# 0062 – Gravity Loading

## Description

A rod with a square cross-section is fixed on the top end according to **Figure 1**. The rod is loaded by its self-weight. For comparison, the example is also modeled with the concentrated force load which value is equal to the gravity. The aim of this verification example is to show the difference between these types of loading although the total loading force is equal. The problem is described by the following set of parameters.

| | | | | | |
|---|---|---|---|---|---|
| Material | Steel | Modulus of Elasticity | $E$ | 210000.000 | MPa |
| | | Poisson's Ratio | $\nu$ | 0.296 | – |
| | | Density | $\rho$ | 7850.000 | $\mathrm{kg/m^3}$ |
| Geometry | Rod | Length | $L$ | 10.000 | m |
| | | Cross-Section Width | $w$ | 20.000 | mm |
| Load | | Gravitational Acceleration | $g$ | 10.000 | $\mathrm{m/s^2}$ |

The self-weight is neglected in the first case. Determine the maximum deformation of the rod $u_{z,\max}$.

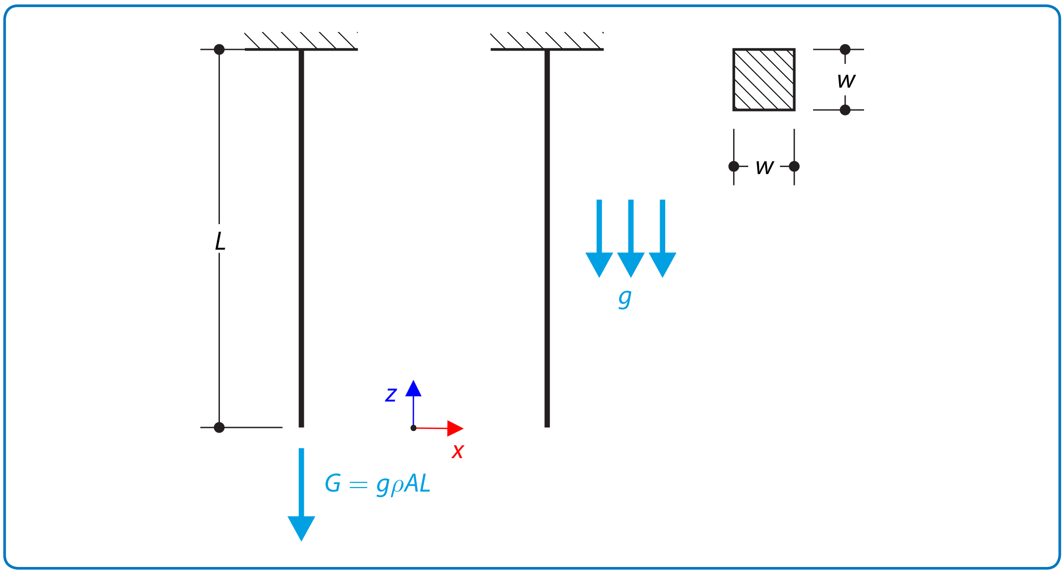


**Figure 1:** Problem Sketch

## Analytical Solution

The equation of equilibrium in one-dimensional case can be written as:

$$\frac{\mathrm{d}\sigma_z}{\mathrm{d}z} + f_z = 0 \tag{62 – 1}$$

where $f_z$ is the volume force in N/m$^3$ and the axial stress $\sigma_z$ is defined by the Hooke's law.

$$\sigma_z = E\varepsilon_z \tag{62 – 2}$$

## Concentrated Force

At first, the loading by the concentrated force is considered. The concentrated force is equal to the gravity.

$$G = g\rho AL = 314.000 \text{ N} \tag{62 – 3}$$

The volume force $f_z$ is equal to zero in this case. Then the strain $\varepsilon_z$ remains constant

$$\frac{\mathrm{d}\sigma_z}{\mathrm{d}z} = \frac{\mathrm{d}(E\varepsilon_z)}{\mathrm{d}z} = \frac{E\mathrm{d}\varepsilon_z}{\mathrm{d}z} = 0 \tag{62 – 4}$$

where the strain $\varepsilon_z$ is defined as:

$$\varepsilon_z = \frac{\mathrm{d}u_z}{\mathrm{d}z} \tag{62 – 5}$$

Then the final differential equation is:

$$\frac{\mathrm{d}^2 u_z}{\mathrm{d}z^2} = 0 \tag{62 – 6}$$

The solution of this differential equation has the form of the following linear function.

$$u_z(z) = C_1 z + C_2, \quad C_1 \in \mathbb{R}, C_2 \in \mathbb{R} \tag{62 – 7}$$

The integration constants $C_1, C_2$ can be obtained from boundary conditions.

$$\frac{\mathrm{d}u_z}{\mathrm{d}z} = \varepsilon_z = \frac{G}{EA} \tag{62 – 8}$$

$$u_z(L) = 0 \tag{62 – 9}$$

Hence,

$$C_1 = \frac{G}{EA} \tag{62 – 10}$$

$$C_2 = -\frac{GL}{EA} \tag{62 – 11}$$

Finally, the formula for the displacement $u_z$ can be completed and the maximum displacement $u_{z,\max}$ calculated.

$$u_z(z) = \frac{G}{EA}(L - z) \tag{62 – 12}$$

$$u_{z,\max} = \frac{GL}{EA} = -0.037\ \text{mm} \tag{62 – 13}$$

## Self-weight

In case of self-weight the volume force is defined as:

$$f_z = \rho g \tag{62 – 14}$$

Using formula **(62 – 1)** again the differential equation of equilibrium can be rewritten as follows:

$$\frac{\mathrm{d}^2 u_z}{\mathrm{d}z^2} = -\frac{\rho g}{E} \tag{62 – 15}$$

Please note that the strain $\varepsilon_z$ is not constant. The deformation $u_z$ is then the following:

$$u_z(z) = -\frac{\rho g}{2E} z^2 + C_1 x + C_2, \quad C_1 \in \mathbb{R}, C_2 \in \mathbb{R} \tag{62 – 16}$$

Boundary conditions in this case are the following:

$$\frac{\mathrm{d}u_z(0)}{\mathrm{d}z} = 0 \tag{62 – 17}$$

$$u_z(L) = 0 \tag{62 – 18}$$

Hence,

$$C_1 = 0 \tag{62 – 19}$$

$$C_2 = -\frac{\rho g}{2E} L^2 \tag{62 – 20}$$

Finally the formula for the displacement $u_z$ can be completed and the maximum displacement $u_{z,\max}$ calculated.

$$u_z(z) = \frac{\rho g}{2E}(L^2 - z^2) \tag{62 – 21}$$

$$u_{z,\max} = \frac{\rho g L^2}{2E} = -0.019 \text{ mm} \tag{62 – 22}$$

## RFEM 5 Settings

- Modeled in RFEM 5.06.1176
- The element size is $l_{FE} = 0.100$ m
- The number of increments is 5
- Isotropic linear elastic material model is used

## Results

| Structure Files | Program | Description |
|---|---|---|
| 0062.01 | RFEM 5 | Concentrated force |
| 0062.02 | RFEM 5 | Self-weight |

| Type of Loading | Analytical Solution | RFEM 5 | |
|---|---|---|---|
| | $u_{z,\max}$ [mm] | $u_{z,\max}$ [mm] | Ratio [-] |
| Concentrated force | -0.037 | -0.037 | 1.000 |
| Self-weight | -0.019 | -0.019 | 1.000 |