Program:	RFEM 5
Category:	Isotropic Linear Elasticity, Geometrically Linear Analysis, Member, Solid
Verification Example:	0082 – Helical Spring

0082 – Helical Spring

Description

Closely coiled helical spring is loaded by a compression force F according to **Figure 1**. The spring has a middle diameter D, the wire diameter d and consists of i turns. The total length of the spring is L. Determine the total deflection of the spring u_x for the member model and one-turn deflection $u_{x,i}$ for the solid model.

Material	Modulus of Elasticity	E	210000.000	MPa
	Poisson's Ratio	ν	0.296	–
Geometry	Middle Diameter	D	30.000	mm
	Wire Diameter	d	3.000	mm
	Number of Turns	i	8	–
	Length	L	50.000	mm
Load	Compression Force	F	50.000	N

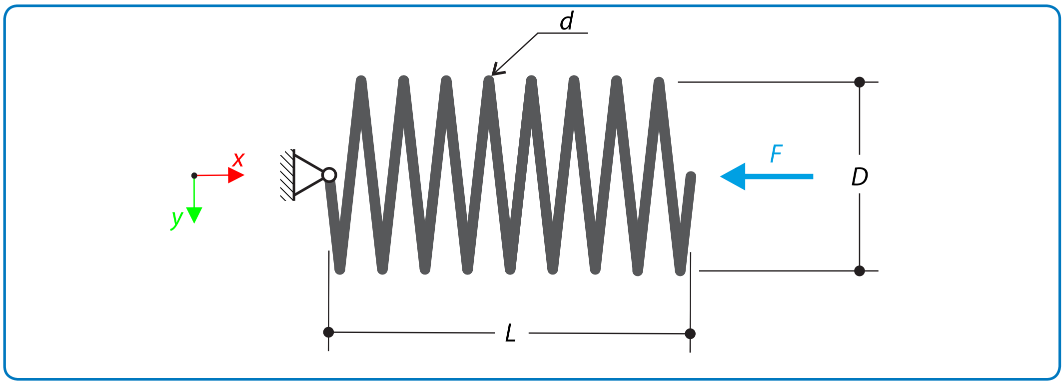

Figure 1: Problem Sketch

Analytical Solution

Analytical solution is based on the spring theory when close coiling (small pitch angle α) and relatively thin wire are assumed, see [1]. Then, the only loading is the torque of the wire

$$M_T = \frac{FD}{2}\cos\alpha \approx \frac{FD}{2}. \qquad (82-1)$$

The deflection u_x of the spring loaded by the concentrated force F can be determined by means of Castigliano's first theorem

$$u_x = \frac{\partial U}{\partial F}, \qquad (82-2)$$

where U is the strain energy. For the torque loading the strain energy is equal to

$$U = \frac{1}{2}\int_L \frac{M_T^2}{GJ}\mathrm{d}s = \frac{1}{2}\frac{M_T^2 L}{GJ} \approx \frac{4F^2D^3i}{Gd^4}. \qquad (82-3)$$

Considering the length of the wire $L = \pi Di$, shear modulus G and the torsional constant $J = \frac{\pi d^4}{32}$. The deflection of the spring is then equal to

$$u_x = \frac{8FD^3i}{Gd^4} \approx 13.169 \text{ mm}. \qquad (82-4)$$

Analogously, the deflection of one turn $u_{x,i}$ is

$$u_{x,i} = \frac{8FD^3}{Gd^4} \approx 1.646 \text{ mm}. \qquad (82-5)$$

This value is used for the comparison in case of a solid model. Due to the large amount of finite elements, only one turn of the spring is modeled.

RFEM 5 Settings

- Modeled in RFEM 5.09.01
- Element size is $l_{FE} = 1.0$ mm (member)
- Element size is $l_{FE} = 0.5$ mm (solid)
- The number of increments is 10
- Isotropic linear elastic material is used
- Shear stiffness of the members is deactivated

Results

Structure File	Program	Entity
0082.01	RFEM 5	Member
0082.02	RFEM 5	Solid

Analytical Solution	RFEM 5	
u_x [mm]	u_x [mm]	Ratio [-]
13.169	13.172	1.000

Analytical Solution	RFEM 5	
$u_{x,i}$ [mm]	$u_{x,i}$ [mm]	Ratio [-]
1.646	1.705	1.036

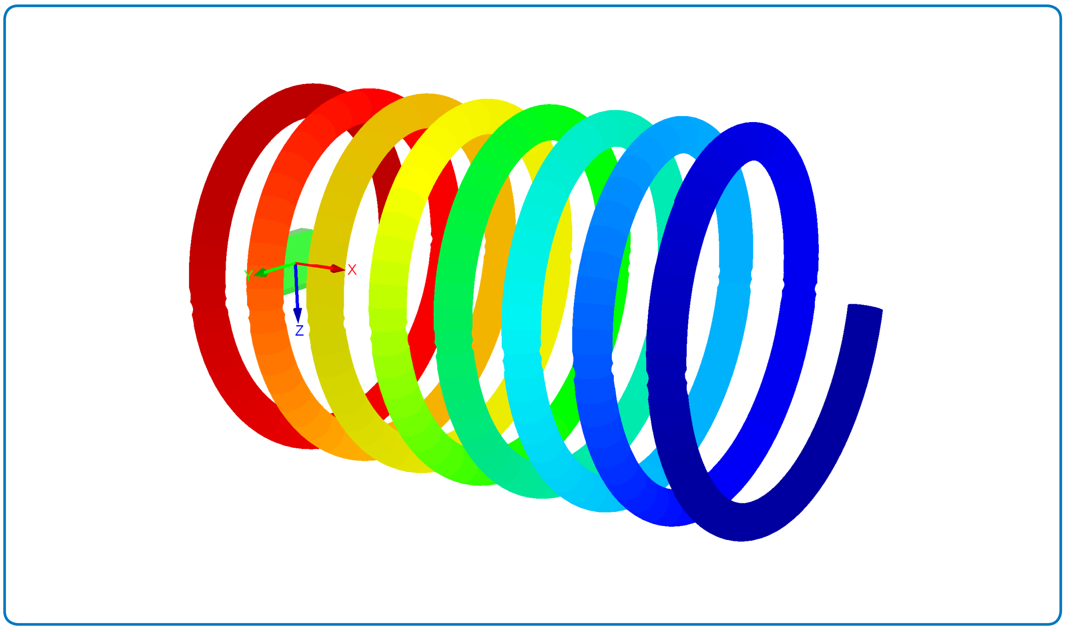

Figure 2: RFEM 5 results on member representation - deflection u_x

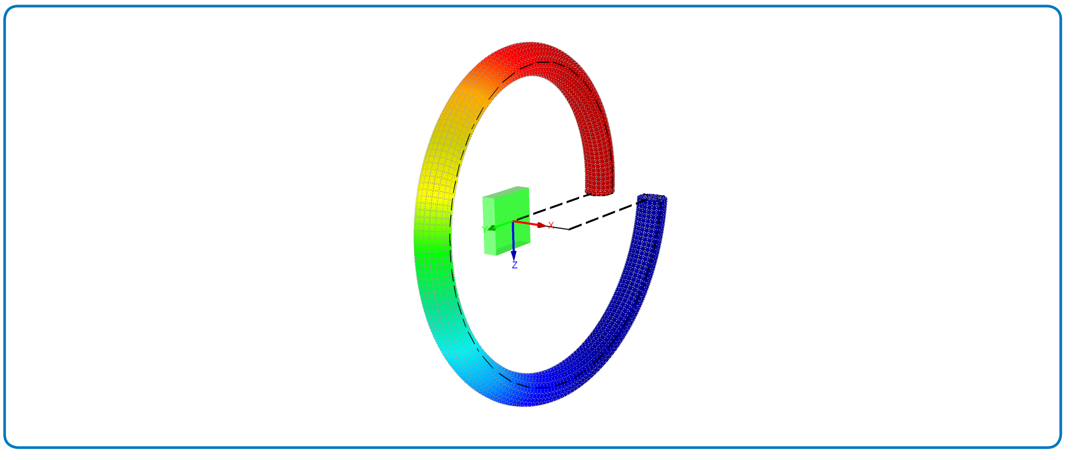

Figure 3: RFEM 5 results on solid representation (one turn of the spring) - deflection $u_{x,i}$

References

[1] DROTSKY, J. *Strength of Materials for Technicians*. Elsevier Science, 2013.