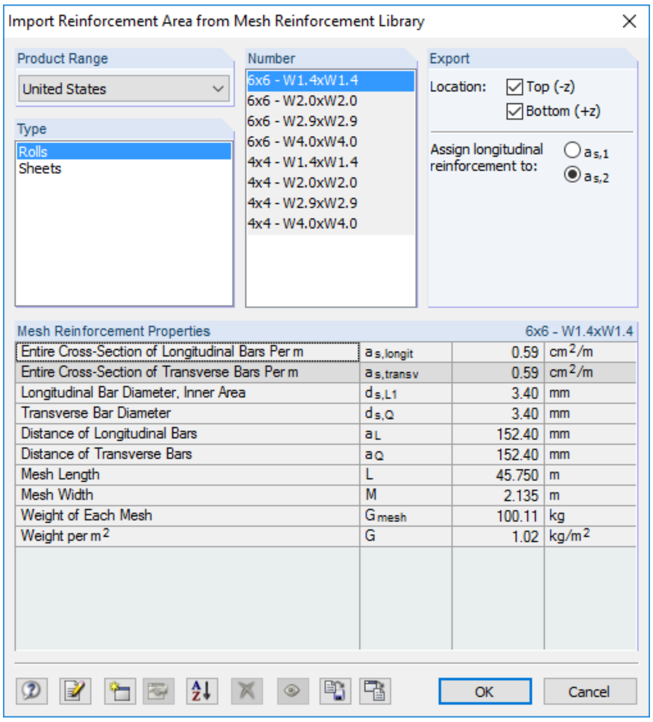
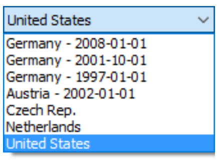


Figure 3.33 *Import Reinforcement Area from Mesh Reinforcement Library* dialog box



First, select the *Product Range* from the drop-down list shown on the left. Then, define the mesh *Type* and select the relevant *Number* in the section on the right. In the section below, you can check the *Mesh Reinforcement Properties*.

The *Export* section allows you to control which input fields of the *Longitudinal Reinforcement* tab the determined reinforcement areas are imported into. The location and reinforcement direction can be specifically defined (or sweepingly, by selecting all check boxes).

Use required reinforcement for design of serviceability

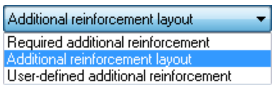
The ideal way to perform the serviceability limit state design would be the following:

- determine the required reinforcement exclusively with the loading of the *Ultimate Limit State* tab
- create a reinforcement drawing including mesh reinforcements and rebars on the basis of the colored result diagrams
- if necessary, use the reinforcement drawing to divide the surfaces into smaller surfaces that have the same provided reinforcement area in each reinforcement direction
- define the provided reinforcement area, rebar spacing, and rebar diameter for each of these surfaces in RF-CONCRETE Surfaces
- calculate once again with the loads of the *Serviceability Limit State* tab

This procedure is cumbersome and contrary to the convention stating that you can determine the reinforcement and perform the serviceability limit state designs at the same time by using the [Calculation] button.

Hence, you can select the *Use required reinforcement for design of serviceability* check box to quickly use a provided reinforcement for the individual surfaces: The program uses the required reinforcement from the ultimate limit state design as the applied reinforcement. You only need to specify the rebar diameter.

Automatic layout of Additional Reinforcement for Serviceability Limit State Design



Additional reinforcement is needed in areas where the statically required reinforcement exceeds the basic reinforcement. Use the drop-down list in this dialog section to specify which additional reinforcement should be used for the serviceability limit state design.

If you select the *Required additional reinforcement* option, the actual $A_{s,req}$ distribution is used as the additional reinforcement in the SLS design.

The *Additional reinforcement layout* is determined as the difference between the greatest statically required reinforcement of all surfaces in the reinforcement group and the defined basic reinforcement:

$$a_{s,additional} = \max a_{s,req} - a_{s,basic}$$

Equation 3.2

Click the button to open a dialog box that illustrates the selected additional reinforcement (see Figure 3.34).

To dimension the additional reinforcement, you only need to specify the rebar diameter.

The reinforcement area can also be specified with *User-defined additional reinforcement*. Just like in the *Provided Basic Reinforcement* section, the program provides libraries for rebars and mesh reinforcements.

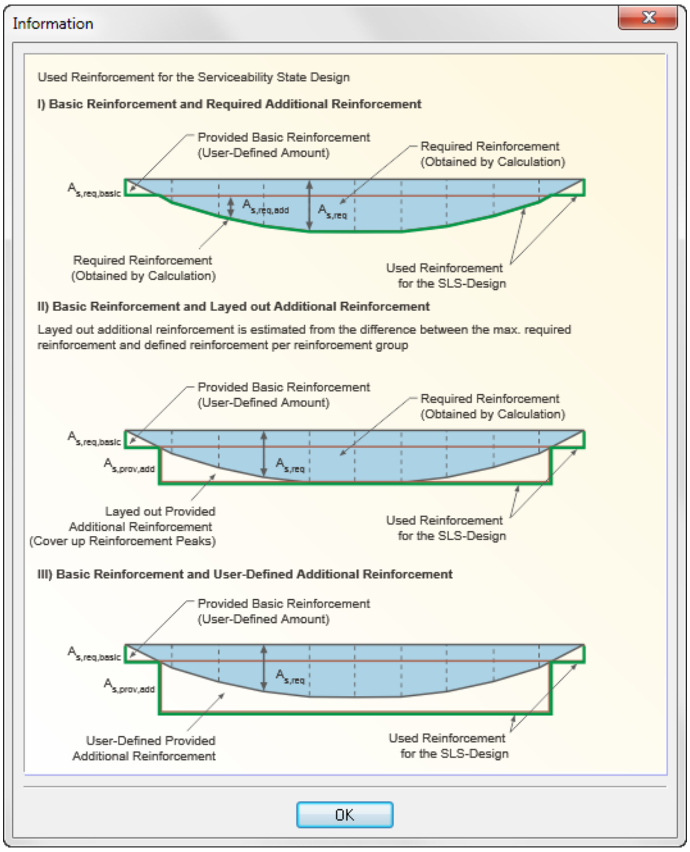


Figure 3.34 Applying additional reinforcement

Manual definition of reinforcement areas

As an alternative to the automatic geometric layout of the additional reinforcement for the serviceability limit state design, the areas covered by the additional reinforcement can also be defined manually.

Details...

To activate this option, click the [Details] button to open the *Details* dialog box. Then, select the *Manual definition of the reinforcement areas* in the *Reinforcement* tab.

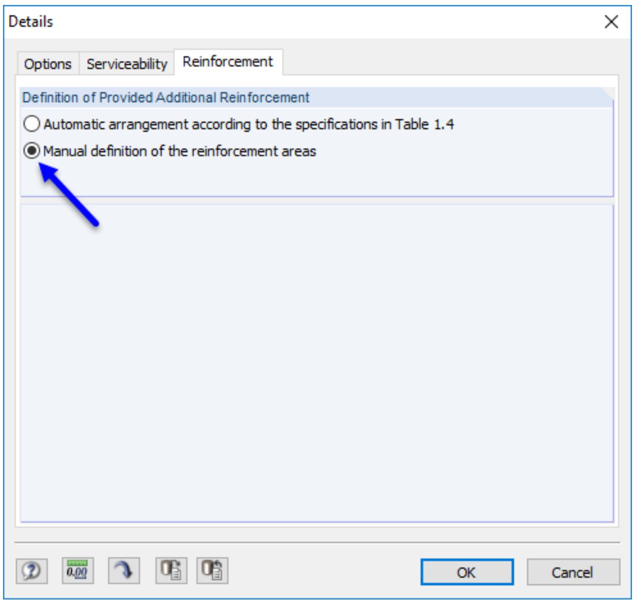


Figure 3.35 Activating manual definition of reinforcement areas in *Details* dialog box

In the *Longitudinal Reinforcement* tab, the *Provided Additional Reinforcement* dialog section then appears instead of the *Additional Reinforcement for Serviceability State Design* section.

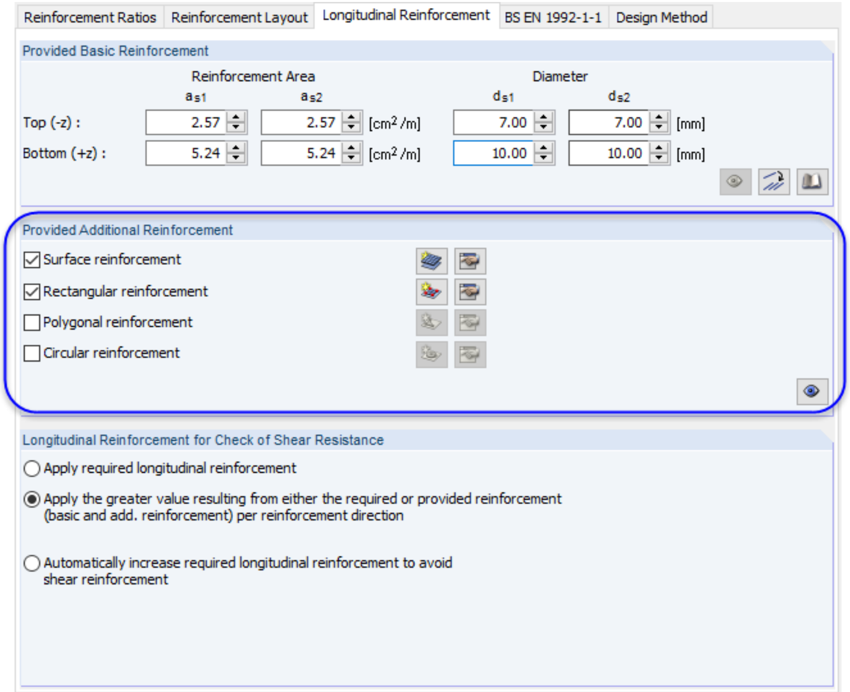


Figure 3.36 Window 1.4 *Reinforcement, Longitudinal Reinforcement* tab

The sections in the tab depend on the designs selected in window 1.1 *General Data*: A pure ultimate limit state design does not require any specific reinforcement settings. You only need to configure which longitudinal reinforcement you want to use for the shear force check. For the serviceability limit state designs, however, you must specify reinforcement areas.

For more information on reinforcement specifications for the serviceability limit state design, see chapter 2.6.3.

The functions are described for a rectangular reinforcement as an example. The explanations analogously apply to surface, polygonal, and circular reinforcements.

Click the [Apply free rectangular reinforcement] button to open the *New Rectangular Reinforcement* dialog box (see Figure 3.37) where you can define the properties and the position of the free reinforcement.

In the *On Surfaces No.* dialog section, you can enter the surfaces that the reinforcement should be used for. If the *All in RG* check box is selected, the new free reinforcement is used for all surfaces of the current reinforcement group.

The *Projection Plane* section determines which plane the reinforcement is applied on.

The *Type of Reinforcement* is either a mesh or rebar reinforcement. You can select the mesh reinforcements in a library (see Figure 3.33), which is opened with the [library] button. For the rebar reinforcement, you can use the [rebar] button to determine the reinforcement area using rebar diameter, rebar spacing, and reinforcement surface (see Figure 3.32).

The *Layout of Reinforcement* section controls the arrangement of the reinforcement. For this, specify the surface side and the direction of the reinforcement or mesh main reinforcement. The concrete cover of the additional reinforcement is taken from the settings in the *Reinforcement Layout* tab; it cannot be changed here.

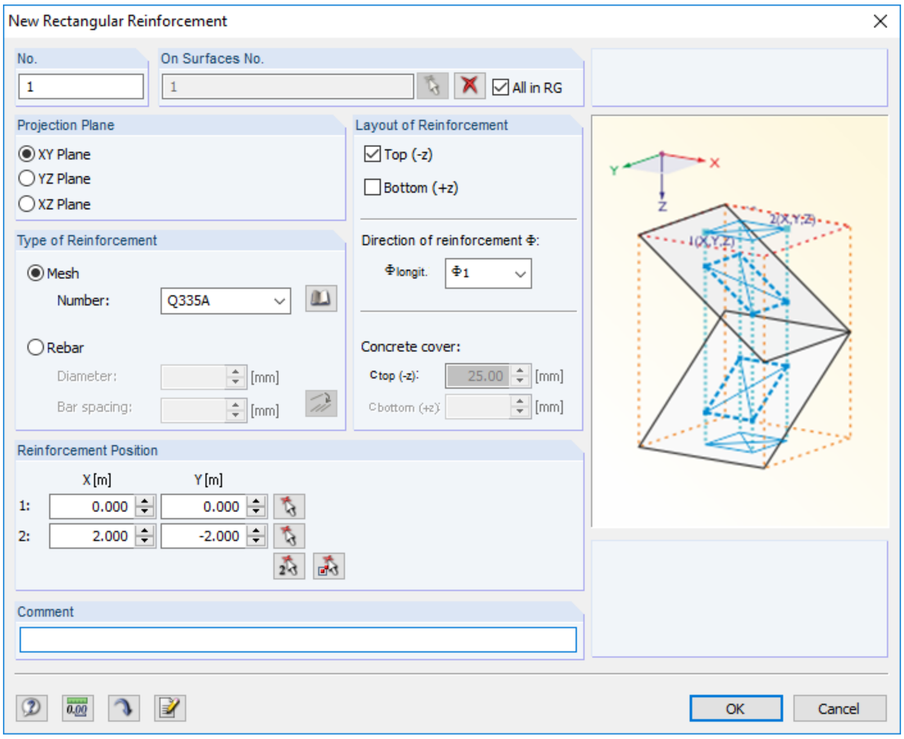
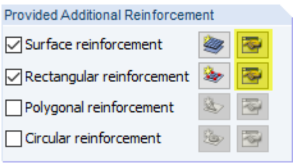


Figure 3.37 *New Rectangular Reinforcement* dialog box

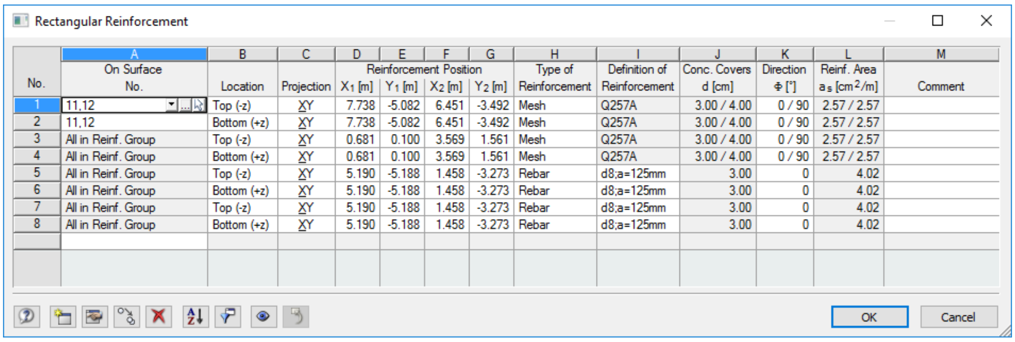
The *Reinforcement Position* – the region of the reinforcement – is defined by the coordinates of two points. They can be entered directly or selected with the [button] button in the work window. You can also draw a rectangular window, either with [button] by selecting two corner points or with [button] by using the rectangle's center point.

Note the following when defining the reinforcement position: The free reinforcement is considered in the finite element if the rectangle includes the element's center.

If two reinforcement surfaces lie on top of one another, the values in the concerned elements are added.



After defining the reinforcement, the [button] button is enabled in the *Provided Additional Reinforcement* section of the tab (see Figure 3.36). It opens a table where you can edit the reinforcement.



Rectangular Reinforcement

No.	A On Surface No.	B Location	C Projection	D Reinforcement Position X_1 [m]	E Y_1 [m]	F X_2 [m]	G Y_2 [m]	H Type of Reinforcement	I Definition of Reinforcement	J Conc. Covers d [cm]	K Direction Φ [°]	L Reinf. Area a_s [cm²/m]	M Comment
1	11,12	Top (-z)	XY	7.738	-5.082	6.451	-3.492	Mesh	Q257A	3.00 / 4.00	0 / 90	2.57 / 2.57	
2	11,12	Bottom (+z)	XY	7.738	-5.082	6.451	-3.492	Mesh	Q257A	3.00 / 4.00	0 / 90	2.57 / 2.57	
3	All in Reinf. Group	Top (-z)	XY	0.681	0.100	3.569	1.561	Mesh	Q257A	3.00 / 4.00	0 / 90	2.57 / 2.57	
4	All in Reinf. Group	Bottom (+z)	XY	0.681	0.100	3.569	1.561	Mesh	Q257A	3.00 / 4.00	0 / 90	2.57 / 2.57	
5	All in Reinf. Group	Top (-z)	XY	5.190	-5.188	1.458	-3.273	Rebar	d8;a=125mm	3.00	0	4.02	
6	All in Reinf. Group	Bottom (+z)	XY	5.190	-5.188	1.458	-3.273	Rebar	d8;a=125mm	3.00	0	4.02	
7	All in Reinf. Group	Top (-z)	XY	5.190	-5.188	1.458	-3.273	Rebar	d8;a=125mm	3.00	0	4.02	
8	All in Reinf. Group	Bottom (+z)	XY	5.190	-5.188	1.458	-3.273	Rebar	d8;a=125mm	3.00	0	4.02	

OK Cancel

Figure 3.38 *Rectangular Reinforcement* table

The buttons in this table have the following functions:

Button	Function
	Creates a new free reinforcement area
	Allows you to edit the selected reinforcement
	Moves or copies the selected reinforcement
	Deletes the selected reinforcement
	Sorts the table entries by position
	Opens the *Reinforcement Filter* dialog box (see Figure 3.39)
	Switches to the RFEM work window for changing the view
	Turns the synchronization on and off in the graphic (see Figure 3.40)

Table 3.3 Buttons in *Rectangular Reinforcement* table

Click the [Filter] button to open the dialog box shown in Figure 3.39. It allows you to filter the table by surface numbers, reinforcement locations, and types of reinforcement. You can get a better overview by hiding particular properties.

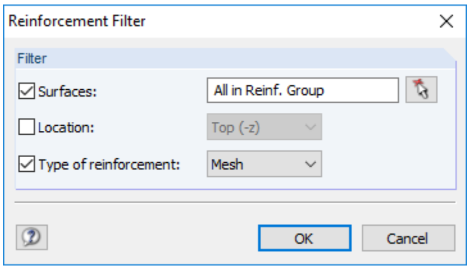


Figure 3.39 *Reinforcement Filter* dialog box

If the [Synchronization] is enabled after the calculation, the graphic only shows the reinforcement areas that are selected in the table. This display is also available for several areas if the row numbers are marked by pressing the [Ctrl] key.

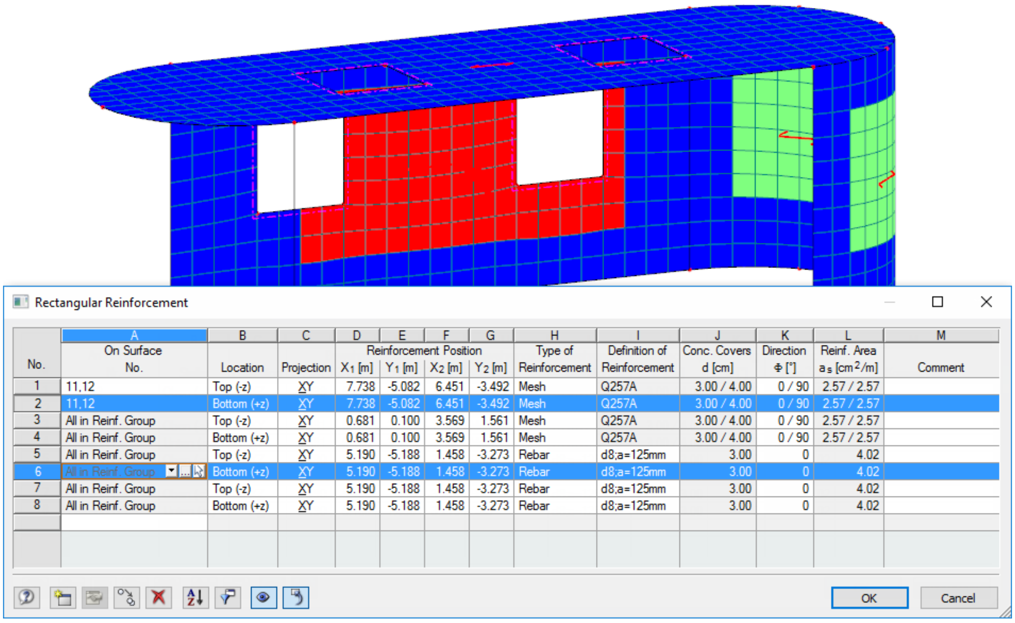


Figure 3.40 Selection synchronization with selection of two reinforcement areas

After the calculation, the *Reinforcement Covering* item appears in the *Results* navigator. With the two options of this item, you can evaluate how the required reinforcement is covered by the additional reinforcement.

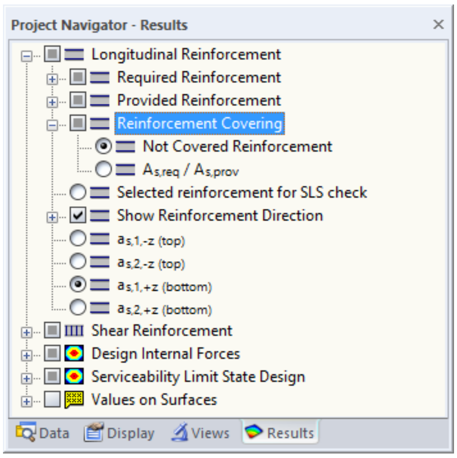


Figure 3.41 Results navigator for selection of *Reinforcement Covering*

When the *Not Covered Reinforcement* option is set, only the areas where a reinforcement is still needed are highlighted in the graphic.

With the display of $A_{s,req}$ / $A_{s,prov}$, any missing or provided reinforcement is quantified via color coding.

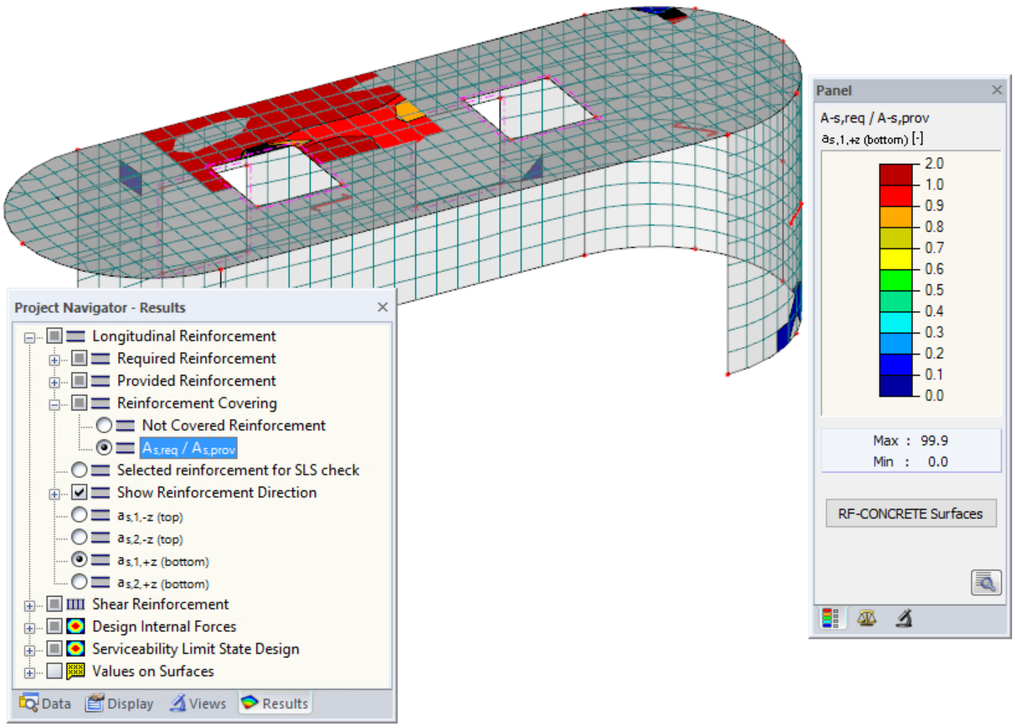


Figure 3.42 Display of ratio of required reinforcement to provided reinforcement

Longitudinal Reinforcement for Check of Shear Resistance

The following options allow you to control which longitudinal reinforcement is applied for the shear force design without shear reinforcement.

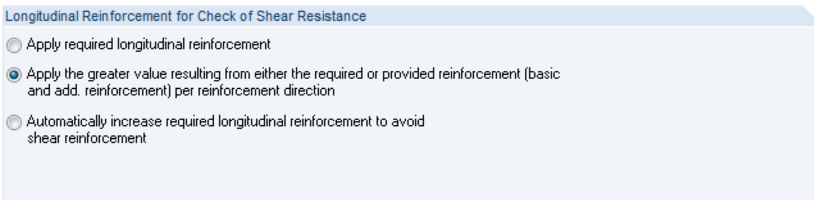


Figure 3.43 Longitudinal reinforcement for check of shear resistance

- *Apply required longitudinal reinforcement*

The check of the shear resistance is carried out with the transformed provided tension reinforcement in the direction of the principal shear force (see chapter 2.4.4.1).

- *Apply the greater value resulting from either the required or provided reinforcement*

For the check of the shear force resistance, either the statically required or the user-defined longitudinal reinforcement is used (see chapter 2.4.4.1).

- *Automatically increase required longitudinal reinforcement to avoid shear reinforcement*

If the required longitudinal reinforcement is not sufficient for the shear force resistance, the longitudinal reinforcement is increased in the main shear force direction until the shear force design is fulfilled without shear reinforcement (see chapter 2.4.4.1).

3.4.4 Standard

The parameters in this tab depend on the standard selected in window 1.1 *General Data*. You have to set standard-specific reinforcement data, hereafter described for EN 1992-1-1.

At the bottom of the tab, the [Default] button is available. You can use it to save the current entries for the standard as new default settings.

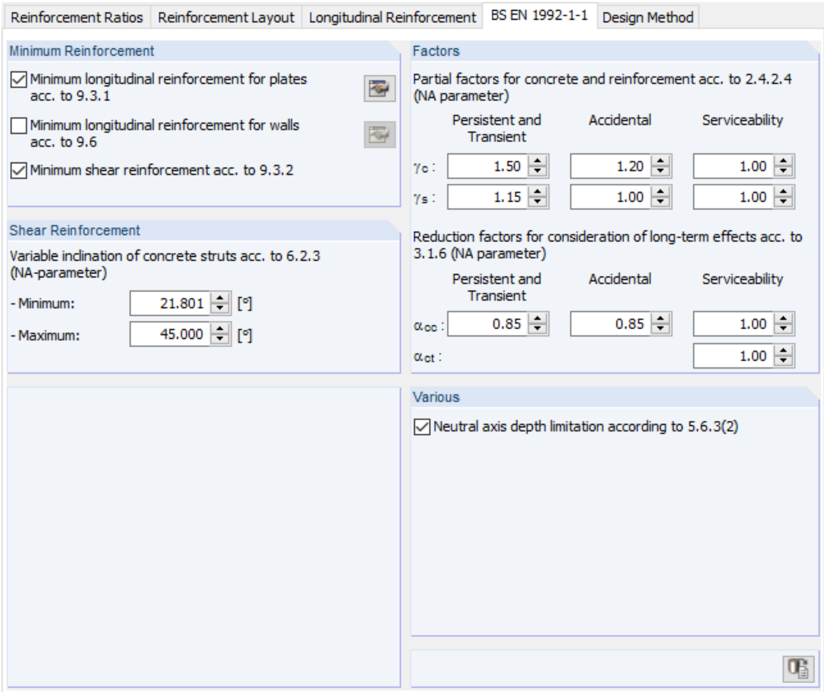


Figure 3.44 Window 1.4 *Reinforcement, BS EN 1992-1-1* tab

Minimum Reinforcement

This section allows you to control which provisions of the standard regarding the minimum reinforcements are to be considered in the design (see chapter 2.3.7).

For plates and walls, use the button to open more dialog boxes where you can set the directions of the minimum and main compression reinforcement.

Plates

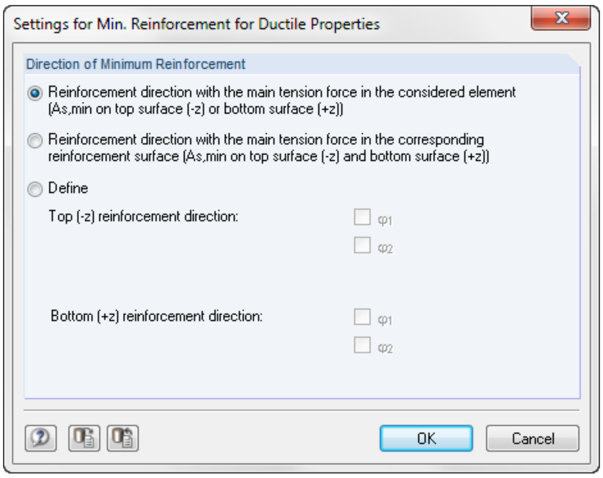


Figure 3.45 *Settings for Min. Reinforcement for Ductile Properties* dialog box

According to EN 1992-1-1, clause 9.3.1, the minimum reinforcement for ensuring a ductile structural component behavior is to be placed in the main span direction of the plate. As the main span direction cannot be found automatically when determining the reinforcement element by element, you can control the reinforcement direction you want to consider the minimum reinforcement in by using the following options:

- *Reinforcement direction with the main tension force in the considered element*

The minimum reinforcement is only considered in the reinforcement direction with the greatest tension force from all reinforcement directions of top surface (-z) and bottom surface (+z): The minimum reinforcement is only placed in **one** direction and on **one** side of the plate.

- *Reinforcement direction with the main tension force in the corresponding reinforcement surface*

For each reinforcement surface, the program searches for the reinforcement direction with the greatest tension force. The minimum reinforcement is then determined on each surface for these directions.

- *Define*

The reinforcement direction in which you want to apply the minimum reinforcement can be specified manually.

Walls

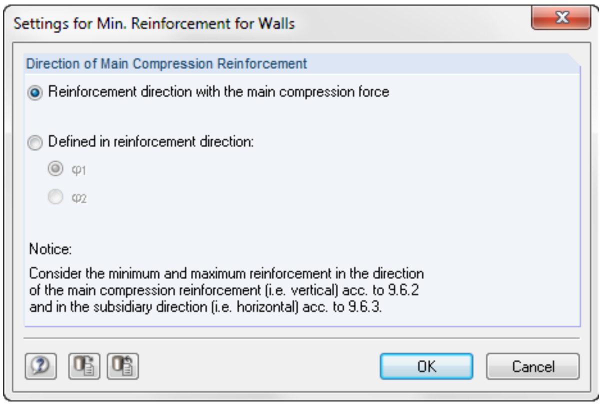


Figure 3.46 *Settings for Min. Reinforcement for Walls* dialog box

You can specify the direction of the main compression reinforcement for determining the minimum longitudinal reinforcement for walls in the direction of the *main compression force* or as *Defined*.

Shear Reinforcement

The two input fields define the allowable zone for the *inclination of concrete struts* θ. The angles are preset according to EN 1992-1-1, clause 6.2.3. You can adjust them, if necessary, but they must not lie outside the allowed limits.

Factors

The upper input fields control the *Partial factors* for concrete γ_c and for reinforcing steel γ_s in the design. The values according to EN 1992-1-1, Table 2.1 are preset for the different design situations.

The *Reduction factors* α_{cc} and α_{ct} take account of long-term effects on the concrete's compressive or tensile strength. These coefficients are governed in EN 1992-1-1, clause 3.1.6 (1) and 3.1.6 (2).

Various

With the check box, you can specify a *Neutral axis depth limitation* according to EN 1992-1-1, clause 5.6.3 (2). In this case, the maximum ratio is x_d / d = 0.45 for concrete up to strength class C50/60 and x_d / d = 0.35 for concrete starting from strength class C55/67.

3.4.5 Design Method

Figure 3.47 Window 1.4 *Reinforcement, Design Method* tab

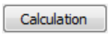
When determining the required reinforcement, the principal internal forces are transformed into design forces (in the direction of the reinforcements) and into a developing concrete compression strut force. The sizes of the design forces depend on the presumed angle of the concrete compression strut that braces the reinforcement mesh.

In the load situations "tension-tension" and "tension-compression" (see Figure 2.18), the design force may become negative in a reinforcement direction for a certain compression strut inclination, which means that compressive forces would occur for the tension reinforcement. By optimizing the design internal forces, the program modifies the direction of the concrete compression strut until the negative design force becomes zero.

During the *Optimization* of the internal forces, the program therefore analyzes which inclination angle of the concrete compression strut leads to the most favorable design result. The design moments are determined iteratively with adjusted inclination angles in order to find the energetically smallest solution with the least required reinforcement. The optimization process is described by example in chapter 2.4.1.

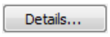
For concrete components subjected to compression such as walls, the optimization may lead to non-designable situations due to failure of the concrete compression strut. The optimization is therefore not recommended for the load situations "compression-compression".

4 Calculation



In RF-CONCRETE Surfaces, the [Calculation] is carried out with the internal forces from RFEM. If no results are available in RFEM yet, the program automatically starts the calculation of the internal forces.

4.1 Details



The *Details* dialog box manages global settings for the analysis approaches. You can open it with the [Details] button that is available in every input window.

4.1.1 Options

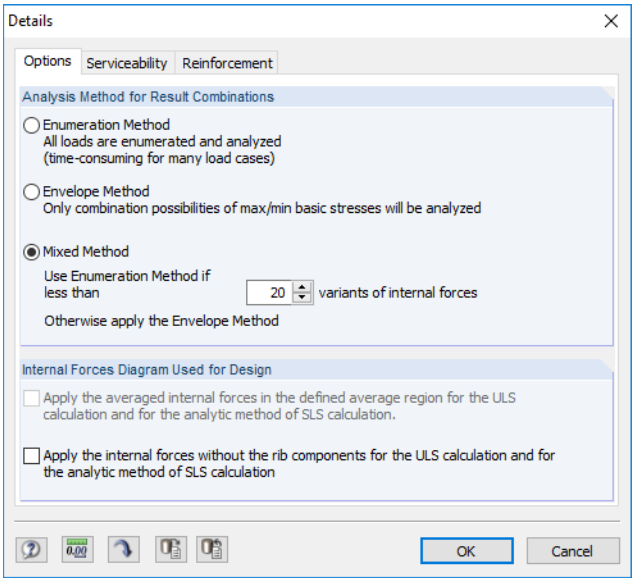


Figure 4.1 *Details* dialog box, *Options* tab

Analysis Method for Result Combinations

This section controls the way the design internal forces of result combinations are included in the calculation. This specification also applies when there are several load cases or load combinations to be analyzed in the design case. The *Mixed Method* is preset: Before the design is carried out, the program finds out if the *Enumeration Method* or the *Envelope Method* needs less computing time.

Enumeration Method

Each load case and load combination selected in window 1.1 *General Data* is designed individually. A reinforcement envelope is calculated from the results. For result combinations, 16 calculations are performed for the RFEM extreme values of the basic internal forces max/min m_x, max/min n_x, max/min m_y, max/min n_y, max/min m_{xy}, max/min n_{xy}, max/min v_x, and max/min v_y.

The enumeration method is precise because every combination is calculated separately and the enveloping reinforcement is determined afterwards. However, it is disadvantageous that the number of the analyzed combinations increases exponentially with the number of load cases as the program proceeds from row to row. If there are 50 selected load combinations, for example, there will also be 50 reinforcement designs. However, the designs cover all possible variants (constellations).

Envelope Method

From the load cases, load combinations, and result combinations selected in window 1.1 *General Data*, the module calculates an internal forces envelope. 16 extreme value variants are analyzed. The difference from the output of extreme values of RFEM result combinations is the following: The add-on module also analyzes extreme value states of the basic stresses that are not only based on the maximum basic internal forces but also on their interaction (for example $m_x + m_{xy}$). With this envelope from 16 variants of extreme values, the determination of the reinforcement is started. Hence, 16 calculation runs are carried out to determine the reinforcement. Even if there is a larger number of load cases, load combinations, or result combinations, the computing time is still adequate.

Since an envelope of internal forces is calculated with 16 extreme values, the most unfavorable variants may potentially not be considered, unlike when load cases are computed row by row. Combinations with load cases whose action directions are orthogonal are regarded as critical. In this case, a control calculation according to the enumeration method is recommended.

Mixed Method

Before the design is carried out, it is analyzed how many designs with the load cases, load combinations, and result combinations selected in window 1.1 *General Data* have to be performed per limit state. As mentioned in the *Enumeration Method* section, each LC or CO is designed separately. For one RC, 16 calculations are required for the extreme value variants of the basic internal forces. If you select one result combination and five load combinations for the design, for example, you get 16 + 5 = 21 calculation runs. As this number is larger than the default of 20 variants of internal forces, the design is carried out with the *Envelope Method*.

In the input field, you can specify the upper limit of the variants that are designed according to the precise enumeration method.

The *Mixed Method* is a compromise between precision of results and computation time.

Internal Forces Diagram Used for Design

Apply the averaged internal forces

For the design, the module normally uses the RFEM internal forces that are averaged surface by surface: RF-CONCRETE Surfaces transforms the moments and axial forces in the directions of the longitudinal reinforcement and then performs the designs (see chapter 2.5.1).

If the check box in this section is selected, the design is carried out with the internal forces available in the *average regions* of RFEM.

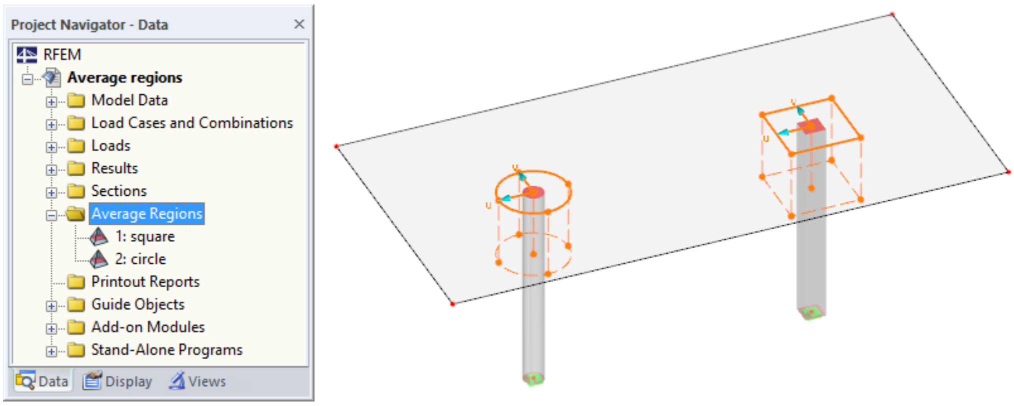


Figure 4.2 Average regions in RFEM

The average regions are described in chapter 9.7.3 of the RFEM manual.

By means of the averaged results, you can reduce singularities and consider local redistribution effects in the model.

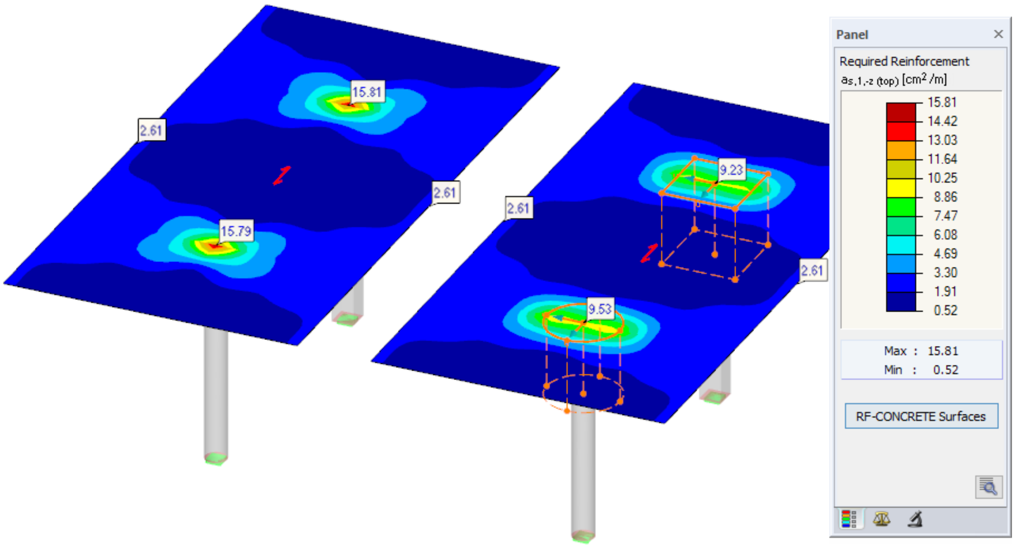


Figure 4.3 Top reinforcement for unaveraged internal forces (left) and average regions (right)

Apply the internal forces without the rib components

In RFEM, you can model a T-beam by using a surface and an eccentrically connected member of the type *rib*. The internal forces of the T-beam from surface component and member are determined as rib internal forces by integration of the surface internal forces.

The check box allows you to control whether the surface internal forces assigned to the rib are considered in the surface design. The design with the rib component is preset.

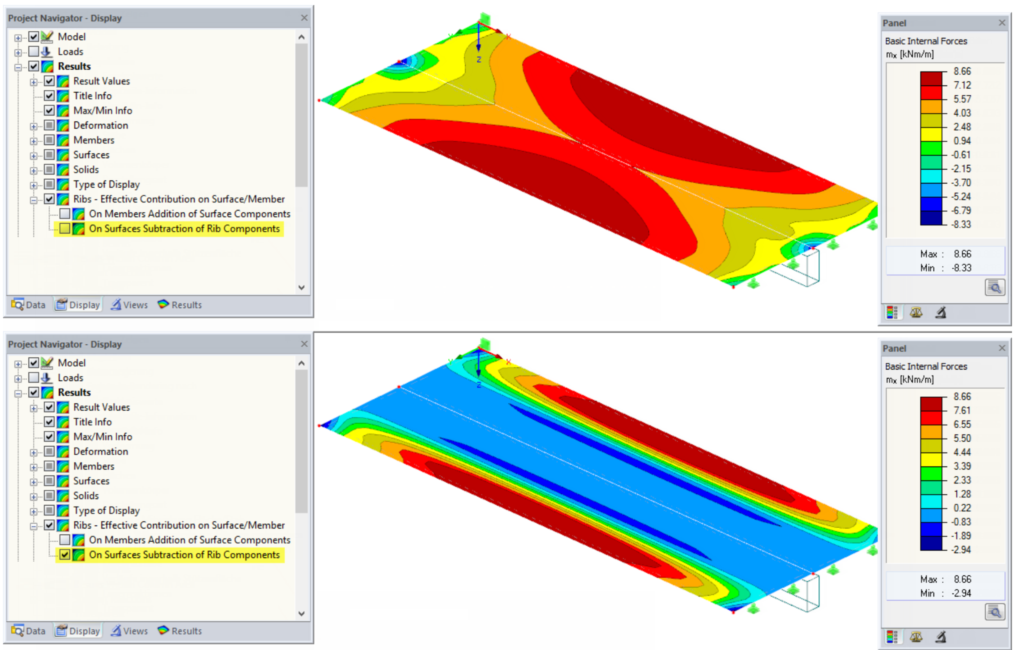


Figure 4.4 Surface internal forces with rib component (above) and without rib component (below)

4.1.2 Serviceability

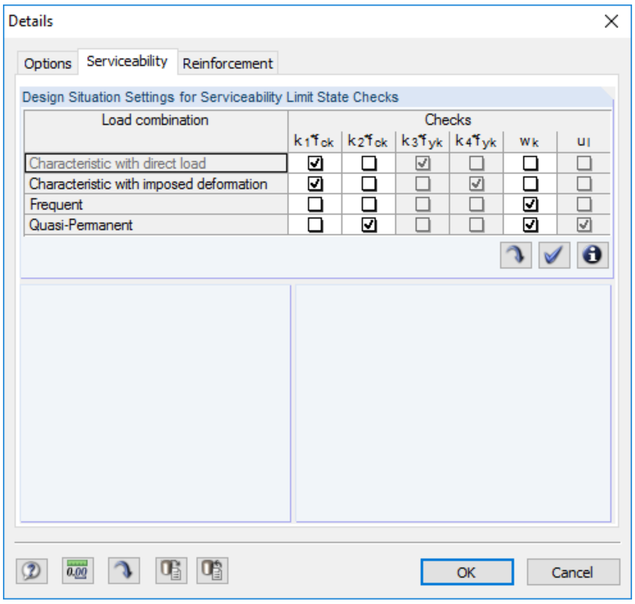


Figure 4.5 *Details* dialog box, *Serviceability* tab

Design Situation Settings for Serviceability Limit State Checks

The table allows you to control which serviceability limit state checks are performed in the individual design situations. Thus, it is possible to calculate different limit values per design situation directly in one concrete case. For example, with the settings in Figure 4.7, the crack widths w_k are only analyzed with loads of the design situations *Frequent* and *Quasi-Permanent*.

Click the [Info] button to display information about the requirements that the limit values of the serviceability limit state design are based on.

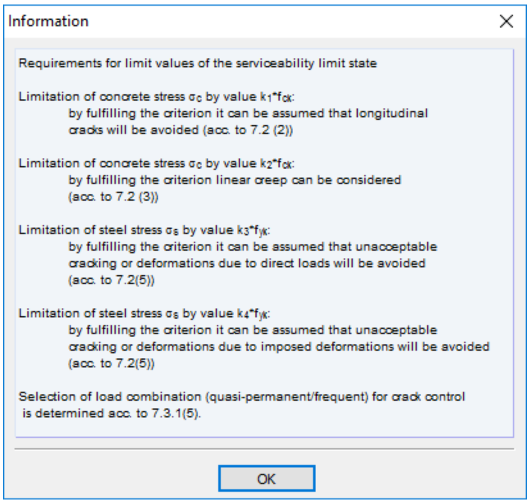


Figure 4.6 *Information* dialog box

Use the [Select or Clear All for Selected Line] button to quickly activate or suppress all checks for a certain design situation.

Click the [Reset to Default Values of the Standard] button to restore the standard's default specifications.

4.1.3 Reinforcement

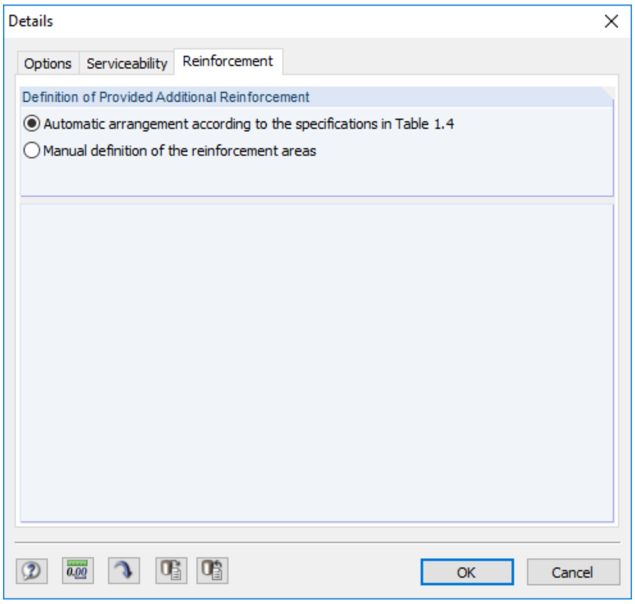


Figure 4.7 *Details* dialog box, *Reinforcement* tab

The *Automatic arrangement according to the specifications in Table 1.4* is preset for the additional reinforcement. This means that the rebars and mesh reinforcements are arranged with the parameters of the *Longitudinal Reinforcement* tab to fulfill the serviceability limit state design.

As an alternative, a *Manual definition of the reinforcement areas* is possible. When the check box is selected, the *Longitudinal Reinforcement* tab in window 1.4 is adjusted accordingly (see Figure 3.36).

The manual definition of reinforcement areas is described in chapter 3.4.3.

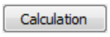
4.2 Check

Before you start the calculation, it is recommended to check if the input data is correct. The [Check] button is available in every input window of RF-CONCRETE Surfaces.

The program checks if the data required for the design is complete and if the references of the data sets are defined sensibly. If the program does not detect any input errors, the following message is displayed.

Figure 4.8 Plausibility check

4.3 Starting the Calculation



You can start the [Calculation] in every input window of RF-CONCRETE Surfaces by clicking the corresponding button.

RF-CONCRETE Surfaces searches for the results of the load cases, load combinations, and result combinations to be designed. If they cannot be found, the RFEM calculation for determining the design-relevant internal forces starts first.

You can also start the calculation in the RFEM user interface: The *To Calculate* dialog box (menu item *Calculate* → *To Calculate*) lists the design cases of the add-on modules such as load cases or load combinations.

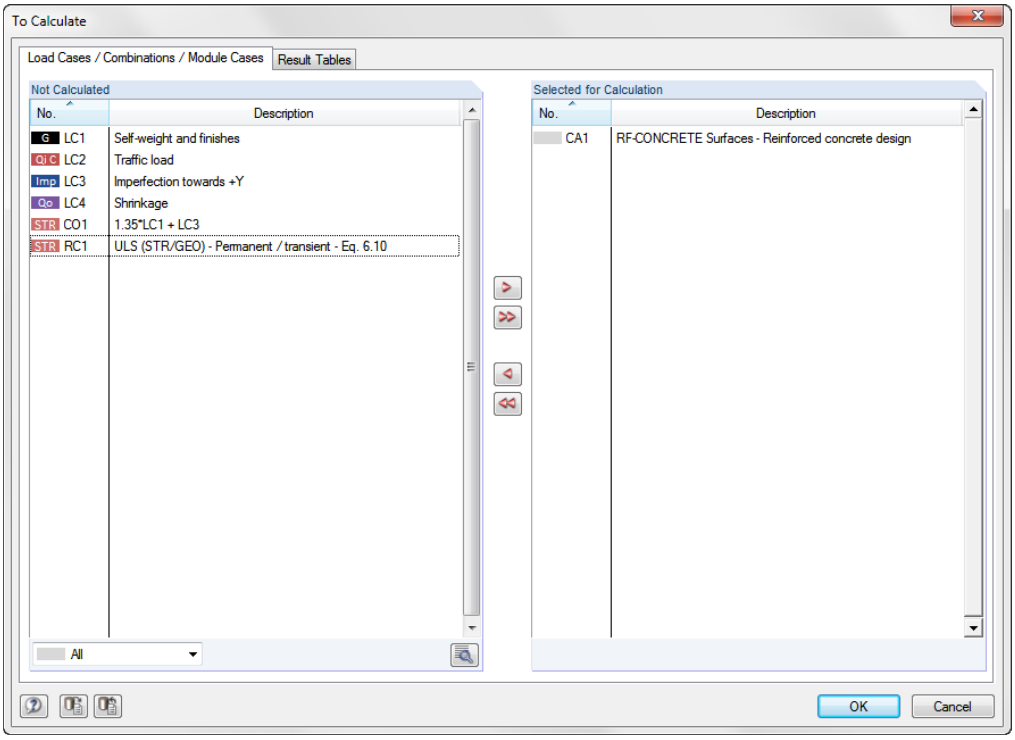
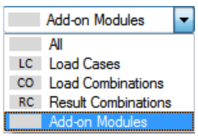


Figure 4.9 *To Calculate* dialog box



If the design cases of RF-CONCRETE Surfaces are missing in the *Not Calculated* list, select *All* or *Add-on Modules* in the drop-down list below.

To transfer the selected RF-CONCRETE Surfaces cases to the list on the right, use [>]. Then, click [OK] to start the calculation.

A design case can also be calculated directly by means of the list in the toolbar: Set the RF-CONCRETE Surfaces case and then click the [Show Results] button.

Figure 4.10 Direct calculation of an RF-CONCRETE Surfaces design case in RFEM

You can subsequently observe the calculation process in a dialog box.

5 Results

Window 2.1 *Required Reinforcement Total* is displayed immediately after the calculation.

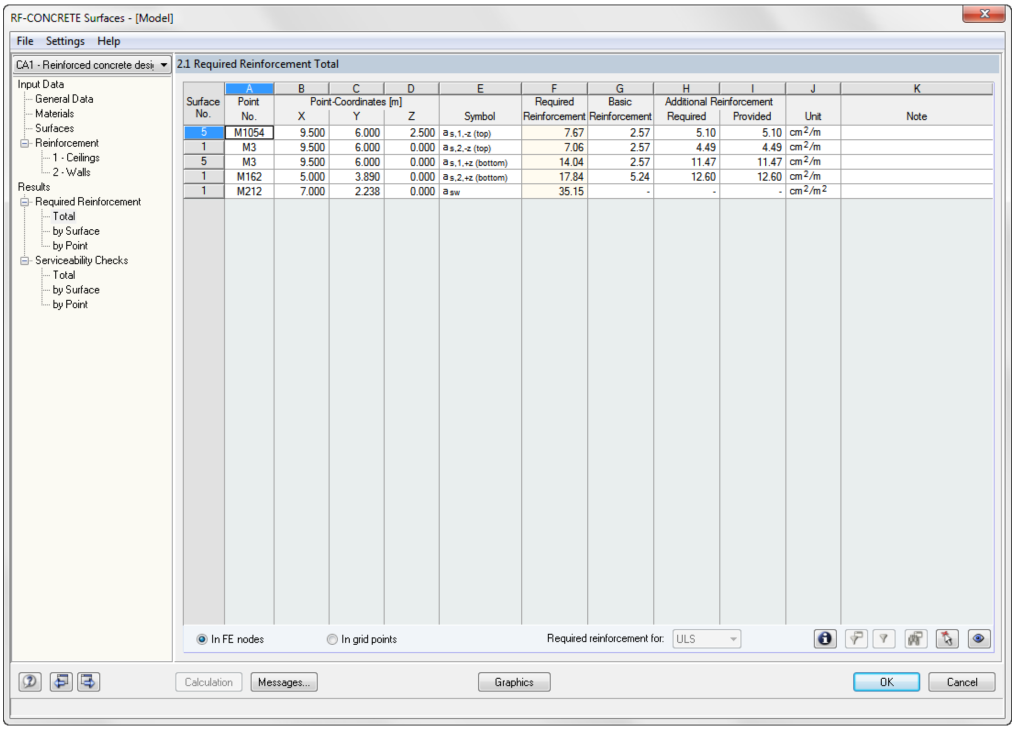


Figure 5.1 Result window

The ultimate limit state designs are sorted in the result windows 2.1 through 2.3 according to various criteria.

Windows 3.1 to 3.3 provide information about the serviceability limit state designs.

You can directly select a window by clicking its entry in the navigator. Use the buttons shown on the left to set the previous or next window. You can also use the function keys [F2] and [F3] to go through the windows.

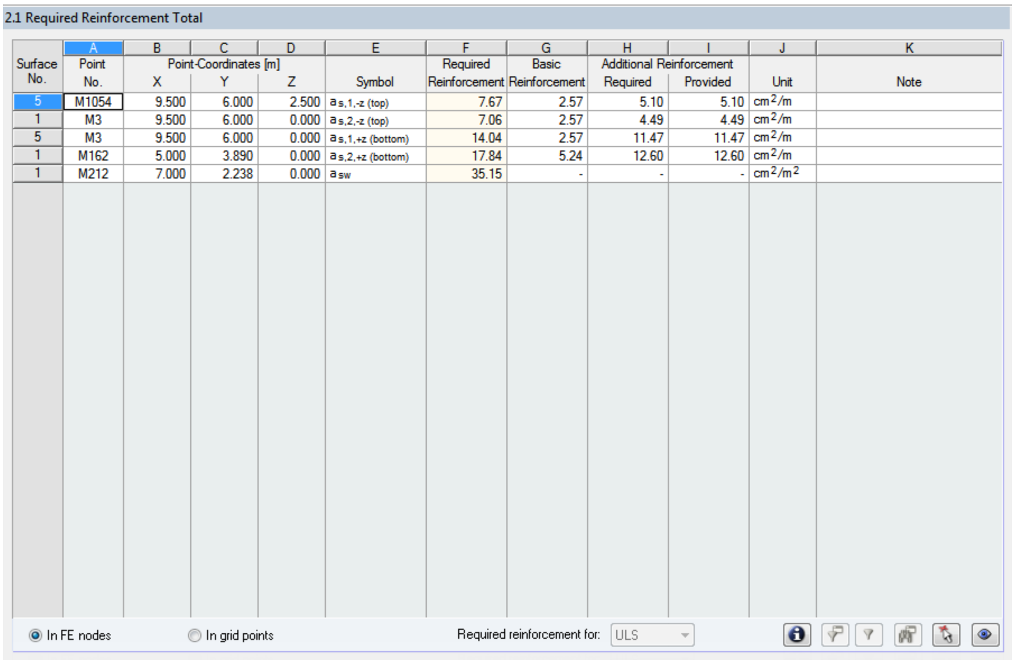
At the bottom of the tables, there are two radio buttons. They control whether the results data is shown *In FE nodes* or *In grid points*. The results of the FE nodes are determined directly by the analysis core and the grid point results by interpolation of the FE node results.

Click [OK] to save the results. RF-CONCRETE Surfaces closes and you return to the main program.

This chapter presents the result windows in order. Evaluating and checking the results is described in chapter 6.

5.1 Required Reinforcement Total

The maximum reinforcement areas of all analyzed surfaces are output, which are determined from the internal forces of the selected load cases, load combinations, and result combinations for the ultimate limit state design.



2.1 Required Reinforcement Total

	A	B	C	D	E	F	G	H	I	J	K
Surface No.	Point No.	Point-Coordinates [m] X	Y	Z	Symbol	Required Reinforcement	Basic Reinforcement	Additional Reinforcement Required	Provided	Unit	Note
5	M1054	9.500	6.000	2.500	$a_{s,1,-z}$ (top)	7.67	2.57	5.10	5.10	cm^2/m	
1	M3	9.500	6.000	0.000	$a_{s,2,-z}$ (top)	7.06	2.57	4.49	4.49	cm^2/m	
5	M3	9.500	6.000	0.000	$a_{s,1,+z}$ (bottom)	14.04	2.57	11.47	11.47	cm^2/m	
1	M162	5.000	3.890	0.000	$a_{s,2,+z}$ (bottom)	17.84	5.24	12.60	12.60	cm^2/m	
1	M212	7.000	2.238	0.000	a_{sw}	35.15	-	-	-	cm^2/m^2	

Figure 5.2 Window 2.1 *Required Reinforcement Total*

Surface No.

This column shows the numbers of the surfaces where the governing points are located.

Point No.

In these FE nodes or grid points, the greatest required reinforcement for each position and direction has been determined. The type of the reinforcement is indicated in column E, *Symbol*.

The FE mesh nodes *M* are generated automatically. In contrast to this, the grid points *G* can be controlled in RFEM, since user-defined result grids are possible for surfaces. The function is described in chapter 8.13 of the RFEM manual.

Point-Coordinates X/Y/Z

The three columns show the coordinates of the respective governing FE nodes or grid points.

Symbol

Column E indicates the type of the reinforcement. For the four (or six) longitudinal reinforcements, it respectively shows the direction (*1*, 2, and possibly 3) and the surface side (*top* and *bottom*).

The reinforcement directions were specified in window 1.4 *Reinforcement* in the *Reinforcement Layout* tab (see chapter 3.4.2).

The top reinforcement is located on the surface side in the direction of the negative local surface axis z (-z), the bottom reinforcement accordingly in the direction of the positive z-axis (+z). Figure 3.28 shows the axis systems of the surfaces.

The shear reinforcement is designated as a_{sw}.

Required Reinforcement

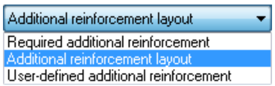
This column displays the reinforcement areas that are required for the ultimate limit state design.

Basic Reinforcement

This column shows the user-defined basic reinforcement specified in the *Longitudinal Reinforcement* tab of window 1.4 *Reinforcement* (see chapter 3.4.3).

Additional Reinforcement

If you only perform the ultimate limit state design, the *Required* column displays the difference between the required reinforcement (column F) and the provided basic reinforcement (column G).



If the serviceability limit state designs have also been performed, you can see the reinforcement areas that are required to fulfill the serviceability limit state designs with the specifications in the *Longitudinal Reinforcement* tab of window 1.4 *Reinforcement* (see chapter 3.4.3). The *Provided* column shows the reinforcement that is available as additional reinforcement for the serviceability limit state design according to the specifications in the *Longitudinal Reinforcement* tab of window 1.4 *Reinforcement*.

Note

The final column indicates non-designable situations or notes referring to design issues. The numbers are clarified in the status bar.

The button shown on the left allows you to view all [Messages] of the current design case. A dialog box with an overview appears.

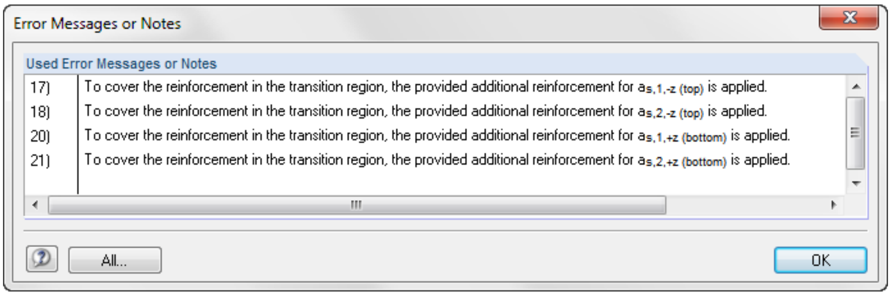


Figure 5.3 *Error Messages or Notes dialog box*

The buttons of window 2.1 are explained in chapter 6.

5.2 Required Reinforcement by Surface

2.2 Required Reinforcement by Surface

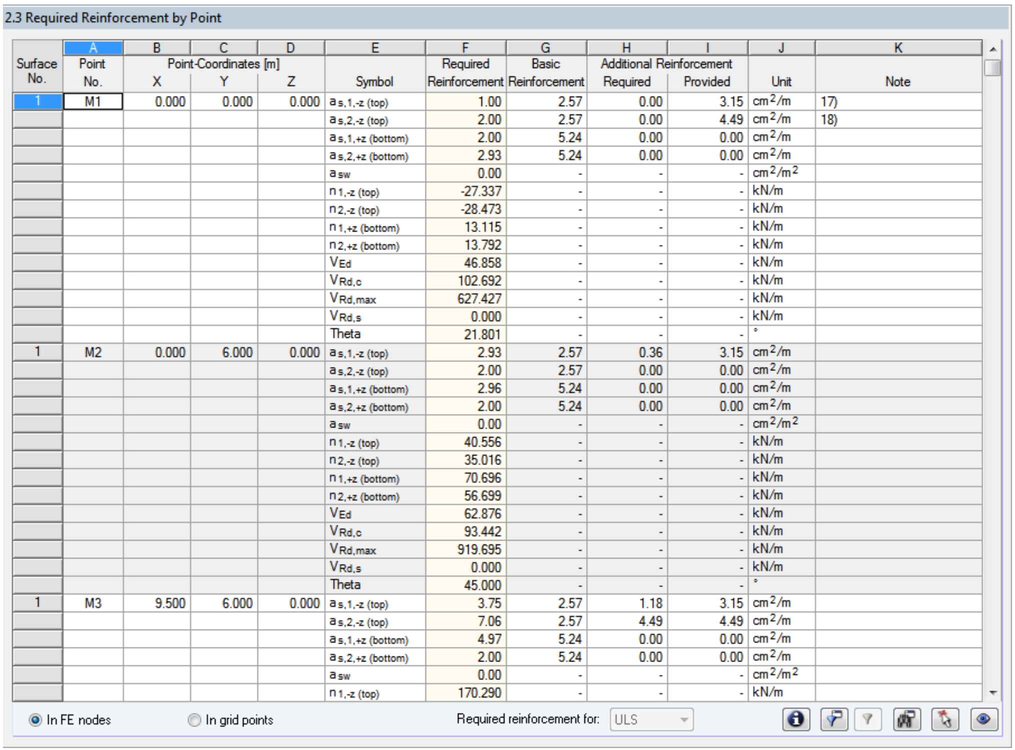
	A	B	C	D	E	F	G	H	I	J	K
Surface No.	Point No.	Point-Coordinates [m] X	Y	Z	Symbol	Required Reinforcement	Basic Reinforcement	Additional Reinforcement Required	Provided	Unit	Note
1	M20	6.000	6.000	0.000	$a_{s,1,-z}$ (top)	5.72	2.57	3.15	3.15	cm²/m	
	M3	9.500	6.000	0.000	$a_{s,2,-z}$ (top)	7.06	2.57	4.49	4.49	cm^2/m	
	M169	5.110	4.000	0.000	$a_{s,1,+z}$ (bottom)	10.69	5.24	5.45	5.45	cm^2/m	
	M162	5.000	3.890	0.000	$a_{s,2,+z}$ (bottom)	17.84	5.24	12.60	12.60	cm^2/m	
	M212	7.000	2.238	0.000	a_{sw}	35.15	-	-	-	cm^2/m^2	
2	M678	9.926	5.785	0.000	$a_{s,1,-z}$ (top)	5.72	2.57	3.15	3.15	cm^2/m	
	M678	9.926	5.785	0.000	$a_{s,2,-z}$ (top)	6.54	2.57	3.97	4.49	cm^2/m	
	M3	9.500	6.000	0.000	$a_{s,1,+z}$ (bottom)	2.00	5.24	0.00	0.00	cm^2/m	
	M74	9.500	4.500	0.000	$a_{s,2,+z}$ (bottom)	2.45	5.24	0.00	0.00	cm^2/m	
	M3	9.500	6.000	0.000	a_{sw}	13.66	-	-	-	cm^2/m^2	
3	M10	0.000	2.500	2.000	$a_{s,1,-z}$ (top)	2.51	2.57	0.00	0.00	cm^2/m	
	M748	0.000	3.000	0.000	$a_{s,2,-z}$ (top)	3.59	2.57	1.02	2.91	cm^2/m	
	M717	0.000	0.000	0.500	$a_{s,1,+z}$ (bottom)	2.53	2.57	0.00	0.00	cm^2/m	
	M717	0.000	0.000	0.500	$a_{s,2,+z}$ (bottom)	0.88	2.57	0.00	0.00	cm^2/m	
	M1	0.000	0.000	0.000	a_{sw}	0.00	-	-	-	cm^2/m^2	
4	M17	9.500	0.000	4.000	$a_{s,1,-z}$ (top)	2.51	2.57	0.00	0.00	cm^2/m	
	M4	9.500	0.000	0.000	$a_{s,2,-z}$ (top)	5.48	2.57	2.91	2.91	cm^2/m	
	M821	9.500	0.000	0.500	$a_{s,1,+z}$ (bottom)	3.84	2.57	1.27	11.47	cm^2/m	
	M878	8.500	0.000	3.500	$a_{s,2,+z}$ (bottom)	2.51	2.57	0.00	0.00	cm^2/m	
	M1	0.000	0.000	0.000	a_{sw}	0.00	-	-	-	cm^2/m^2	
5	M1054	9.500	6.000	2.500	$a_{s,1,-z}$ (top)	7.67	2.57	5.10	5.10	cm^2/m	
	M821	9.500	0.000	0.500	$a_{s,2,-z}$ (top)	3.38	2.57	0.81	2.91	cm^2/m	
	M3	9.500	6.000	0.000	$a_{s,1,+z}$ (bottom)	14.04	2.57	11.47	11.47	cm^2/m	
	M3	9.500	6.000	0.000	$a_{s,2,+z}$ (bottom)	3.96	2.57	1.39	1.39	cm^2/m	
	M3	9.500	6.000	0.000	a_{sw}	8.78	-	-	-	cm^2/m^2	

In FE nodes   In grid points   Required reinforcement for: ULS

Figure 5.4 Window 2.2 *Required Reinforcement by Surface*

This window lists the maximum reinforcement areas that are required for each of the designed surfaces. The columns are explained in chapter 5.1.

5.3 Required Reinforcement by Point

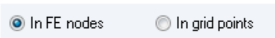


2.3 Required Reinforcement by Point

Surface No.	Point No.	Point-Coordinates [m] X	Y	Z	Symbol	Required Reinforcement	Basic Reinforcement	Additional Reinforcement Required	Provided	Unit	Note
1	M1	0.000	0.000	0.000	$a_{s,1,-z\,(top)}$	1.00	2.57	0.00	3.15	cm^2/m	17)
					$a_{s,2,-z\,(top)}$	2.00	2.57	0.00	4.49	cm^2/m	18)
					$a_{s,1,+z\,(bottom)}$	2.00	5.24	0.00	0.00	cm^2/m	
					$a_{s,2,+z\,(bottom)}$	2.93	5.24	0.00	0.00	cm^2/m	
					a_{sw}	0.00	-	-	-	cm^2/m^2	
					$n_{1,-z\,(top)}$	-27.337	-	-	-	kN/m	
					$n_{2,-z\,(top)}$	-28.473	-	-	-	kN/m	
					$n_{1,+z\,(bottom)}$	13.115	-	-	-	kN/m	
					$n_{2,+z\,(bottom)}$	13.792	-	-	-	kN/m	
					V_{Ed}	46.858	-	-	-	kN/m	
					$V_{Rd,c}$	102.692	-	-	-	kN/m	
					$V_{Rd,max}$	627.427	-	-	-	kN/m	
					$V_{Rd,s}$	0.000	-	-	-	kN/m	
					Theta	21.801	-	-	-	°	
1	M2	0.000	6.000	0.000	$a_{s,1,-z\,(top)}$	2.93	2.57	0.36	3.15	cm^2/m	
					$a_{s,2,-z\,(top)}$	2.00	2.57	0.00	0.00	cm^2/m	
					$a_{s,1,+z\,(bottom)}$	2.96	5.24	0.00	0.00	cm^2/m	
					$a_{s,2,+z\,(bottom)}$	2.00	5.24	0.00	0.00	cm^2/m	
					a_{sw}	0.00	-	-	-	cm^2/m^2	
					$n_{1,-z\,(top)}$	40.556	-	-	-	kN/m	
					$n_{2,-z\,(top)}$	35.016	-	-	-	kN/m	
					$n_{1,+z\,(bottom)}$	70.696	-	-	-	kN/m	
					$n_{2,+z\,(bottom)}$	56.699	-	-	-	kN/m	
					V_{Ed}	62.876	-	-	-	kN/m	
					$V_{Rd,c}$	93.442	-	-	-	kN/m	
					$V_{Rd,max}$	919.695	-	-	-	kN/m	
					$V_{Rd,s}$	0.000	-	-	-	kN/m	
					Theta	45.000	-	-	-	°	
1	M3	9.500	6.000	0.000	$a_{s,1,-z\,(top)}$	3.75	2.57	1.18	3.15	cm^2/m	
					$a_{s,2,-z\,(top)}$	7.06	2.57	4.49	4.49	cm^2/m	
					$a_{s,1,+z\,(bottom)}$	4.97	5.24	0.00	0.00	cm^2/m	
					$a_{s,2,+z\,(bottom)}$	2.00	5.24	0.00	0.00	cm^2/m	
					a_{sw}	0.00	-	-	-	cm^2/m^2	
					$n_{1,-z\,(top)}$	170.290	-	-	-	kN/m	

In FE nodes · In grid points · Required reinforcement for: ULS

Figure 5.5 Window 2.3 *Required Reinforcement By Point*



This result window lists the maximum reinforcement areas for all FE nodes / grid points of each surface. The columns are explained in chapter 5.1.

In addition to the longitudinal and shear reinforcements, the table displays design-relevant values of the actions and resistances. For EN 1992-1-1, these are the following:

Symbol	**Meaning**
$n_{1,-z}$ (top)	Axial or membrane force for designing the reinforcement in the first reinforcement direction at the surface's top side
$n_{2,-z}$ (top)	Axial or membrane force for designing the reinforcement in the second reinforcement direction at the surface's top side
$n_{1,+z(bottom)}$	Like $n_{1,-z(top)}$, but for surface bottom
$n_{2,+z(bottom)}$	Like $n_{2,-z(top)}$, but for surface bottom
$m_{1,-z(top)}$ $m_{2,-z(top)}$	Only for model type 2D - XY ($u_Z/\varphi_X/\varphi_Y$): moment for designing the reinforcement in the first or second reinforcement direction at the surface's top side
$m_{1,-z(bottom)}$ $m_{2,-z(bottom)}$	Like $m_{1,-z(top)}$ / $m_{2,-z(top)}$, but for surface bottom
V_{Ed}	Design value of applied shear force

$V_{Rd,c}$	Shear force resistance without shear reinforcement
$V_{Rd,max}$	Shear force resistance of concrete compression strut
$V_{Rd,s}$	Shear force resistance of shear reinforcement
Theta	Inclination angle of concrete compression strut

Table 5.1 Output values in window 2.3 for EN 1992-1-1

The search function that you can access with the button shown on the left allows you to quickly find FE nodes and grid points (see Figure 6.7).

5.4 Serviceability Design Total

The upper part of the window provides a summary of the governing serviceability limit state designs. The lower part displays the intermediate results of the current FE node or grid point (of the entry selected above) including all design-relevant parameters. You can expand the chapters with ⊞ and reduce them with ⊟ .

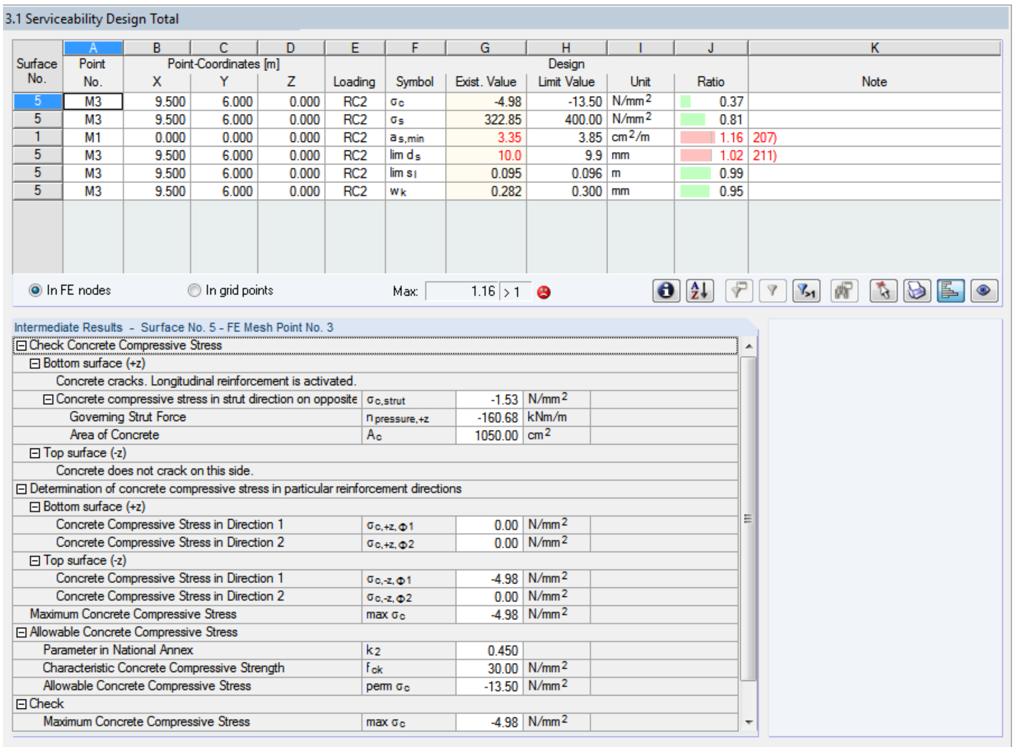


Figure 5.6 Window 3.1 *Serviceability Design Total*

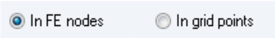
Figure 5.6 shows the result window of an analytical serviceability limit state check. Chapter 5.7 describes the result windows that appear after a nonlinear serviceability limit state calculation has been carried out.

The method of check is defined in the *Serviceability Limit State* tab of window 1.1 *General Data* (see Figure 3.8).

Surface No.

This column shows the numbers of the surfaces where the governing points are located.

Point No.



These FE nodes or grid points provide the maximum ratios for the required checks. The type of check is specified in column F, *Symbol*.

The FE mesh nodes *M* are generated automatically. The grid points *G* can be controlled in RFEM (see chapter 8.12 of the RFEM manual).

Point-Coordinates X/Y/Z

The three columns show the coordinates of the governing FE nodes or grid points.

Loading

Column E displays the load cases, load combinations, and result combinations whose internal forces lead to the greatest ratio in the respective serviceability limit state design.

Symbol

Column F shows the type of the serviceability limit state design. In the analytical method, up to six design types are displayed. They are described in chapter 2.6.4 by means of an example.

The symbols have the following meaning:

Type	Design SLS
σ_c	Limitation of concrete compressive stress (→ chapter 2.6.4.7) according to specifications in window 1.3 *Surfaces* (see Figure 3.19)
σ_s	Limitation of reinforcing steel stress (→ chapter 2.6.4.8) according to specifications in window 1.3 *Surfaces* (see Figure 3.18)
$a_{s,min}$	Minimum reinforcement for crack width limitation (→ chapter 2.6.4.9) according to specifications in window 1.3 *Surfaces* (see Figure 2.97)
lim d_s	Limitation of rebar diameter (→ chapter 2.6.4.10) according to specifications in window 1.4 *Reinforcement* (see Figure 3.31)
lim s_l	Limitation of rebar spacing (→ chapter 2.6.4.11) according to specifications in window 1.4 *Reinforcement* (see Figure 3.31)
w_k	Limitation of crack width (→ chapter 2.6.4.12) according to specifications in window 1.3 *Surfaces* (see Figure 3.18)

Table 5.2 Serviceability limit state designs according to analytical method

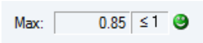
Existing Value

This column displays the values that are governing among all examined surfaces for the serviceability limit state designs.

Limit Value

The limit values are determined from the standard specifications and the load situation. The determination of limit values is described in chapter 2.6.4.

Ratio



Column J shows the ratio of existing value (column G) to limit value (column H). Ratios greater than 1 mean that the design is not fulfilled. The length of the colored bar depicts the respective design ratio graphically.

For the serviceability limit state design, not all check types have to be fulfilled (see explanation in Figure 3.11).

Note

The final column indicates non-designable situations or notes referring to design issues. The numbers are clarified in the status bar.

The button shown on the left allows you to view all [Messages] of the current design case. A dialog box with an overview appears (see Figure 5.3).

The buttons are described in chapter 6.

5.5 Serviceability Design by Surface

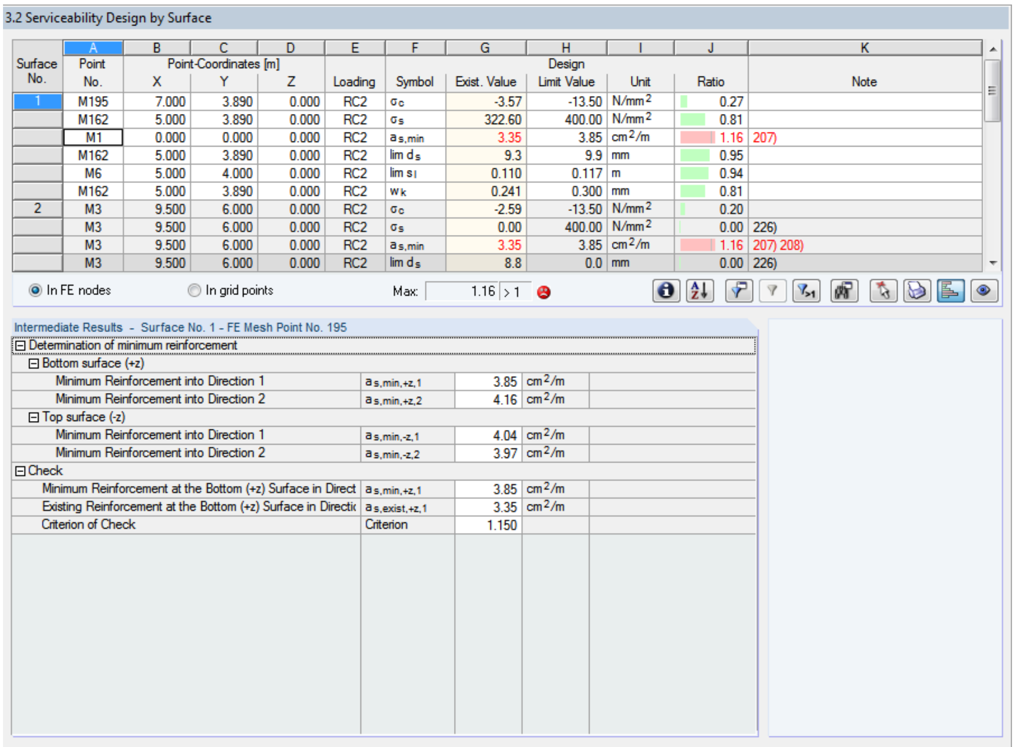


Figure 5.7 Window 3.2 *Serviceability Design by Surface*

This window lists the maximum ratios of each designed surface that are obtained for the serviceability limit state designs. The columns are explained in chapter 5.4.

5.6 Serviceability Design by Point

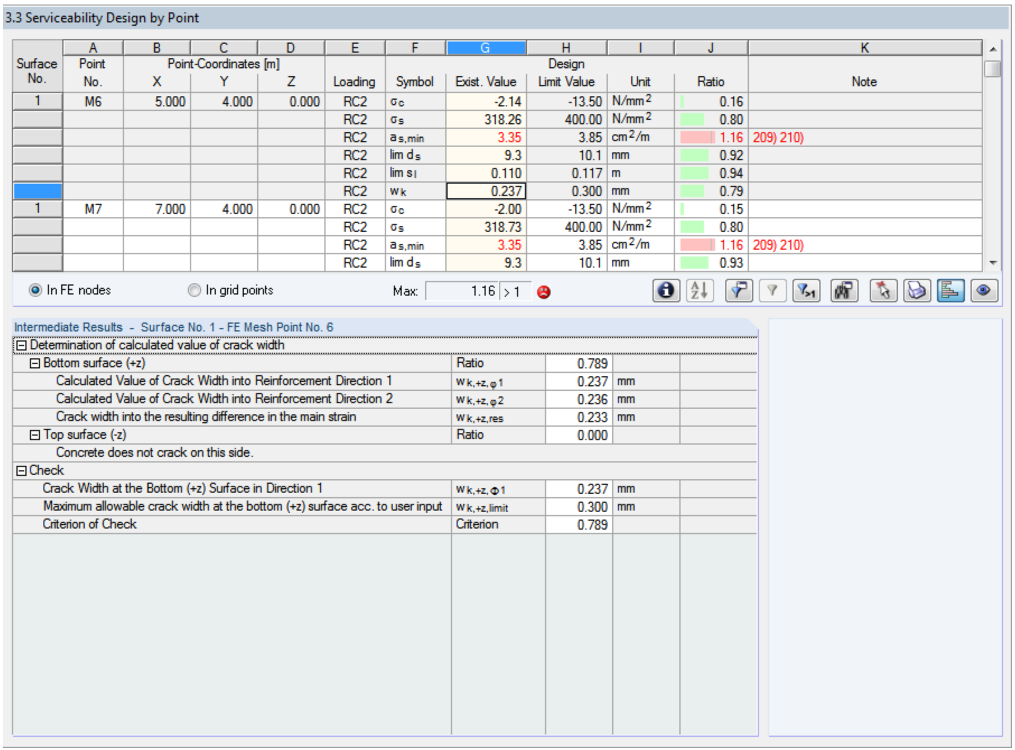
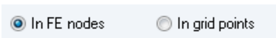


Figure 5.8 Window 3.3 *Serviceability Design by Point*



This result window lists the maximum ratios for all FE nodes / grid points of each surface. The columns are explained in chapter 5.4.

The search function that you can access with the button shown on the left allows you to quickly find FE nodes and grid points (see Figure 6.7).

5.7 Nonlinear Calculation Total

The upper part of the window provides a summary of the governing serviceability limit state designs. The lower part displays the intermediate results of the current FE node or grid point (of the entry selected above) including all design-relevant parameters. You can expand the chapters with ⊞ and reduce them with ⊟.

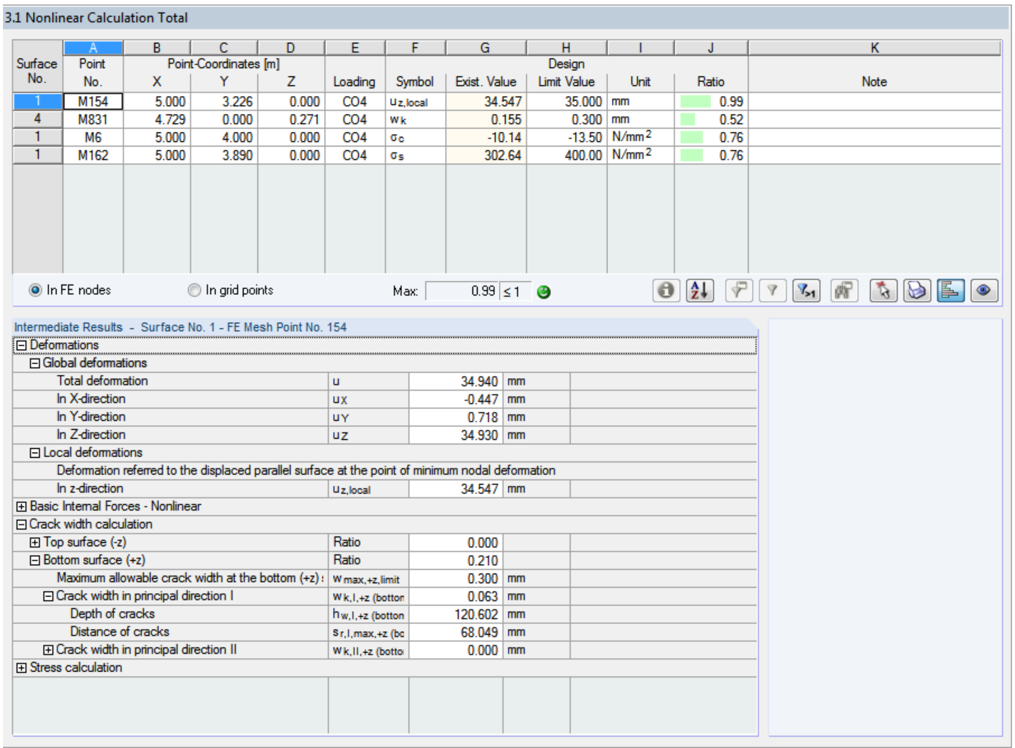


Figure 5.9 Window 3.1 *Nonlinear Calculation Total*

Figure 5.9 shows the result window of a nonlinear serviceability limit state design. The method of check is defined in the *Serviceability* tab of window 1.1 *General Data* (see Figure 3.8).

The columns are described in chapter 5.4.

The symbols signify the following designs:

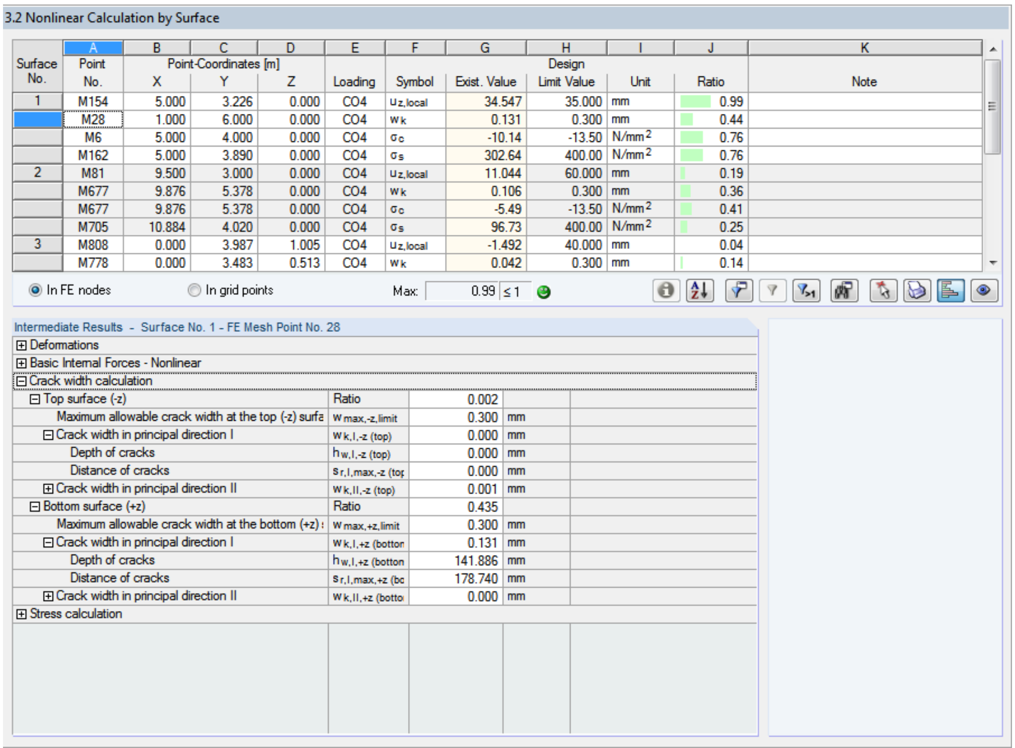
Symbol	**Design SLS**
$u_{z,local}$	Deformation in cracked state (→ chapter 2.8.2.4) according to specifications in window 1.3 *Surfaces*
w_k	Limitation of crack width (→ chapter 2.6.4.12) according to specifications in window 1.3 *Surfaces* (see Figure 3.18)
σ_c	Limitation of concrete compressive stress (→ chapter 2.6.4.7) according to specifications in window 1.3 *Surfaces* (see Figure 3.19)
σ_s	Limitation of reinforcing steel stress (→ chapter 2.6.4.8) according to specifications in window 1.3 *Surfaces* (see Figure 3.18)

Table 5.3 Serviceability limit state designs according to the nonlinear method

The deformations, crack widths, and stresses represent the results in cracked sections (state II).

The crack widths w_k for the intermediate results refer to the reinforcement directions. For example, the value for $w_{k,I,-z(top)}$ represents the crack width for the first reinforcement direction at the top side of the surface; the crack runs perpendicular to reinforcement direction 1.

5.8 Nonlinear Calculation by Surface



3.2 Nonlinear Calculation by Surface

Surface No.	A Point No.	B Point-Coordinates [m] X	C Y	D Z	E Loading	F Symbol	G Exist. Value	H Design Limit Value	I Unit	J Ratio	K Note
1	M154	5.000	3.226	0.000	CO4	$u_{z,local}$	34.547	35.000	mm	0.99	
	M28	1.000	6.000	0.000	CO4	w_k	0.131	0.300	mm	0.44	
	M6	5.000	4.000	0.000	CO4	σ_c	-10.14	-13.50	N/mm²	0.76	
	M162	5.000	3.890	0.000	CO4	σ_s	302.64	400.00	N/mm²	0.76	
2	M81	9.500	3.000	0.000	CO4	$u_{z,local}$	11.044	60.000	mm	0.19	
	M677	9.876	5.378	0.000	CO4	w_k	0.106	0.300	mm	0.36	
	M677	9.876	5.378	0.000	CO4	σ_c	-5.49	-13.50	N/mm²	0.41	
	M705	10.884	4.020	0.000	CO4	σ_s	96.73	400.00	N/mm²	0.25	
3	M808	0.000	3.987	1.005	CO4	$u_{z,local}$	-1.492	40.000	mm	0.04	
	M778	0.000	3.483	0.513	CO4	w_k	0.042	0.300	mm	0.14	

In FE nodes   In grid points   Max: 0.99 ≤ 1

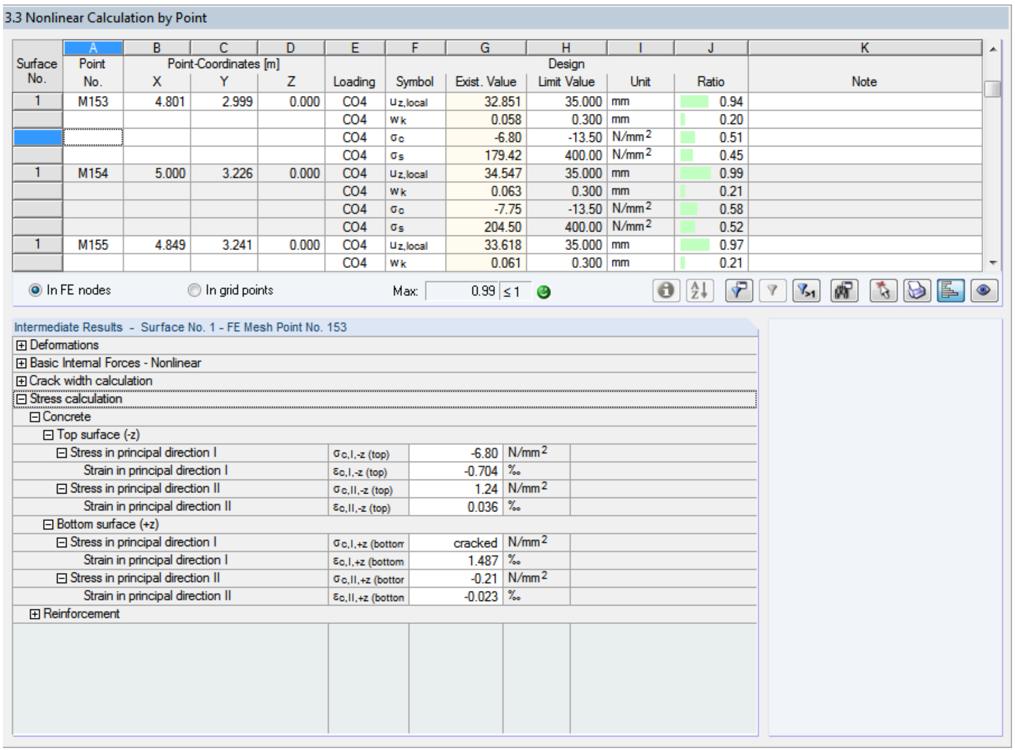
Intermediate Results - Surface No. 1 - FE Mesh Point No. 28

⊞ Deformations			
⊞ Basic Internal Forces - Nonlinear			
⊟ Crack width calculation			
⊟ Top surface (-z)	Ratio	0.002	
Maximum allowable crack width at the top (-z) surfa	$w_{max,-z,limit}$	0.300	mm
⊟ Crack width in principal direction I	$w_{k,I,-z\ (top)}$	0.000	mm
Depth of cracks	$h_{w,I,-z\ (top)}$	0.000	mm
Distance of cracks	$s_{r,I,max,-z\ (top}$	0.000	mm
⊞ Crack width in principal direction II	$w_{k,II,-z\ (top)}$	0.001	mm
⊟ Bottom surface (+z)	Ratio	0.435	
Maximum allowable crack width at the bottom (+z) :	$w_{max,+z,limit}$	0.300	mm
⊟ Crack width in principal direction I	$w_{k,I,+z\ (bottom}$	0.131	mm
Depth of cracks	$h_{w,I,+z\ (bottom}$	141.886	mm
Distance of cracks	$s_{r,I,max,+z\ (bo}$	178.740	mm
⊞ Crack width in principal direction II	$w_{k,II,+z\ (botto}$	0.000	mm
⊞ Stress calculation			

Figure 5.10 Window 3.2 *Nonlinear Calculation by Surface*

This window lists the maximum ratios of each designed surface that are determined in the serviceability limit state designs. The columns are described in chapter 5.4 and chapter 5.7.

5.9 Nonlinear Calculation by Point


3.3 Nonlinear Calculation by Point

Surface No.	Point No.	X	Y	Z	Loading	Symbol	Exist. Value	Design Limit Value	Unit	Ratio	Note
1	M153	4.801	2.999	0.000	CO4	$u_{z,local}$	32.851	35.000	mm	0.94	
					CO4	w_k	0.058	0.300	mm	0.20	
					CO4	σ_c	-6.80	-13.50	N/mm²	0.51	
					CO4	σ_s	179.42	400.00	N/mm²	0.45	
1	M154	5.000	3.226	0.000	CO4	$u_{z,local}$	34.547	35.000	mm	0.99	
					CO4	w_k	0.063	0.300	mm	0.21	
					CO4	σ_c	-7.75	-13.50	N/mm²	0.58	
					CO4	σ_s	204.50	400.00	N/mm²	0.52	
1	M155	4.849	3.241	0.000	CO4	$u_{z,local}$	33.618	35.000	mm	0.97	
					CO4	w_k	0.061	0.300	mm	0.21	

In FE nodes · In grid points · Max: 0.99 ≤ 1

Intermediate Results - Surface No. 1 - FE Mesh Point No. 153

Deformations			
Basic Internal Forces - Nonlinear			
Crack width calculation			
Stress calculation			
Concrete			
Top surface (-z)			
Stress in principal direction I	$\sigma_{c,I,-z\,(top)}$	-6.80	N/mm²
Strain in principal direction I	$\varepsilon_{c,I,-z\,(top)}$	-0.704	‰
Stress in principal direction II	$\sigma_{c,II,-z\,(top)}$	1.24	N/mm²
Strain in principal direction II	$\varepsilon_{c,II,-z\,(top)}$	0.036	‰
Bottom surface (+z)			
Stress in principal direction I	$\sigma_{c,I,+z\,(bottom)}$	cracked	N/mm²
Strain in principal direction I	$\varepsilon_{c,I,+z\,(bottom)}$	1.487	‰
Stress in principal direction II	$\sigma_{c,II,+z\,(bottom)}$	-0.21	N/mm²
Strain in principal direction II	$\varepsilon_{c,II,+z\,(bottom)}$	-0.023	‰
Reinforcement			

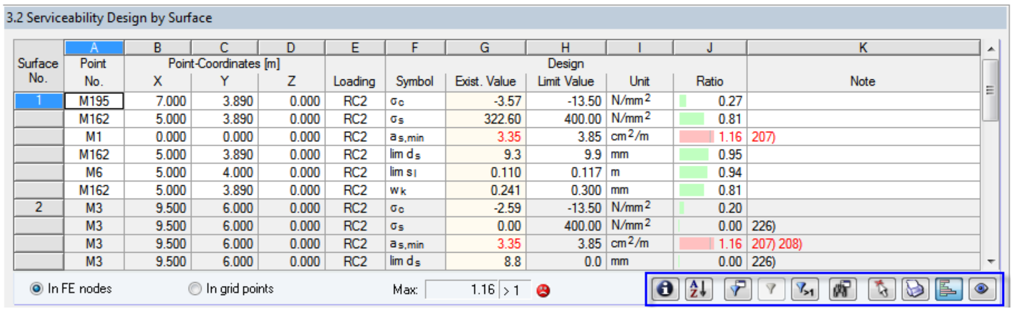
Figure 5.11 Window 3.3 *Nonlinear Calculation by Point*

This result window lists the maximum ratios for all FE nodes / grid points of each surface. The columns are described in chapter 5.4 and chapter 5.7.

The search function that you can access with the button shown on the left allows you to quickly find FE nodes and grid points (see Figure 6.7).

6 Result Evaluation

You can evaluate the design results in different ways. The buttons below the upper table can help you with this.



Surface No.	A Point No.	B Point-Coordinates [m] X	C Y	D Z	E Loading	F Design Symbol	G Exist. Value	H Limit Value	I Unit	J Ratio	K Note
1	M195	7.000	3.890	0.000	RC2	σ_c	-3.57	-13.50	N/mm²	0.27	
	M162	5.000	3.890	0.000	RC2	σ_s	322.60	400.00	N/mm²	0.81	
	M1	0.000	0.000	0.000	RC2	$a_{s,min}$	3.35	3.85	cm²/m	1.16	207)
	M162	5.000	3.890	0.000	RC2	lim d_s	9.3	9.9	mm	0.95	
	M6	5.000	4.000	0.000	RC2	lim s_l	0.110	0.117	m	0.94	
	M162	5.000	3.890	0.000	RC2	w_k	0.241	0.300	mm	0.81	
2	M3	9.500	6.000	0.000	RC2	σ_c	-2.59	-13.50	N/mm²	0.20	
	M3	9.500	6.000	0.000	RC2	σ_s	0.00	400.00	N/mm²	0.00	226)
	M3	9.500	6.000	0.000	RC2	$a_{s,min}$	3.35	3.85	cm²/m	1.16	207) 208)
	M3	9.500	6.000	0.000	RC2	lim d_s	8.8	0.0	mm	0.00	226)

Figure 6.1 Buttons for result evaluation

The buttons have the following functions:

Button	Description	Function
	Details	Opens the *Design Details* dialog box → chapter 6.1
	Sort Results	Sorts the results by maximum ratios (column J) or maximum values (column G) → chapter 6.3
	Filter	Opens the *Filter Points* dialog box for selecting FE nodes or grid points according to specific criteria → chapter 6.3
	Designable Results	Hides rows with non-designable situations
	Exceeding	Only displays rows with a ratio greater than 1 (design not fulfilled)
	Find	Opens the *Find Point* dialog box to search for a specific result row → chapter 6.3
	Select Surface	Allows you to graphically select a surface to show its results in the table
	Print	Prints the intermediate results of the current FE node or grid point into the printout report
	Show Color Bars	Displays or hides the colored relation scales in the result windows

	View Mode	Jumps to the RFEM work window to change the view

Table 6.1 Buttons in the result windows

6.1 Design Details

Click the [Info] button, which is available in all result windows, to view the design details of the selected FE node or grid point, i.e. the point whose table row the cursor is placed in.

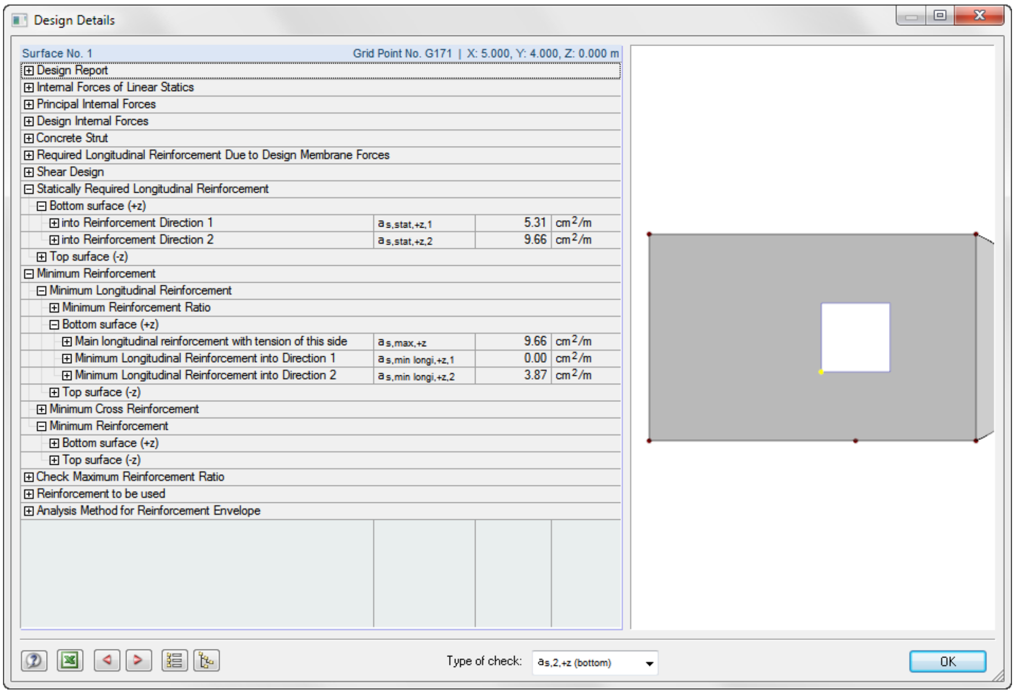
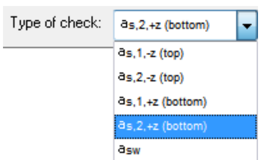


Figure 6.2 *Design Details* dialog box for ultimate limit state design

The design details are listed in a tree structure. You can expand the chapters with ⊞ and close them with ⊟ . The buttons shown on the left allow you to [Close] and [Open] the sub-chapters in the directory tree.

In the graphic on the right, the location of the point is shown in the model.



The following details are output in the ultimate limit state design (see chapter 2.5):

- Design Report
- Internal Forces of Linear Statics
- Principal Internal Forces
- Design Internal Forces
- Concrete Strut
- Required Longitudinal Reinforcement
- Shear Design
- Statically Required Longitudinal Reinforcement
- Minimum Reinforcement
- Check of Maximum Reinforcement Ratio

- Reinforcement to be used
- Analysis Method for Reinforcement Envelope

The design details depend on the selected *Type of check*. Use the list at the bottom of the dialog box to select the displayed results.

In the serviceability limit state design, numerous intermediate results are shown in the lower part of the windows (see Figure 5.6). Click the button to view a detailed list of the design details available for the current point. This option is only available for results according to the analytical method.

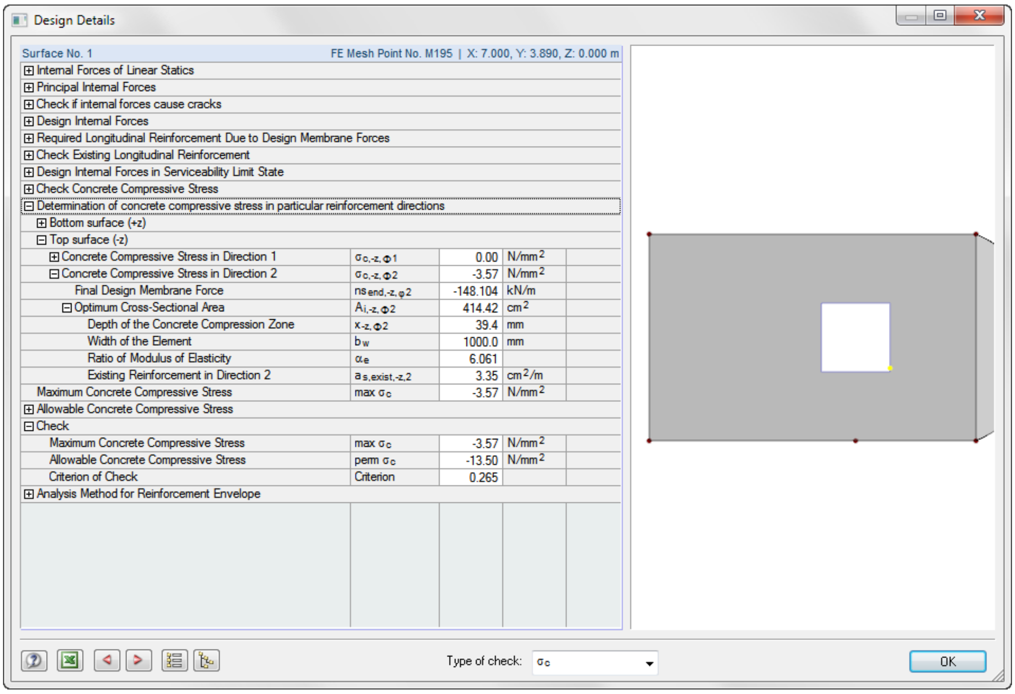
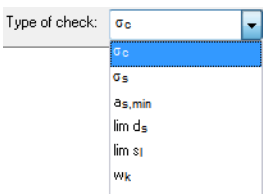


Figure 6.3 *Design Details* dialog box for serviceability limit state design



All the design details that are relevant for each type of check are displayed in a tree structure. Use the list at the bottom of the dialog box to control the displayed results.

Method of check	**Type of check**	
Analytical	σ_c σ_s $a_{s,min}$ lim d_s lim s_l w_k	→ see Table 5.2

Click to go to the previous FE node or grid point, or use to set the next point.

6.2 Results on RFEM Model

You can also evaluate the design results in the RFEM work window.

RFEM background graphic and view mode

The RFEM work window in the background helps in finding the location of an FE node or grid point in the model: An arrow in the background graphic indicates the point selected in the result window of RF-CONCRETE Surfaces; the surface is highlighted in the selection color.

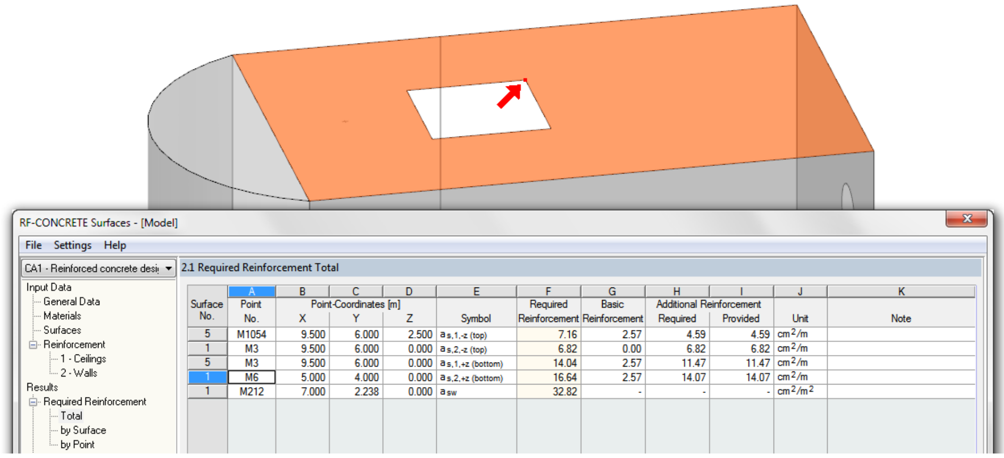
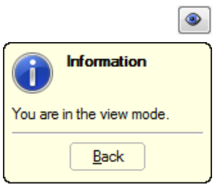


Figure 6.4 Surface and current FE node highlighted in the RFEM model



If you cannot improve the display by moving the RF-CONCRETE Surfaces window, click the [Jump to graphic] button to activate the *view mode*: The program hides the window so that you can adjust the view in the RFEM work window. The view mode provides the functions of the *View* menu, such as zooming, moving, or rotating the view. The marking arrow remains visible.

To return to RF-CONCRETE Surfaces, click [Back].

RFEM work window

You can also check the reinforcements and design ratios graphically on the RFEM model: Click the [Graphics] button to exit the design module. The work window of RFEM now displays all design results such as the internal forces of a load case.

Results navigator

Reinforcement direction

The *Results* navigator is adjusted to the RF-CONCRETE Surfaces module: You can graphically display the results of the longitudinal reinforcements for each reinforcement direction and layer, the shear reinforcement, the design internal forces, or the ratios and detailed results of the serviceability limit state designs.

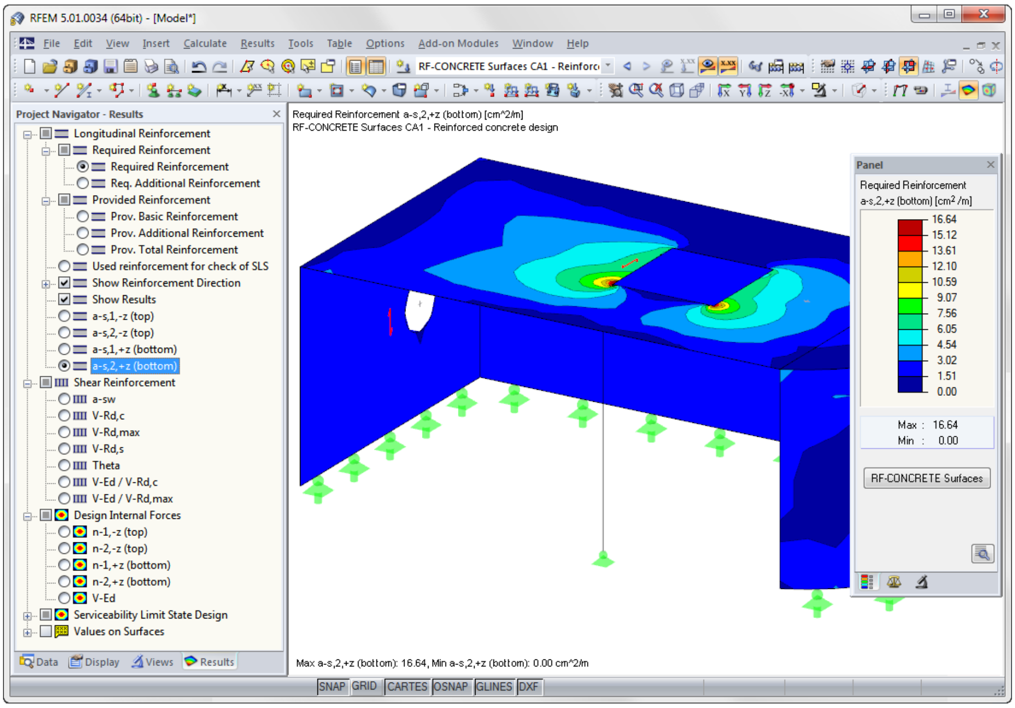
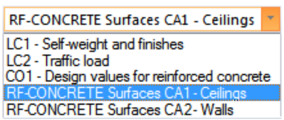


Figure 6.5 RFEM work window with *Results* navigator for RF-CONCRETE Surfaces

Analogous to the display of internal forces, the [Show Results] button shows or hides the display of the design results.

Since the RFEM tables serve no function for the evaluation of the design results, they can be hidden.



You can set the design cases in the list of the RFEM menu bar.

Panel

The color panel with the usual control options is available for the evaluation. The functions are described in chapter 3.4.6 of the RFEM manual. In the second tab, you can set the *Display Factors* for the reinforcements, internal forces, or design ratios. The third tab of the panel allows you to display the results of selected surfaces (see chapter 9.9.3 of the RFEM manual).

Values on surfaces

You can use all options provided by RFEM to display the result values of the reinforcements and design ratios on the surfaces. These functions are described in chapter 9.4 of the RFEM manual. The following figure shows the *Bottom (+z) Reinforcement* group that must be placed in addition to the basic reinforcement. The values are respectively applied in reinforcement direction 1 and 2.

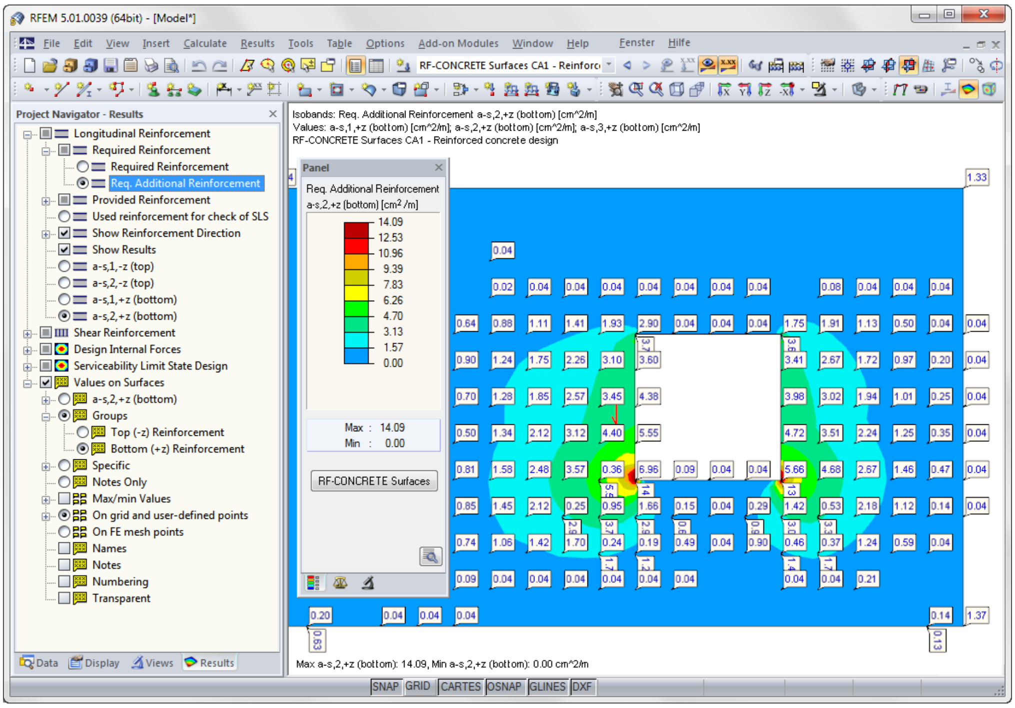


Figure 6.6 *Bottom (+z) Reinforcement* group for required additional reinforcement

You can transfer the graphics of the design results into the printout report (see chapter 7.2).

To return to the add-on module, click the [RF-CONCRETE Surfaces] panel button.

6.3 Filter for Results

The result windows of RF-CONCRETE Surfaces allow you to select the results according to various criteria. In addition, you can use the filter options described in chapter 9.9 of the RFEM manual in order to evaluate the design results graphically.

The options provided by the *Visibilities* (see RFEM manual, chapter 9.9.1) can also be used for RF-CONCRETE Surfaces in order to filter the surfaces for the evaluation.

You can also use the *Sections* in the RFEM model or create new ones (see RFEM manual, chapter 9.6.1), which allow you to selectively evaluate the results. By using the smoothing function, you can redistribute the reinforcement peaks that stem from singularities.

Finding points

The result windows 2.2 and 2.3 (reinforcement), as well as 3.2 and 3.3 (serviceability) provide a search function for FE nodes and grid points. Click the button shown on the left (see Figure 6.1) to open the following dialog box.

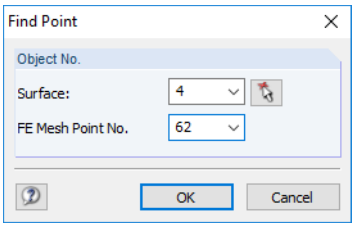


Figure 6.7 *Find Point* dialog box

First, enter the number of the surface manually or click to select it graphically. Then you can enter the number of the grid point or FE node or select it in the list.

After clicking [OK], the result row of this point is set in the current window.

Sorting results

By default, the windows 3.1 and 3.2 show the results arranged by the maximum design ratios: The decisive factor for this is table column J.

You can also sort the results by the existing values available in column G. The greatest ratio of the deformation, for example, does not necessarily have to be the maximum deformation because the limit values can be defined differently for each surface. You can use the button to switch between the two types of arrangement.

Filtering points

The button shown on the left is available in the result windows 2.2 and 2.3 (reinforcement), as well as 3.2 and 3.3 (serviceability). It opens the following dialog box.

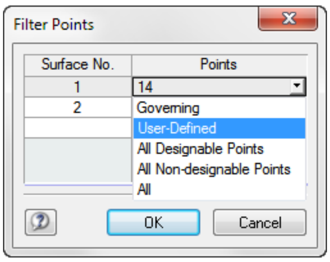
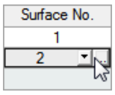


Figure 6.8 *Filter Points* dialog box



In the *Surface No.* column, you can enter the the desired surface number or select it graphically in the RFEM work window. To access this function, click into the input field and use the button.

The *Points* column provides several filter criteria. In addition to all designable or non-designable points, you can select only the points that are *Governing*: These points provide the largest reinforcement areas or ratios for the respective types of check. Point numbers can also be *User-Defined*.

Showing only designable or non-designable results

The two buttons shown on the left allow you to only display designable results or failed designs in the windows. Thus you can, for example, hide failed designs due to singularities or more closely analyze the causes of design problems.

Filtering results in the work window

The reinforcements and design ratios can be used as filter criteria in the RFEM work window, which can be accessed with the [Graphics] button. For this, the panel must be displayed. Should it not be active, it can be displayed by selecting

View → Control Panel

in the RFEM menu or by using the corresponding toolbar button.

The panel is described in chapter 3.4.6 of the RFEM manual. You can set the filter settings for the results in the first panel tab (color spectrum).

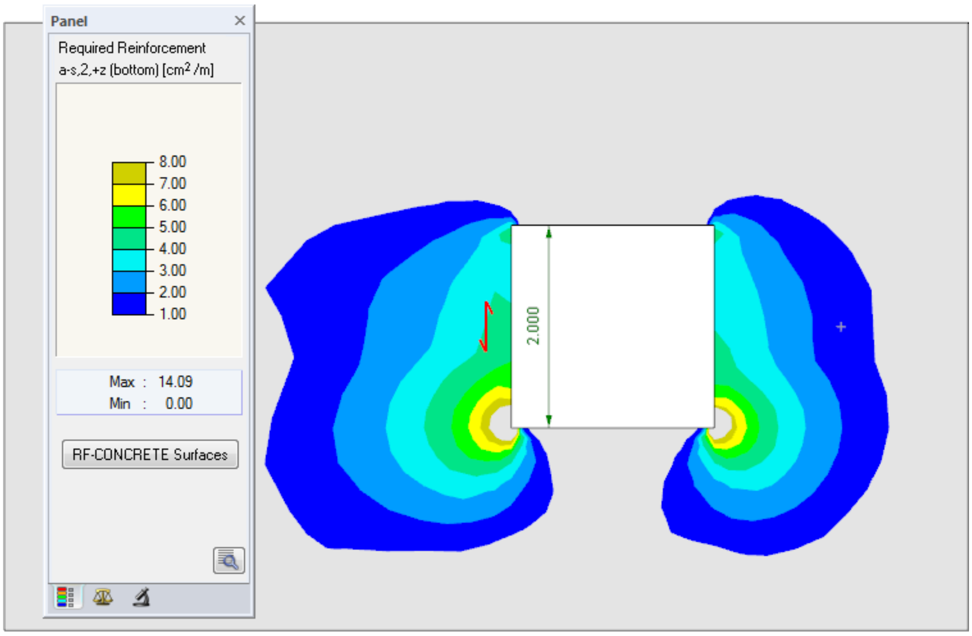


Figure 6.9 Filtering the additional reinforcement with an adjusted color spectrum

As shown in the figure above, you can set the panel's value scale to only display reinforcements larger than 1.00 cm^2/m. The color scale is set in such a way that a color range covers 1.00 cm^2/m and that the maximum value of 8.00 cm^2/m suppresses effects of singularities.

The following chapter 6.4 explains how you can adjust the color and value spectra to the diameters and spacings of the rebars.

To display the grid point or FE node values in the graphics, the usual control functions of RFEM are available. They are described in chapter 9.4 of the RFEM manual.

Filtering surfaces in the work window

In the *Filter* tab of the control panel, you can specify the numbers of selected surfaces to display their filtered results. This function is described in chapter 9.9.3 of the RFEM manual.

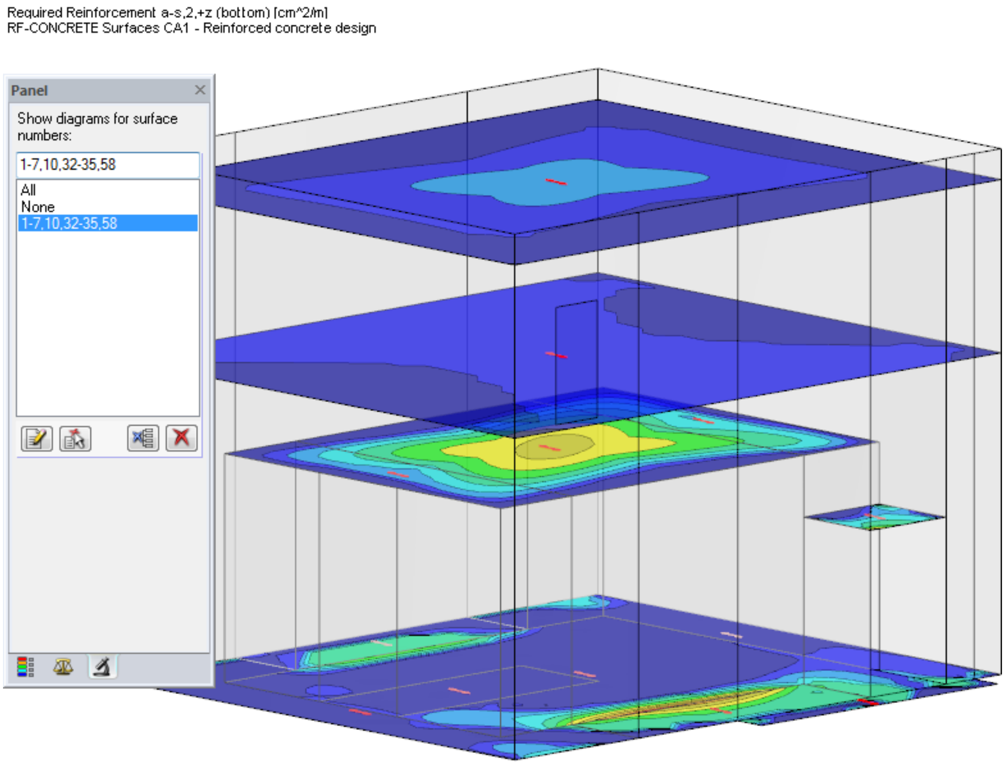


Figure 6.10 Surface filter for reinforcement of floor slab and ceilings

In contrast to the visibility function, the model is displayed completely in the graphic. Figure 6.10 shows the reinforcement of the horizontal surfaces of a building. The remaining surfaces are shown in the model but are displayed without reinforcements.

6.4 Configuring the Panel

The reinforcement results can be graphically displayed as isobands or isolines. By default, a twelve-colored value spectrum between the minimum and maximum value is used. You can also adjust this value spectrum with regard to the definition of the reinforcement in order to prepare the graphical results for a reinforcement drawing, for example.

To adjust the panel, double-click one of the colors. Alternatively, you can use the button in the panel: In the subsequent *Options* dialog box, you can also click the button to access the dialog box for changing the ranges of colors and values.

In the *Edit Isoband Value and Color Spectra* dialog box, you can click the button again to open the *Edit Value Spectrum with Reinforcement Definition* dialog box.

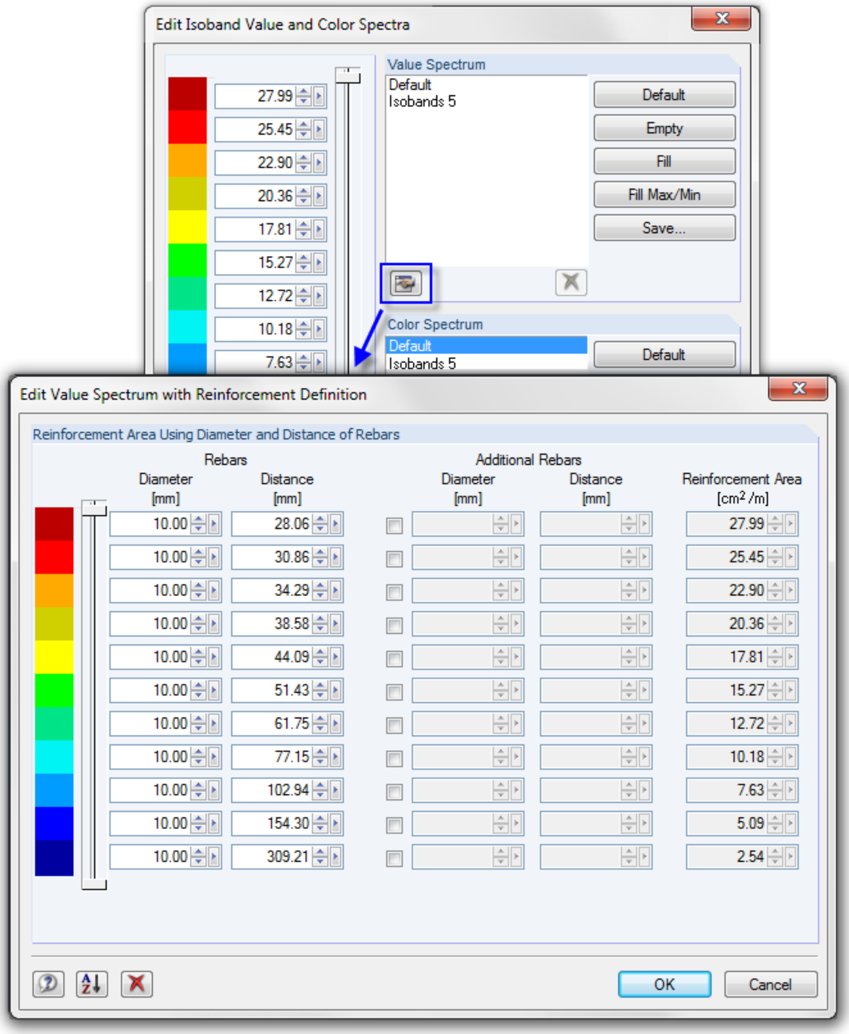


Figure 6.11 *Edit Isoband Value and Color Spectra and Edit Value Spectrum with Reinforcement Definition dialog boxes*

This dialog box determines the reinforcement area per meter from the *Diameter* and the *Distance* of the rebars. In the *Additional Rebars* columns, you can assign additional rebar diameters and distances (see Figure 6.12). With them, you can set user-defined reinforcement specifications, which can be used for a reinforcement drawing.

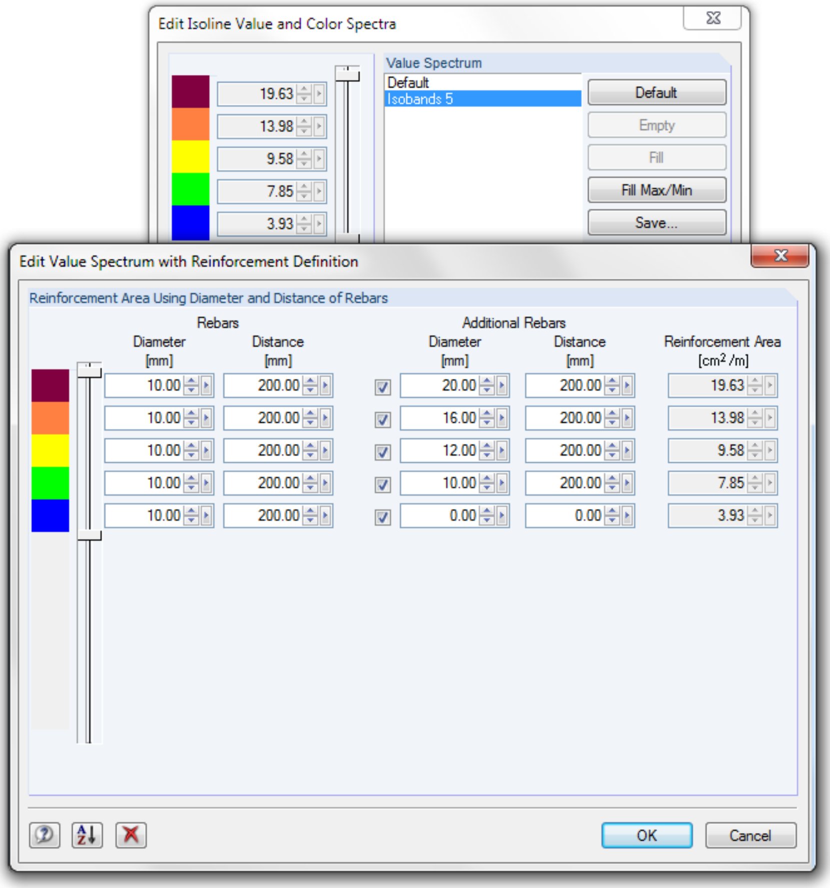


Figure 6.12 *Edit Value Spectrum with Reinforcement Definition* dialog box with rebar diameters and distances

Click [OK] to import the reinforcement areas that result from the defined rebar diameters and rebar distances into the *Edit Isoband Value and Color Spectra* dialog box.

In the panel, the diameters of the rebars appear, along with the related distances that are to be provided for the individual value ranges.

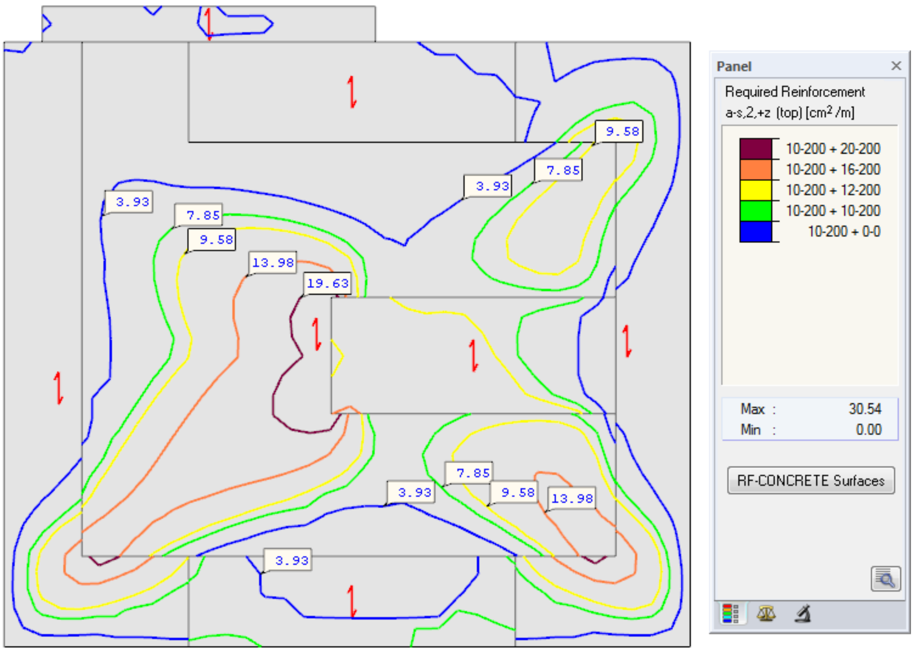


Figure 6.13 Graphic and panel with user-defined reinforcement areas

7 Printout

7.1 Printout Report

Like in RFEM, you can generate a printout report for the data of the RF-CONCRETE Surfaces add-on module where graphics and descriptions can be added. The selection in the printout report determines the design module's data included in the final printout.

The printout report is described in the RFEM manual. Chapter 10.1.3.5 *Selecting data of add-on modules* explains how to prepare the input and output data of add-on modules for the printout.

A special selection option is available for the intermediate results of the serviceability limit state designs performed according to the analytical method. In the *Points* column, you can select all designable or non-designable points or just the *Governing* points: These points provide the greatest reinforcement areas or design ratios. Point numbers can also be *User-Defined*.

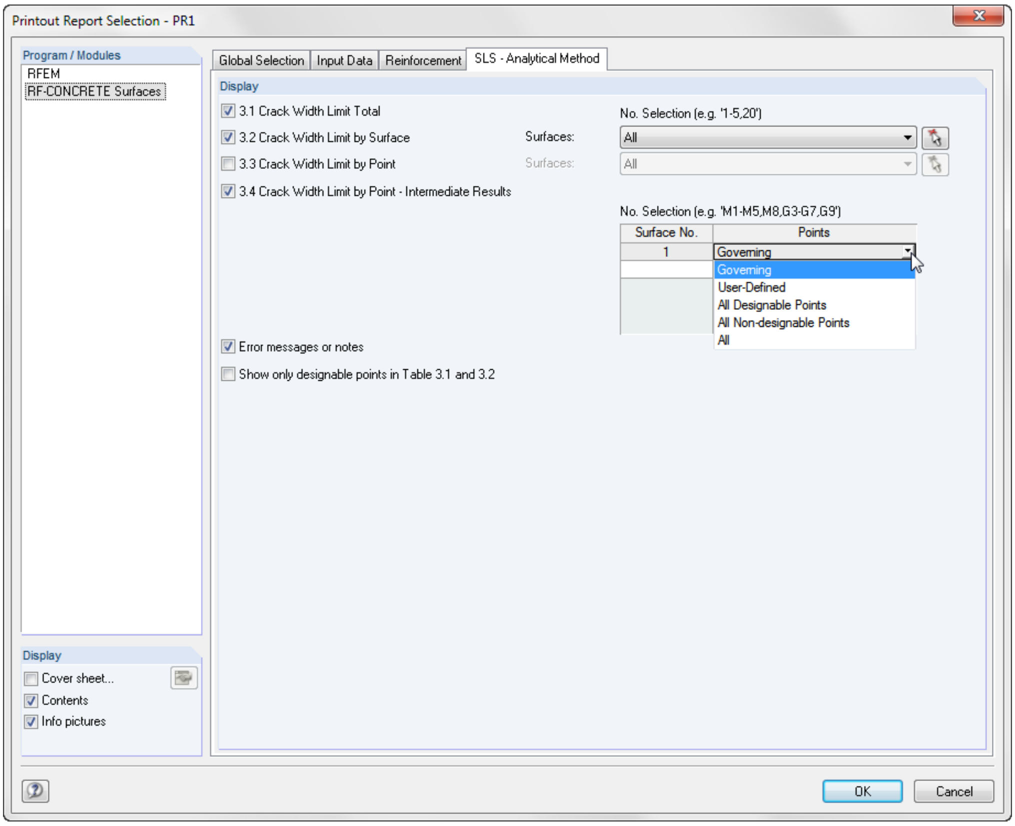


Figure 7.1 *Printout Report Selection dialog box, SLS - Analytical Method tab*

For large structural systems with numerous design cases, it is recommended to split the data into several printout reports for a clear overview.

7.2 Graphic Printout

In RFEM, you can transfer every image shown in the work window into the printout report or send it directly to a printer. Thus, you can also prepare the reinforcements and design ratios shown on the RFEM model for the printout.

Printing graphics is described in chapter 10.2 of the RFEM manual.

Analyses on the RFEM model

To print the current graphic of the design ratios, select

File → Print Graphic

in the menu or use the corresponding toolbar button.

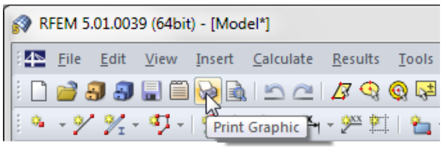


Figure 7.2 *Print Graphic* button in RFEM toolbar

The following dialog box opens.

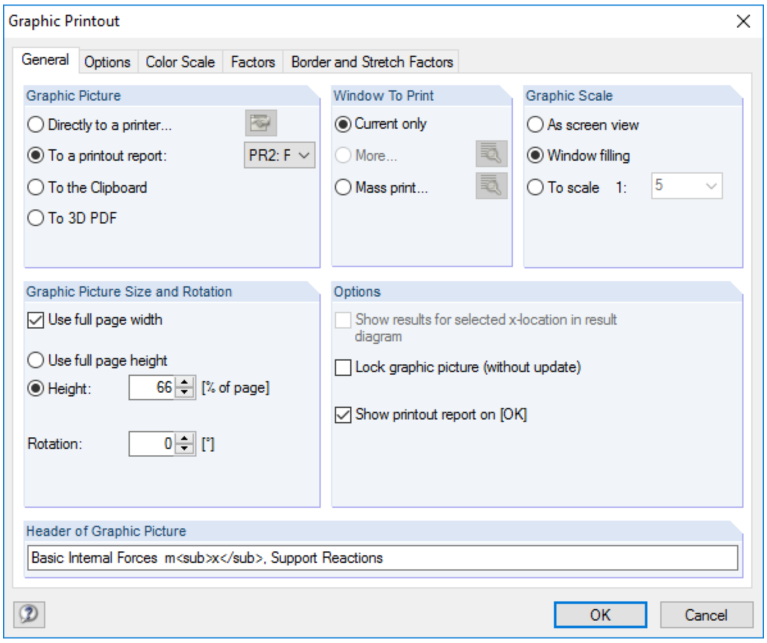
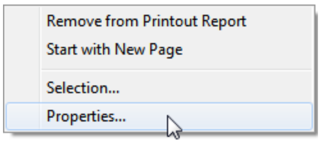


Figure 7.3 *Graphic Printout* dialog box, *General* tab

This dialog box is described in chapter 10.2 of the the RFEM manual.

You can use the drag-and-drop function as usual to move a graphic to another position within the printout report.



To retroactively adjust a graphic in the printout report, right-click the corresponding entry in the report navigator. The *Properties* option in the shortcut menu once more opens the *Graphic Printout* dialog box where you can adjust the settings.

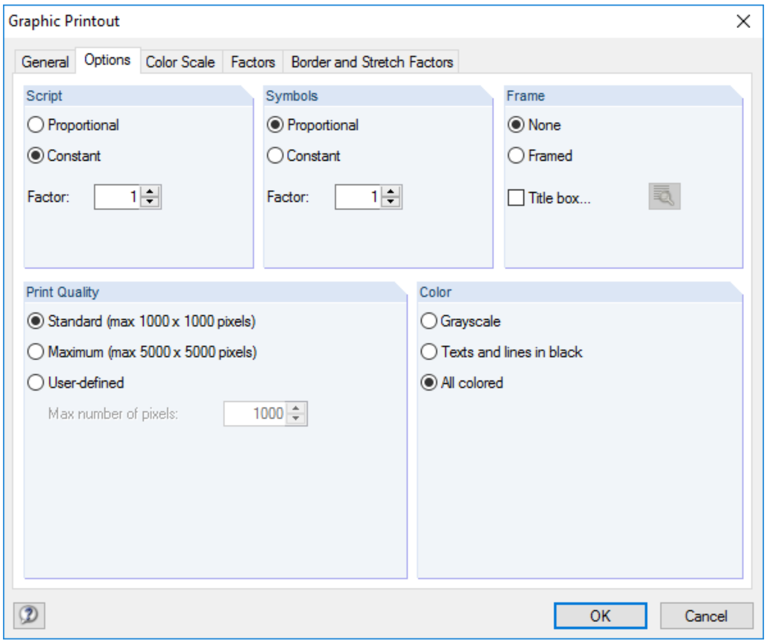


Figure 7.4 *Graphic Printout* dialog box, *Options* tab

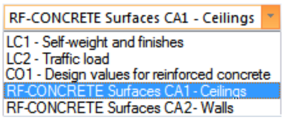
8 General Functions

This chapter describes useful menu functions and presents export options for the designs.

8.1 Design Cases

Design cases allow you to group surfaces for the designs or analyze variants (e.g. modified materials or reinforcement specifications, nonlinear analysis).

Analyzing a surface in different design cases is no problem.



The design cases of RF-CONCRETE Surfaces can also be accessed in RFEM by using the load case list in the toolbar.

Creating a new design case

To create a new design case in RF-CONCRETE Surfaces, use the menu item

File → New Case.

The following dialog box appears.

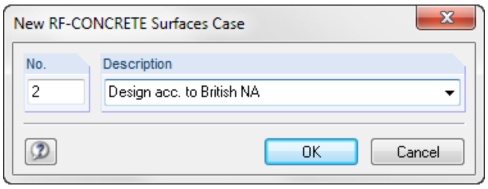


Figure 8.1 *New RF-CONCRETE Surfaces Case dialog box*

In this dialog box, you can enter an unassigned *No.* for the new design case. A *Description* makes the selection in the load case list easier.

After clicking [OK], the RF-CONCRETE Surfaces window 1.1 *General Data* opens where you can enter the design data.

Renaming a design case

To change the description of a design case in RF-CONCRETE Surfaces, select the menu option

File → Rename Case.

The following dialog box appears.

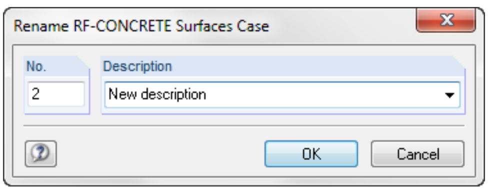


Figure 8.2 *Rename RF-CONCRETE Surfaces Case dialog box*

In this dialog box, you can specify a different *Description* as well as a different *No.* for the design case.

Copying a design case

To copy the input data of the current design case in RF-CONCRETE Surfaces, use the menu item

File → Copy Case.

The following dialog box appears.

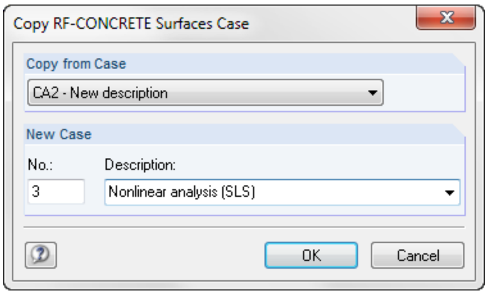


Figure 8.3 *Copy RF-CONCRETE Surfaces Case* dialog box

Define the *No.* and, if necessary, a *Description* for the new case.

Deleting a design case

To delete a design case in RF-CONCRETE Surfaces, use the menu option

File → Delete Case.

The following dialog box appears.

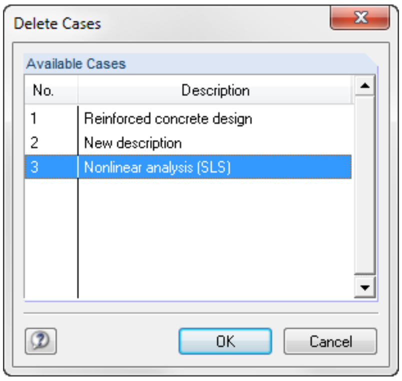


Figure 8.4 *Delete Cases* dialog box

You can select the design case in the *Available Cases* list. To delete the selected case, click [OK].

8.2 Units and Decimal Places

The units and decimal places for RFEM and the add-on modules are managed together. In RF-CONCRETE Surfaces, you can open the dialog box for adjusting the units with the menu item

Settings → Units and Decimal Places.

The dialog box familiar from RFEM appears. RF-CONCRETE Surfaces is preset in the *Program / Module* list.

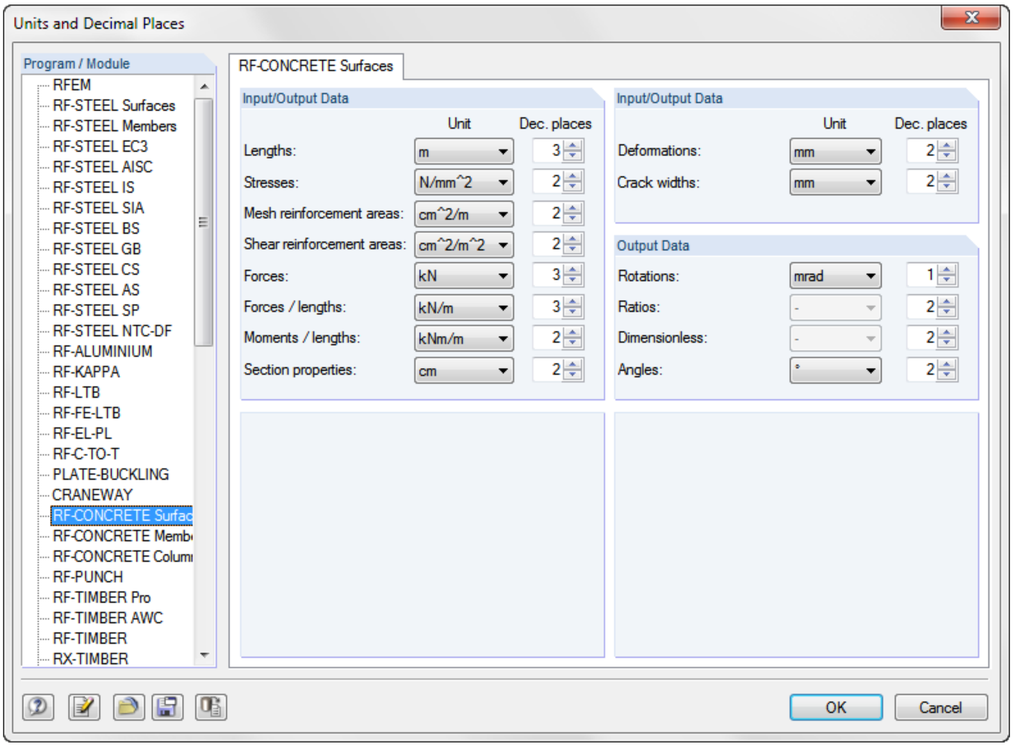


Figure 8.5 *Units and Decimal Places* dialog box

The settings can be saved as a user profile and reused in other models. These functions are described in chapter 11.1.3 of the RFEM manual.

8.3

Exporting the Results

The results of RF-CONCRETE Surfaces can also be used in other programs.

Clipboard

You can copy cells selected in the result windows to the clipboard with [Ctrl]+[C] and subsequently insert them with [Ctrl]+[V] into a word processing program, for example. The headers of the table columns are not transferred.

Printout report

The data of RF-CONCRETE Surfaces can be printed into the printout report (see chapter 7.1) where they can be exported with the menu item

File → Export to RTF.

This function is described in chapter 10.1.11 of the RFEM manual.

Excel

RF-CONCRETE Surfaces allows you to directly export data to MS Excel or into the CSV format. You can access this function by selecting the menu option

File → Export Tables.

The following export dialog box opens.

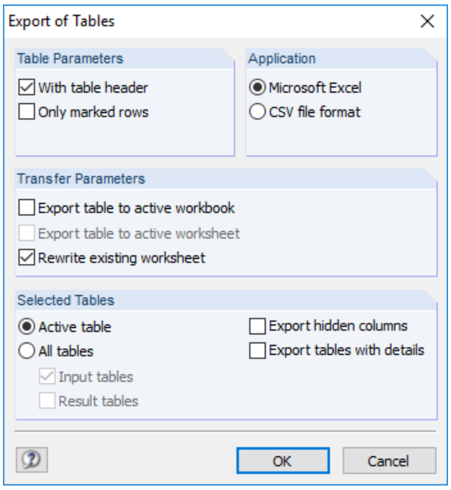
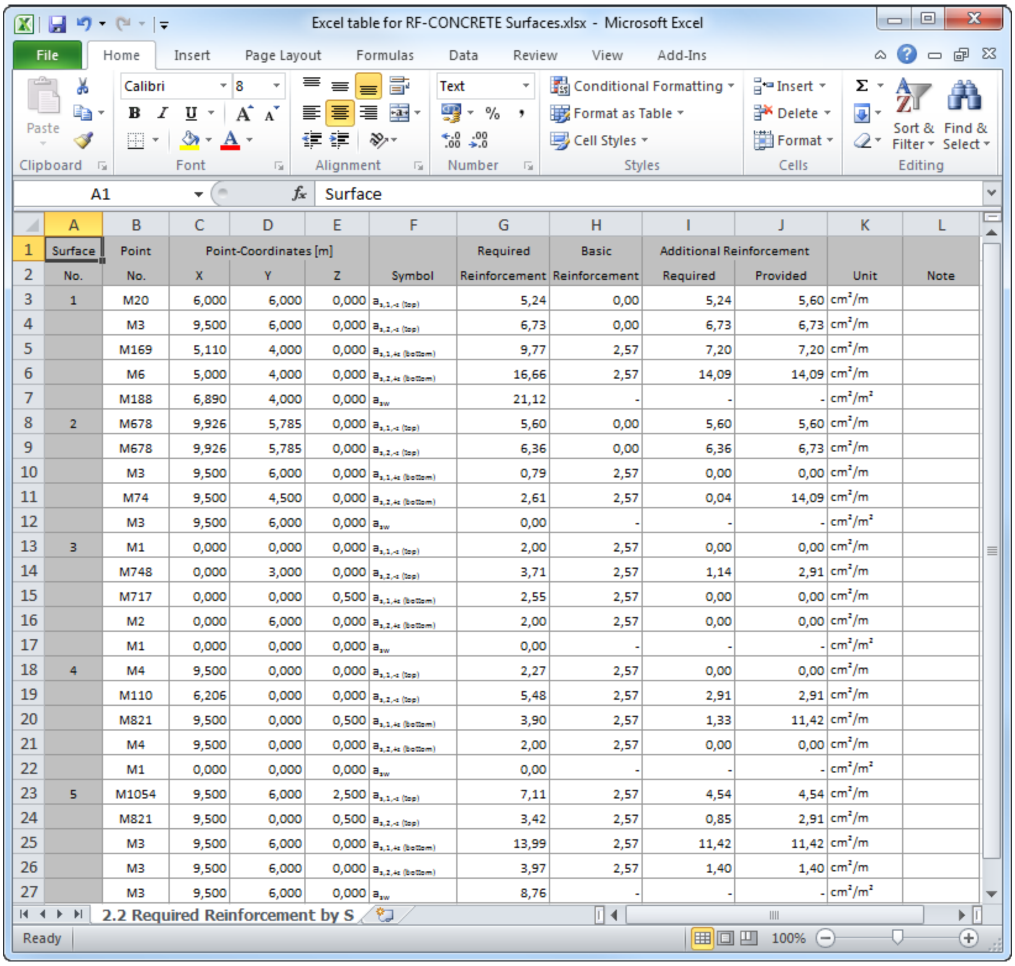


Figure 8.6 *Export of Tables dialog box*

When the selection is complete, click [OK] to start the export. Excel is started automatically, meaning you do not need to open the program first.



	Surface No.	Point No.	Point-Coordinates [m] X	Y	Z	Symbol	Required Reinforcement	Basic Reinforcement	Additional Reinforcement Required	Provided	Unit	Note
3	1	M20	6,000	6,000	0,000	$a_{s,1,-z\ (top)}$	5,24	0,00	5,24	5,60	cm²/m	
4		M3	9,500	6,000	0,000	$a_{s,2,-z\ (top)}$	6,73	0,00	6,73	6,73	cm²/m	
5		M169	5,110	4,000	0,000	$a_{s,1,+z\ (bottom)}$	9,77	2,57	7,20	7,20	cm²/m	
6		M6	5,000	4,000	0,000	$a_{s,2,+z\ (bottom)}$	16,66	2,57	14,09	14,09	cm²/m	
7		M188	6,890	4,000	0,000	a_{sw}	21,12	-	-	-	cm²/m²	
8	2	M678	9,926	5,785	0,000	$a_{s,1,-z\ (top)}$	5,60	0,00	5,60	5,60	cm²/m	
9		M678	9,926	5,785	0,000	$a_{s,2,-z\ (top)}$	6,36	0,00	6,36	6,73	cm²/m	
10		M3	9,500	6,000	0,000	$a_{s,1,+z\ (bottom)}$	0,79	2,57	0,00	0,00	cm²/m	
11		M74	9,500	4,500	0,000	$a_{s,2,+z\ (bottom)}$	2,61	2,57	0,04	14,09	cm²/m	
12		M3	9,500	6,000	0,000	a_{sw}	0,00	-	-	-	cm²/m²	
13	3	M1	0,000	0,000	0,000	$a_{s,1,-z\ (top)}$	2,00	2,57	0,00	0,00	cm²/m	
14		M748	0,000	3,000	0,000	$a_{s,2,-z\ (top)}$	3,71	2,57	1,14	2,91	cm²/m	
15		M717	0,000	0,000	0,500	$a_{s,1,+z\ (bottom)}$	2,55	2,57	0,00	0,00	cm²/m	
16		M2	0,000	6,000	0,000	$a_{s,2,+z\ (bottom)}$	2,00	2,57	0,00	0,00	cm²/m	
17		M1	0,000	0,000	0,000	a_{sw}	0,00	-	-	-	cm²/m²	
18	4	M4	9,500	0,000	0,000	$a_{s,1,-z\ (top)}$	2,27	2,57	0,00	0,00	cm²/m	
19		M110	6,206	0,000	0,000	$a_{s,2,-z\ (top)}$	5,48	2,57	2,91	2,91	cm²/m	
20		M821	9,500	0,000	0,500	$a_{s,1,+z\ (bottom)}$	3,90	2,57	1,33	11,42	cm²/m	
21		M4	9,500	0,000	0,000	$a_{s,2,+z\ (bottom)}$	2,00	2,57	0,00	0,00	cm²/m	
22		M1	0,000	0,000	0,000	a_{sw}	0,00	-	-	-	cm²/m²	
23	5	M1054	9,500	6,000	2,500	$a_{s,1,-z\ (top)}$	7,11	2,57	4,54	4,54	cm²/m	
24		M821	9,500	0,000	0,500	$a_{s,2,-z\ (top)}$	3,42	2,57	0,85	2,91	cm²/m	
25		M3	9,500	6,000	0,000	$a_{s,1,+z\ (bottom)}$	13,99	2,57	11,42	11,42	cm²/m	
26		M3	9,500	6,000	0,000	$a_{s,2,+z\ (bottom)}$	3,97	2,57	1,40	1,40	cm²/m	
27		M3	9,500	6,000	0,000	a_{sw}	8,76	-	-	-	cm²/m²	

Figure 8.7 Result in *Excel*

CAD applications

The reinforcement areas determined in RF-CONCRETE Surfaces can also be used in CAD applications. RFEM provides interfaces with the following programs:

- Glaser (format *.fem)
- Strakon (format *.cfe)
- Nemetschek (FEM format for Allplan *.asf)
- Engineering Structural Format (format *.esf)

You can access the export function with the RFEM menu item

File → Export.

The *Export* dialog box opens where you can select the appropriate interface (see Figure 8.8). This dialog box is described in chapter 12.5.2 of the RFEM manual.

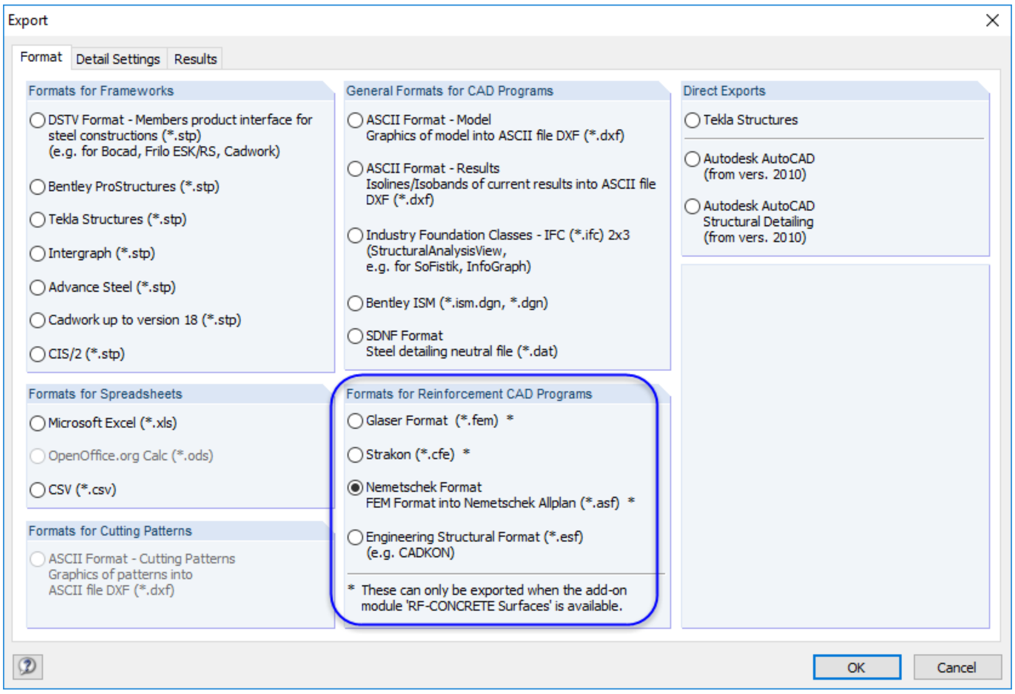


Figure 8.8 *Export* RFEM dialog box, *Format* tab

Depending on the interface, there can be additional tabs with settings for controlling the export of the reinforcements.

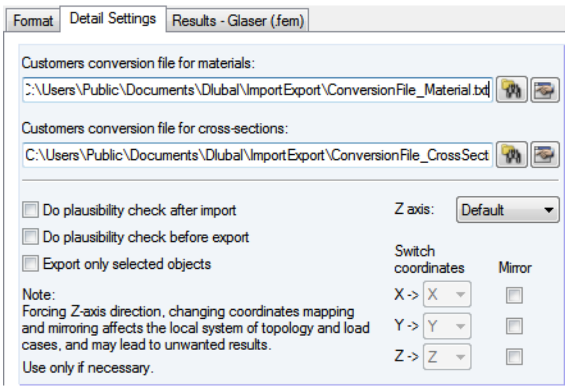


Figure 8.9 *Export* RFEM dialog box, *Detail Settings* tab for Nemetschek format

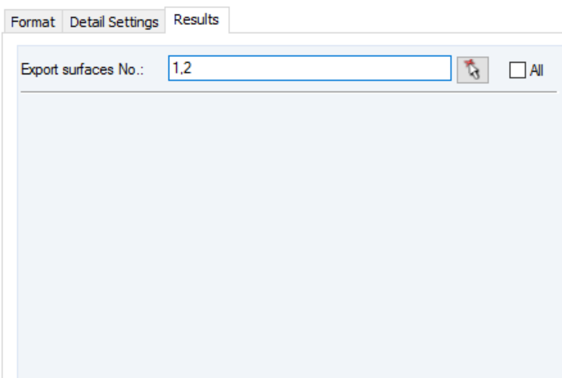


Figure 8.10 *Export* RFEM dialog box, *Results* tab for Nemetschek format

After clicking [OK], another dialog box for the *Selection* of the relevant results appears.

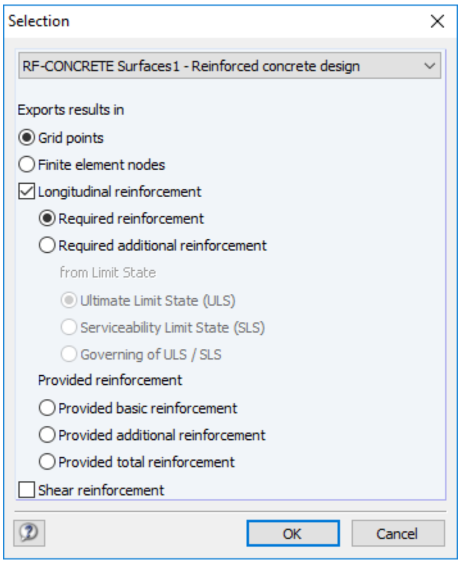


Figure 8.11 *Selection* RFEM dialog box

9 Literature

[1] Deutscher Ausschuss für Stahlbeton, Heft 217: Tragwirkung orthogonaler Bewehrungsnetze beliebiger Richtung in Flächentragwerken aus Stahlbeton (von Theodor Baumann). Verlag Ernst & Sohn, Berlin, 1972.

[2] Jörg Schlaich and Kurt Schäfer. Konstruieren im Stahlbetonbau. Betonkalender, 1993.

[3] Deutscher Beton-Verein e.V.: Beispiele zur Bemessung von Betontragwerken nach EC2. Bauverlag, Wiesbaden/Berlin, 1994.

[4] Quast, Ulrich. Zur Mitwirkung des Betons in der Zugzone. Beton und Stahlbetonbau, Heft 10, 1981.

[5] Pfeiffer, Uwe. Die nichtlineare Berechnung ebener Rahmen aus Stahl- oder Spannbeton mit Berücksichtigung der durch das Aufreißen bedingten Achsendehnung. Cuviller Verlag, Göttingen, 2004.

[6] Quast, Ulrich. Zum nichtlinearen Berechnen im Stahlbeton- und Spannbetonbau. Beton und Stahlbetonbau, Heft 9 und Heft 10, 1994.

[7] Eurocode 2: Design of concrete structures - Part 1-1: General rules and rules for buildings; EN 1992-1-1:2011-01

[8] DIN 1045: Beton- und Stahlbetonbau — Bemessung und Ausführung. Beuth Verlag GmbH, Berlin, 1988.

[9] DIN 1045-1: Concrete, reinforced and prestressed concrete structures - Part 1: Design and construction. Beuth Verlag GmbH, Berlin, 2001.

[10] DIN V ENV 1992-1-1: Planung von Stahlbeton- und Spannbetontragwerken — Teil 1: Grundlagen und Anwendungsregeln für den Hochbau. Beuth Verlag GmbH, Berlin, 1992

[11] ACI 318-19, Building Code Requirements for Structural Concrete and Commentary

[12] CSA A23.3:19, Design of Concrete Structures

[13] Reymendt, Jörg. DIN 1045 neu, Anwendung und Beispiele. Papenberg Verlag, Frankfurt, 2001.

[14] Avak, Ralf. Stahlbetonbau in Beispielen, DIN 1045 und Europäische Normung — Teil 2: Konstruktion-Platten-Treppen-Fundamente. Werner Verlag, Düsseldorf, 1992.

[15] Avak, Ralf. Stahlbetonbau in Beispielen, DIN 1045 und Europäische Normung — Teil 2: Bemessung von Flächentragwerken, Konstruktionspläne für Stahlbetonbauteile. Werner Verlag, Düsseldorf, 2. Auflage, 2002.

[16] Lang, Christian, Meiswinkel, Rüdiger und Wittek, Udo. Bemessung von Stahlbetonplatten mit dem nichtlinearen Verfahren nach DIN 1045-1. Beton- und Stahlbetonbau 95, Heft 5, 2000.

[17] Meiswinkel, Rüdiger. Nichtlineare Nachweisverfahren von Stahlbeton-Flächentragwerken. Beton- und Stahlbetonbau 96, Heft 1, 2000.

[18] Rahm, Heiko. Modellierung und Berechnung von Alterungsprozessen bei Stahlbeton-Flächentragwerken. Universität Kaiserslautern, 2002.

[19] Albert, A. (2018). Schneider - Bautabellen für Ingenieure mit Berechnungshinweisen und Beispielen (23rd ed.). Cologne: Bundesanzeiger.