0010 – One-dimensional Orthotropic Plasticity - 4 Columns

Description

Four columns with width d, depth d, height h and distance d between them are oriented in the direction of the Z-axis. They are fixed at the bottom and connected by the rigid block at the top. Block is loaded by the pressure p in the Z-direction and modeled by an elastic material with high modulus of elasticity E_r. Outer columns are modeled as orthotropic elastic material and inner columns as orthotropic elastic-plastic material with the same elastic parameters as outer columns and with plasticity properties defined according to the Tsai-Wu plasticity theory. Material fibers are oriented by angles -45° and 45° (**Figure 1**). Assuming only small deformations theory and neglecting structure's self-weight, determine its maximum deflection.

Material	Columns	Modulus of Elasticity	$E_x = E_y$	3000.000	MPa
			E_z	11000.000	MPa
		Poisson's Ratio	$\nu_{xy} = \nu_{xz} = \nu_{yz}$	0.000	–
		Shear Modulus	$G_{xy} = G_{xz} = G_{yz}$	5500.000	MPa
	Inner Columns - Plasticity	Tensile Plastic Strength	$f_{t,x} = f_{t,z}$	3.000	MPa
			$f_{t,y}$	2.121	MPa
		Compressive Plastic Strength	$f_{c,x} = f_{c,z}$	3.000	MPa
			$f_{c,y}$	2.121	MPa
		Shear Tensile Plastic Strength	$f_{v,xy} = f_{v,xz} = f_{v,yz}$	99999.000	MPa
	Block	Modulus of Elasticity	E_r	20000000.000	MPa
		Poisson's Ratio	ν_r	0.000	–
Geometry	Column	Height	h	1.000	m
		Depth Width Distance	d	0.050	m
	Block	Height	d	0.050	m
		Width	$7d$	0.350	m
Load		Pressure	p	4.571	MPa

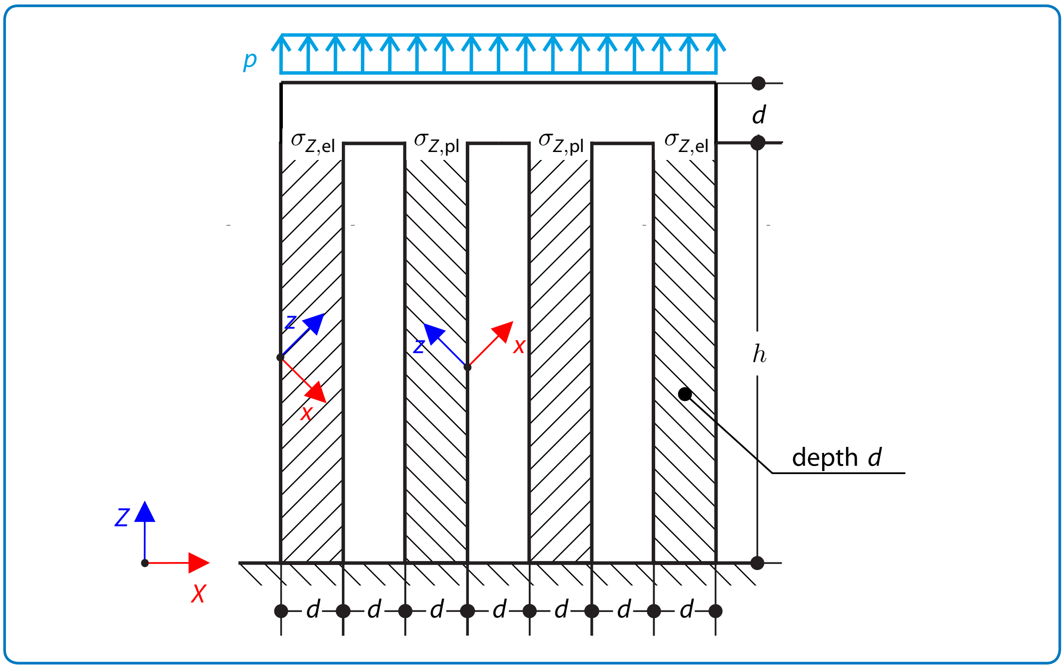

Figure 1: Problem sketch

Analytical Solution

Linear Analysis

Formula for the maximum displacement can be evaluated with the use of the transformed stiffness matrix into the loading direction (see verification example 0007 for the detailed description):

$$u_{\max} = \sigma_L h \left(\frac{\sin^4 \beta}{E_x} + \frac{\cos^4 \beta}{E_z} + \frac{\sin^2 \beta \cos^2 \beta}{G_{xz}} \right) \tag{10 – 1}$$

where σ_L is the loading pressure per one column:

$$\sigma_L = \frac{7pd^2}{4d^2} = \frac{7}{4}p \tag{10 – 2}$$

Nonlinear Analysis

The maximum deformation of the structure can be obtained by:

$$u_{\max} = h\varepsilon = h\frac{\sigma_{\text{el}}}{E_{\text{eff}}} \tag{10 – 3}$$

where E_{eff} is the corresponding effective modulus of elasticity in the Z-direction:

$$E_{\text{eff}} = \frac{\sigma_L}{u_{Z,\max}} \tag{10 – 4}$$

where $u_{Z,\max}$ is the elastic deformation of one column in the *Z*-direction and can be described same as in **(10 – 1)**:

$$u_{Z,\max} = \sigma_{\text{el}} h \left(\frac{\sin^4 \beta}{E_x} + \frac{\cos^4 \beta}{E_z} + \frac{\sin^2 \beta \cos^2 \beta}{G_{xz}} \right) \tag{10 – 5}$$

where σ_{el} is the stress in the elastic column:

$$\sigma_{\text{el}} = 2\sigma_{\text{L}} - \sigma_{Z,\text{el}} \tag{10 – 6}$$

where $\sigma_{Z,\text{el}}$ is the stress in the plastic column, which under these circumstances can be expressed according to the Tsai-Wu surface condition as follows (see verification example 0009 for the detailed description):

$$\sigma_{Z,\text{el}} = \sqrt{2f_{\text{t},z} f_{\text{c},z}} \tag{10 – 7}$$

Substituting those formulae into the equation **(10 – 3)**, the maximum deformation can be obtained:

$$u_{\max} = h^2 \left(\frac{\sin^4 \beta}{E_x} + \frac{\cos^4 \beta}{E_z} + \frac{\sin^2 \beta \cos^2 \beta}{G_{xz}} \right) \left(\frac{7}{2} p - \sqrt{2f_{\text{t},z} f_{\text{c},z}} \right) = 1.781 \text{ mm} \tag{10 – 8}$$

RFEM 5 Settings

- Modeled in version RFEM 5.03.0050
- The element size is $l_{\text{FE}} = 0.025$ m
- Geometrically linear analysis is considered
- The number of increments is 5

Results

Structure File	Entity	Material Model
0010.01	Solid	Orthotropic Plastic 3D
0010.02	Plate	Orthotropic Plastic 2D
0010.03	Solid	Orthotropic Elastic 3D
0010.04	Plate	Orthotropic Elastic 2D

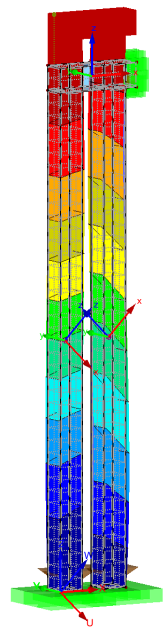

Figure 2: Model and results in RFEM 5

As can be seen from the following comparisons, good agreements of analytical result and outputs from RFEM were achieved.

Linear Analysis

Analytical Solution	RFEM 5 Orthotropic Elastic 3D		RFEM 5 Orthotropic Elastic 2D	
u_{max} [mm]	u_{max} [mm]	Ratio [-]	u_{max} [mm]	Ratio [-]
1.212	1.206	0.995	1.206	0.995

Nonlinear Analysis

Analytical Solution	RFEM 5 Orthotropic Plastic 3D		RFEM 5 Orthotropic Plastic 2D	
u_{max} [mm]	u_{max} [mm]	Ratio [-]	u_{max} [mm]	Ratio [-]
1.781	1.772	0.995	1.773	0.996