Program: RFEM 5, RFEM 6

Category: Geometrically Linear Analysis, Isotropic Linear Elasticity, Member, Plate, Solid

Verification Example: 0035 – Mixed Dimensional Coupling

0035 – Mixed Dimensional Coupling

Description

Prove that coupling of different dimensional elements doesn't affect the results. A cantilever with a rectangular cross-section is fixed at one end and loaded at the other with forces F_x and F_z. Neglecting it's self-weight and assuming only small deformations, determine cantilever's maximum deflections u_x, u_z and $u_{\max}$.

Material	Linear Elastic	Modulus of Elasticity	E	200.000	GPa
		Poisson's ratio	ν	0.000	–
Geometry	Cantilever	Length	L	1.000	m
		Width	b	0.100	m
		Height	h	0.010	m
Load	Force	x-direction	F_x	1000.000	kN
		z-direction	F_z	0.100	kN

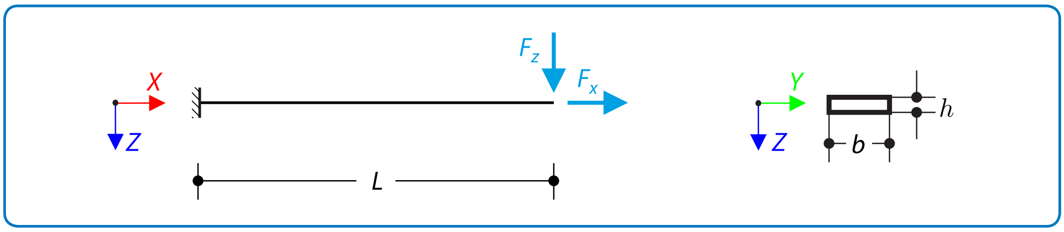

Figure 1: Problem sketch

Analytical Solution

Total maximum deflection of the cantilever can be obtained as:

$$u_{\max} = \sqrt{u_x^2 + u_z^2} \tag{35 – 1}$$

where u_x and u_z are maximum deflections at the free end in the given directions. Deflection in the x-direction can be obtained according to the principle of the virtual forces, while considering virtual force δF_x acting at the end of the cantilever in the direction of the displacement u_x:

$$\sigma_x = \int_L \frac{\delta N N}{EA} \mathrm{d}x \tag{35 – 2}$$

where $A = bh$ is the cross-section area, δN is the virtual normal force caused by the virtual force δF_x. Integrating the equation **(35 – 2)**, formula for the deflection u_x can be given as follows:

$$u_x = \frac{F_x L}{Ebh} = 5.000 \text{ mm} \tag{35 – 3}$$

Deflection in the z-direction can be obtained similarly by considering virtual force δF_z acting at the end of the cantilever in the direction of the displacement u_z:

$$u_z = \int_L \frac{\delta M_z M_z}{EI_y} + \beta \frac{\delta Q_z Q_z}{GA} \mathrm{dx} \tag{35 – 4}$$

where $I_y = \frac{bh^3}{12}$ is the second moment of inertia, $G = \frac{E}{2(1+\nu)} = \frac{E}{2}$ is a shear modulus, β is a parameter dependent on the shape of the cross-section, in the case of the rectangular cross-section it is equal to $\beta = 1.2$, δM and δQ are virtual bending moment and shear force respectively caused by the virtual force δF_z. Integrating the equation **(35 – 4)**, formula for the deflection u_z can be given as follows:

$$u_z = \frac{4F_z L^3}{Ebh^3} + \beta \frac{2F_z L}{Ebh} = 20.001 \text{ mm} \tag{35 – 5}$$

Finally total maximum deflection can be evaluated:

$$u_{\max} = \sqrt{u_x^2 + u_z^2} = 20.617 \text{ mm} \tag{35 – 6}$$

RFEM Settings

- Modeled in version RFEM 5.26 and RFEM 6.01
- The element size is $l_{\text{FE}} = 0.010$ m
- Geometrically linear analysis is considered
- The Mindlin plate theory is used
- Isotropic linear elastic material model is used
- Shear stiffness of members is activated

Results

Structure File	Entities
0035.01	Plate & Member
0035.02	Solid & Member
0035.03	Solid & Plate

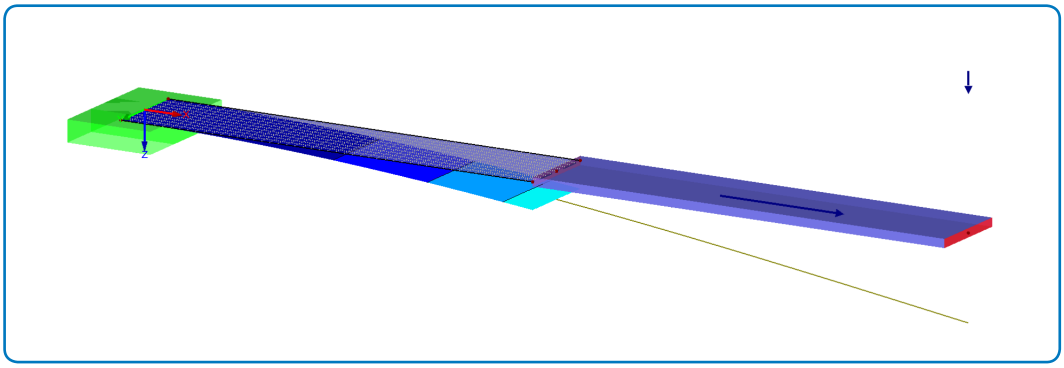

Figure 2: Plate & Member

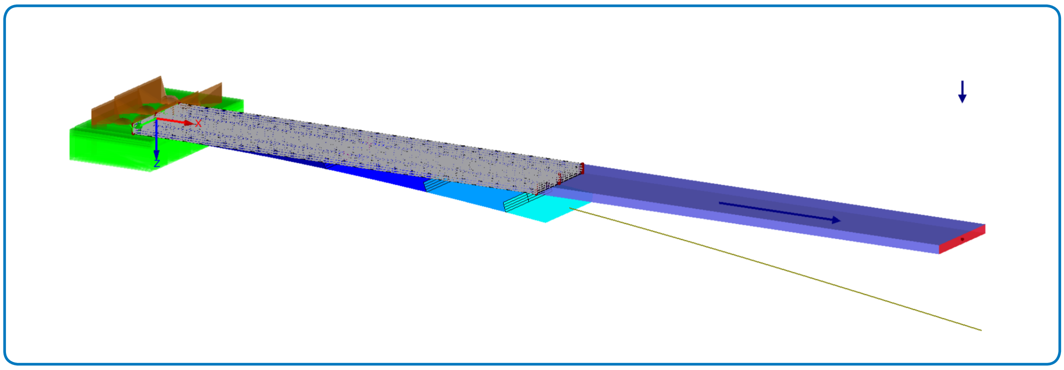

Figure 3: Solid & Member

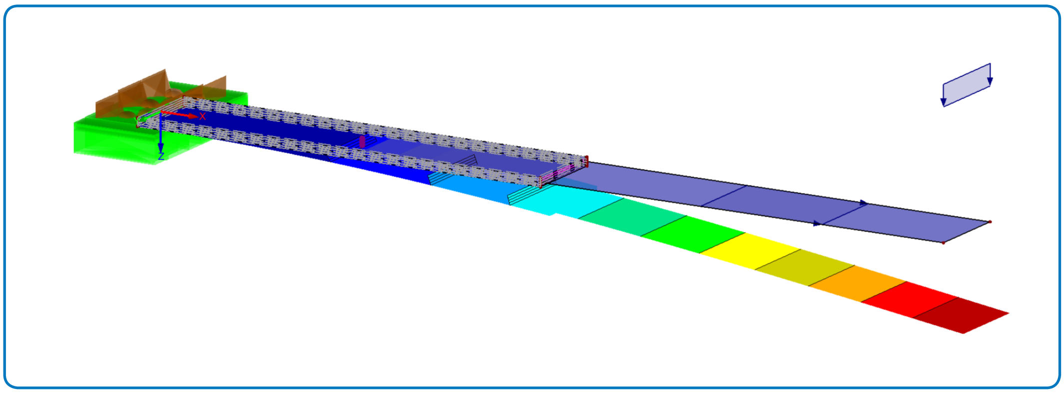

Figure 4: Solid & Plate

As can be seen from the table below, excellent agreements of analytical solution with numerical simulations were achieved for the coupling of plate and member.

Quantity	Analytical Solution	RFEM 5 Plate & Member		RFEM 5 Solid & Member		RFEM 5 Solid & Plate	
	[mm]	[mm]	Ratio [-]	[mm]	Ratio [-]	[mm]	Ratio [-]
u_x	5.000	5.000	1.000	5.018	1.004	5.000	1.000
u_z	20.001	20.001	1.000	20.002	1.000	20.002	1.000
u	20.617	20.617	1.000	20.621	1.000	20.617	1.000

Quantity	Analytical Solution	RFEM 6 Plate & Member		RFEM 6 Solid & Member		RFEM 6 Solid & Plate	
	[mm]	[mm]	Ratio [-]	[mm]	Ratio [-]	[mm]	Ratio [-]
u_x	5.000	5.000	1.000	5.018	1.004	5.000	1.000
u_z	20.001	20.001	1.000	20.002	1.000	20.002	1.000
u	20.617	20.616	1.000	20.622	1.000	20.617	1.000