0094 – Buckling of a Circular Ring

Description

A thin circular ring of rectangular cross-section $b \times h$ is exposed to an external pressure p according to **Figure 1**. Determine the critical load p_{cr} and corresponding load factor f for in-plane buckling. The problem is described by the following parameters.

Material	Steel	Modulus of Elasticity	E	210000.000	MPa
		Poisson's Ratio	ν	0.300	–
Geometry	Ring	Radius	R	0.500	m
	Cross-section	Width	h	3.000	mm
		Height	b	30.000	mm
Load		Pressure	p	1.000	N/mm

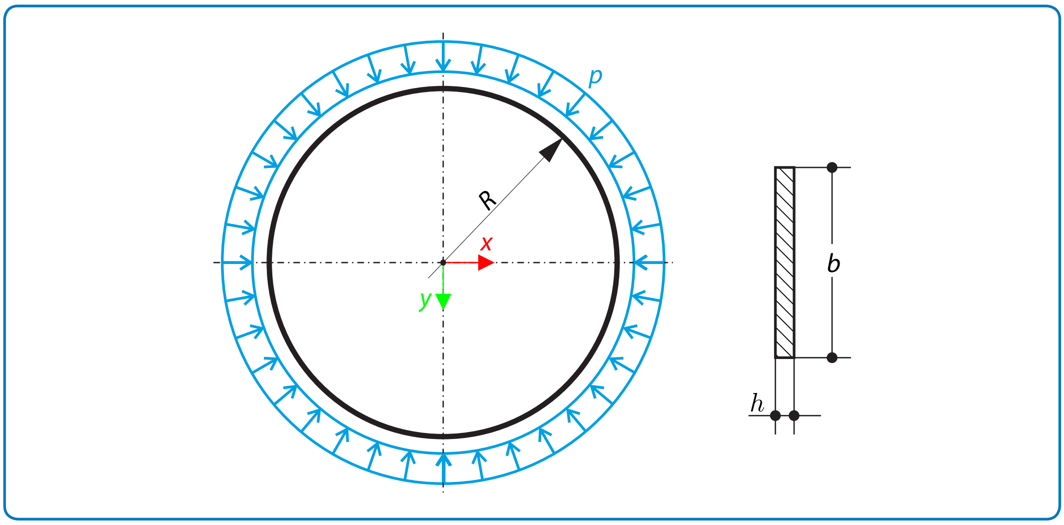

Figure 1: Problem Sketch

Analytical Solution

The analytical solution is based on the theory introduced in [1]. The deflection curve of a thin bar with a circular center line is defined by means of the following differential equation

$$\frac{\mathrm{d}^2}{\mathrm{d}\varphi^2} w + w = -\frac{MR^2}{EI}, \qquad (94-1)$$

where w is the deflection, φ is the angular coordinate, I is the cross-section moment of inertia[1] and M is the bending moment magnitude. This can be derived from the **Figure 2**, where S is the compressive force in the ring and M_0 is the bending moment at any cross-section.

[1] For the rectangular cross-section the moment of inertia is $I = \frac{1}{12} b h^3$.

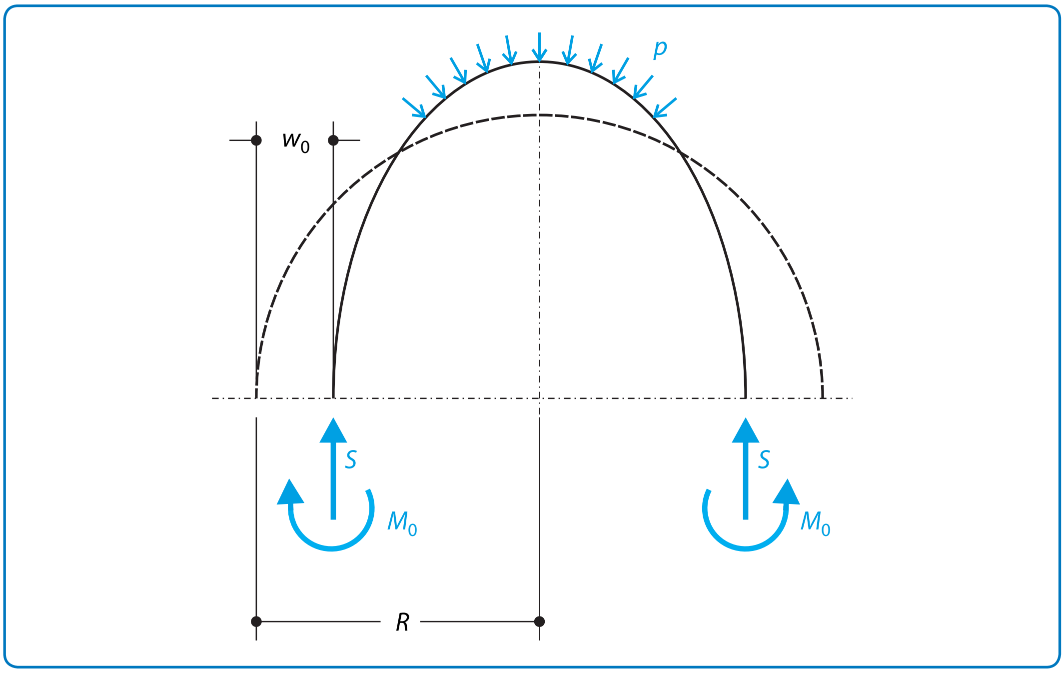

Figure 2: Schema for the bending moment magnitude derivation

$$M = M_0 - pR\,(w_0 - w) \qquad (94-2)$$

The differential equation **(94 – 1)** can be then modified into the following form

$$\frac{d^2}{d\varphi^2}w + \alpha^2 w = -\frac{pR^3 w_0 - M_0 R^2}{EI}, \qquad (94-3)$$

where the coefficient α is equal to

$$\alpha^2 = 1 + \frac{pR^3}{EI}. \qquad (94-4)$$

The general solution is then

$$w = C_1 \sin \alpha\varphi + C_2 \cos \alpha\varphi + \frac{pR^3 w_0 - M_0 R^2}{EI + pR^3}. \qquad (94-5)$$

Real constants C_1 and C_2 are obtained from the conditions of zero rotation at points corresponding to $\varphi = 0$ and $\varphi = \frac{\pi}{2}$

$$\left(\frac{d}{d\varphi}w\right)_{\varphi=0}, \qquad (94-6)$$

$$\left(\frac{d}{d\varphi}w\right)_{\varphi=\frac{\pi}{2}}. \qquad (94-7)$$

From the first condition follows that $C_1 = 0$ and the second gives

$$\sin \alpha \frac{\pi}{2} = 0. \qquad (94-8)$$

The smallest root of this equation is $\alpha = 2$, hence, from **(94 – 4)**, the critical value of the pressure p_{cr} is determined

$$p_{cr} = \frac{3EI}{R^3} \approx 0.340\ \text{N/mm}. \qquad (94-9)$$

The load factor f is then

$$f = \frac{p_{cr}}{p} \approx 0.340. \qquad (94-10)$$

RFEM 5 and RSTAB 8 Settings

- Modeled in RFEM 5.16.01 and RSTAB 8.16.01
- Element size is $l_{\text{FE}} = 0.005$ m
- The number of increments is 10
- Isotropic linear elastic material is used
- Lanczos method is used for eigenvalue analysis

Results

Structure File	Program	Method	Entity
0094.01	RFEM 5 - RF-STABILITY	Nonlinear Analysis	Member
0094.02	RFEM 5	Postcritical Analysis	Member
0094.03	RFEM 5	Postcritical Analysis	Shell

Model	Analytical Solution	RFEM 5 / RSTAB 8	
	f [-]	f [-]	Ratio [-]
RF-STABILITY, Nonlinear Analysis	0.340	0.348	1.024
RFEM 5, Postcritical Analysis*, Member		0.337	0.991
RFEM 5, Postcritical Analysis*, Shell		0.346	1.018

* Remark: The postcritical analysis (modified Newton-Raphson method) is used as a variant to the solution in add-on modules for buckling. The critical force or the load factor can be approximately determined from the deflection behaviour. It is convenient to use the incrementally increasing

loading with refinement of the last load increment. Small imperfections are added to the initial geometry to reach the instability.

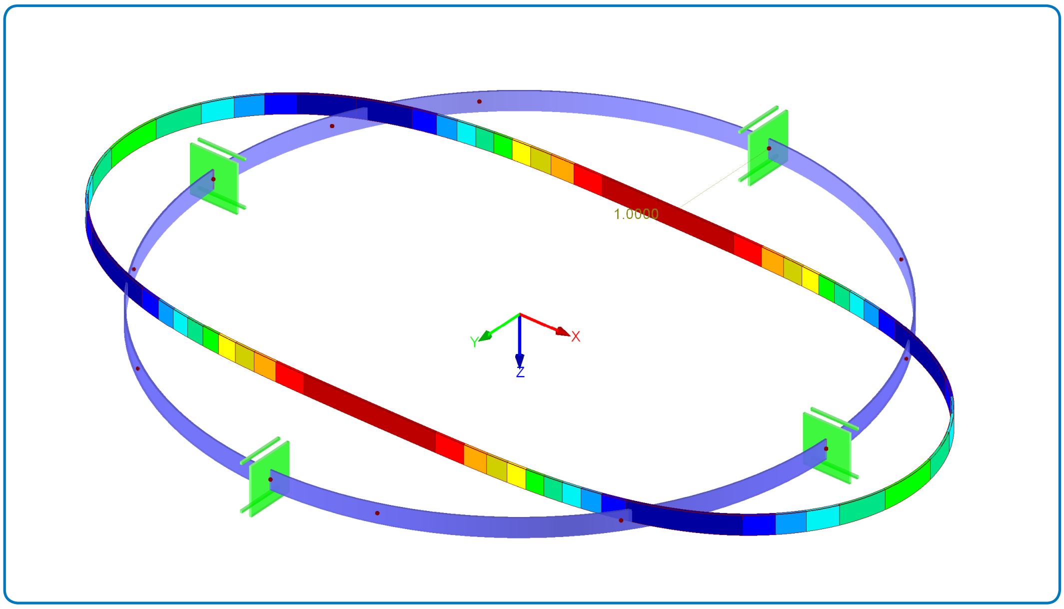

Figure 3: Buckled shape in RFEM 5 - RF-STABILITY

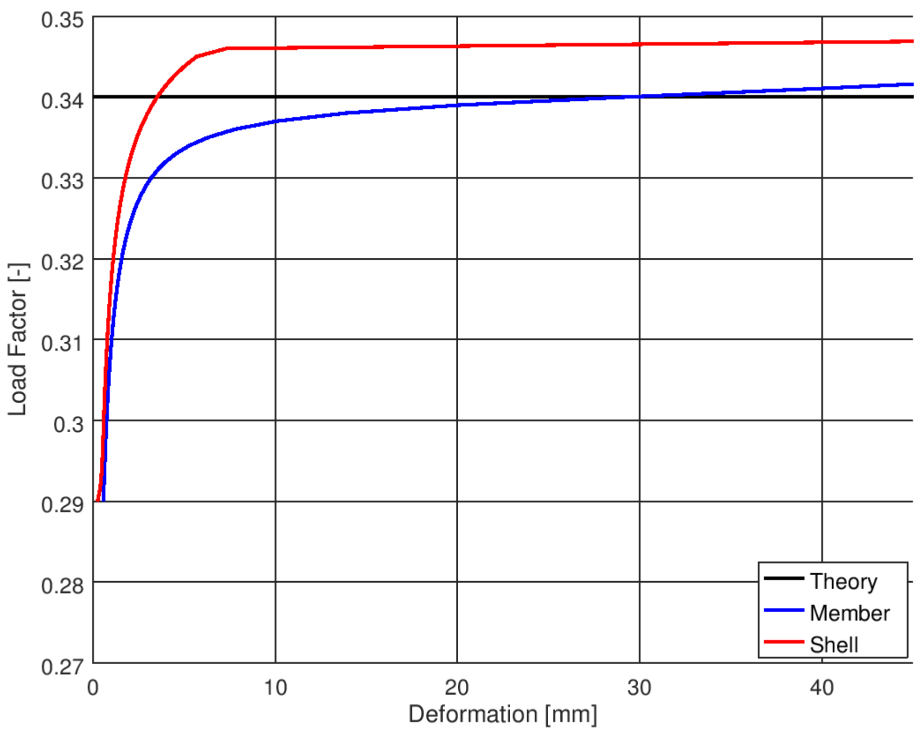

Figure 4: Comparison of postcritical analysis results for member and shell entity

References

[1] TIMOSHENKO, S. and GERE, J. *Theory of Elastic Stability*. McGraw-Hill Book Company, 1963.