

0099 – Spatial Bending with Warping

Description

A structure made of I-profile is embedded according to the the **Figure 1**[1]. The axial rotation φ_x is restricted on the both ends by means of the fork supports¹. The structure is loaded by two transverse forces in its middle. The problem is described by the following set of parameters.

Material	Steel	Modulus of Elasticity	E	210000.000	MPa
		Poisson's Ratio	ν	0.300	—
Geometry	Structure	Length	L	6.000	m
	Cross-Section	Height	h	400.000	mm
		Width	b	180.000	mm
		Web Thickness	s	10.000	mm
		Flange Thickness	t	14.000	mm
Load	Transverse Forces	F_y		3.000	kN
		F_z		194.000	kN

The self-weight is neglected in this example. Determine the maximum deflections of the structure $u_{y,\max}$ and $u_{z,\max}$, maximum rotation $\varphi_{x,\max}$, maximum bending moments $M_{y,\max}$ and $M_{z,\max}$ and maximum torsional moments $M_{T,\max}$, $M_{T_{pri},\max}$, $M_{T_{sec},\max}$ and $M_{\omega,\max}$.

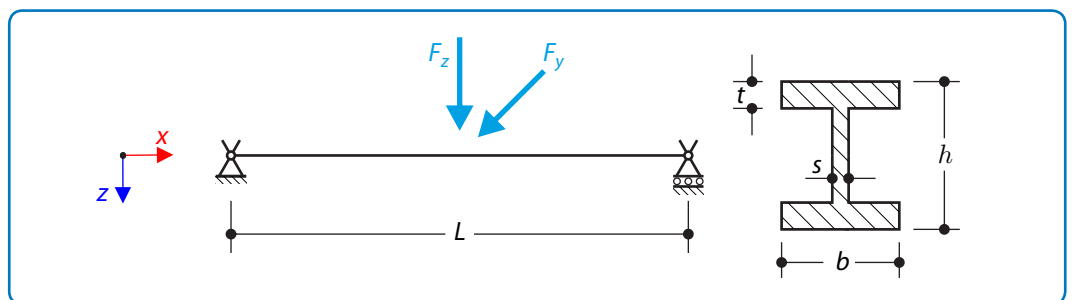


Figure 1: Problem sketch

Analytical Solution

Analytical solution is not available, results given in [1] are taken as a reference.

¹ The axial rotation is restricted in the fork support while the warping is enabled.

RFEM and RSTAB Settings

- Modeled in version RFEM 5.26.02, RSTAB 8.26.02 and RFEM 6.01, RSTAB 9.01
- The element size is $l_{FE} = 0.025$ m
- Isotropic linear elastic material model is used
- Second-Order Analysis is used
- Torsional Warping and Steel Design add-on is used in RFEM 6 and RSTAB 9

Results

Structure Files	Program
0099.01	RFEM 5 – RF-FE-LTB, RFEM 6
0099.02	RSTAB 8 – FE-LTB, RSTAB 9

Following graphs illustrate the behaviour of selected deformations and internal forces calculated in RSTAB 9. Maximum values are sorted and compared in result tables.

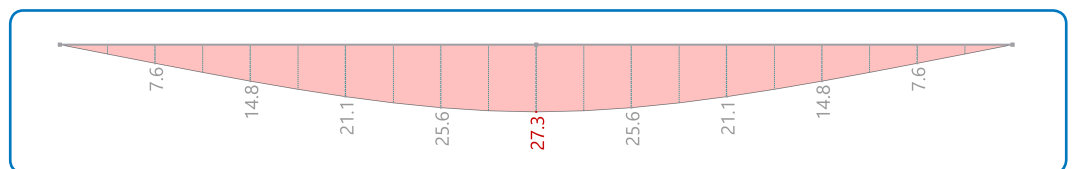


Figure 2: RSTAB 9 results - deformation u_y [mm]

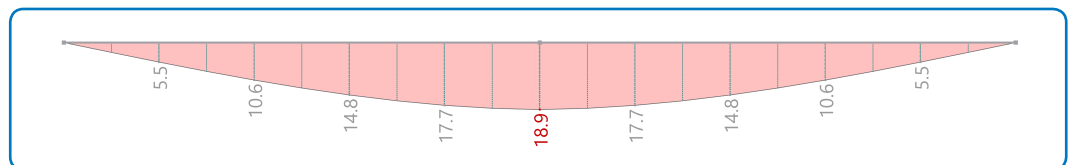


Figure 3: RSTAB 9 results - deformation u_z [mm]

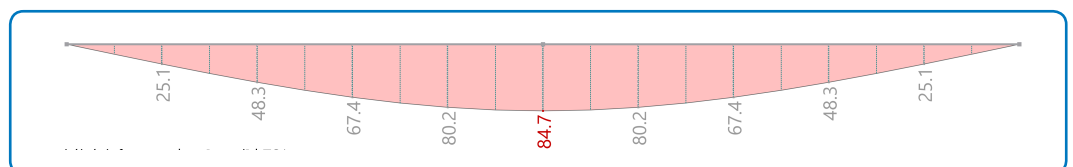


Figure 4: RSTAB 9 results - deformation φ_x [mrad]

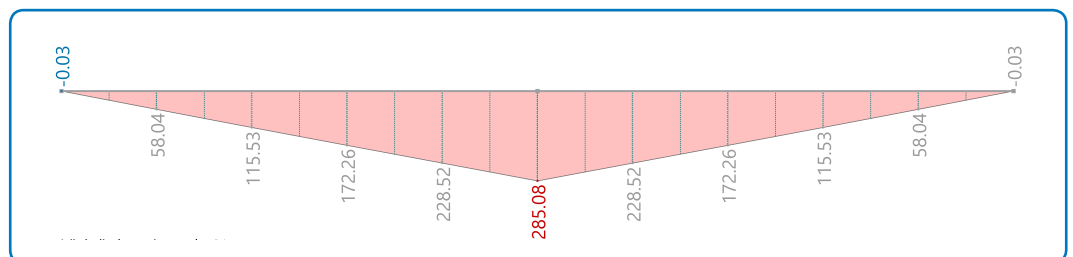


Figure 5: RSTAB 9 results - bending moment M_y [kNm]

Verification Example: 0099 – Spatial Bending with Warping

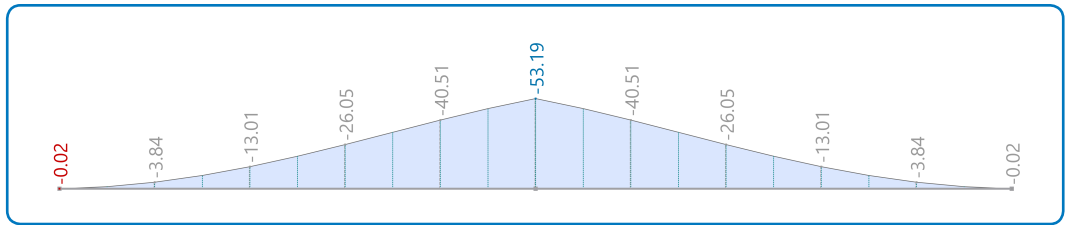


Figure 6: RSTAB 9 results - bending moment M_z [kNm]

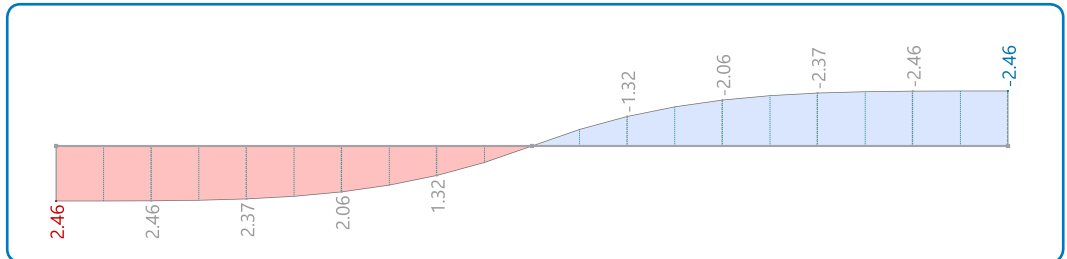


Figure 7: RSTAB 9 results - torsional moment M_T [kNm]

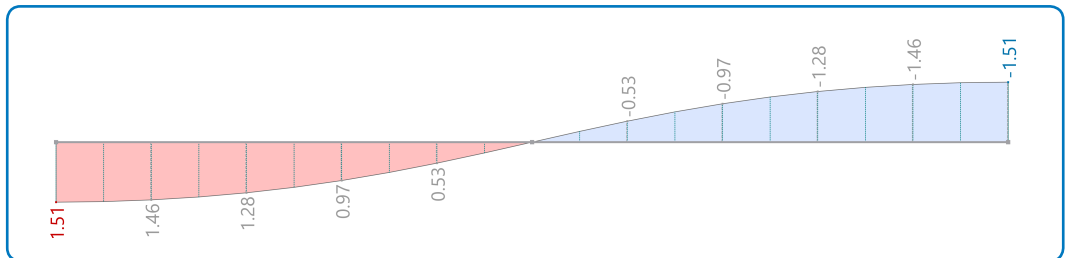


Figure 8: RSTAB 9 results - primary torsional moment $M_{T_{pri}}$ [kNm]

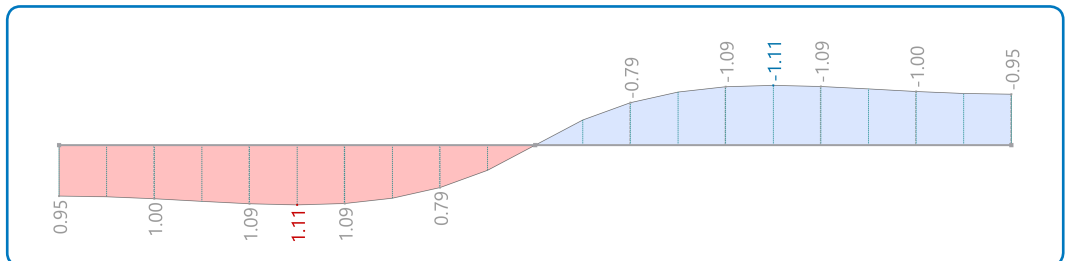


Figure 9: RSTAB 9 results - secondary torsional moment $M_{T_{sec}}$ [kNm]

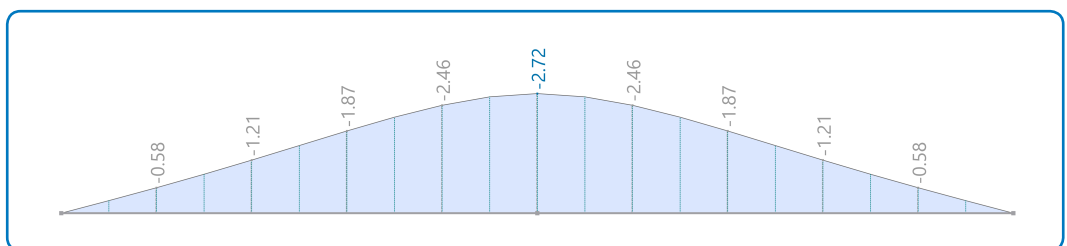


Figure 10: RSTAB 9 results - warping moment M_ω [kNm]

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Quantity	S3D [2]	RFEM 5, RF-FE-LTB	Ratio	RSTAB 8, FE-LTB	Ratio [-]
$u_{y,max}$ [mm]	25.0	28.1	1.124	28.1	1.124
$u_{z,max}$ [mm]	19.8	18.9	0.954	18.9	0.954
$\varphi_{x,max}$ [mrad]	81.0	87.8	1.084	87.9	1.084
$M_{y,max}$ [kNm]	290.0	289.0	0.997	289.0	0.997
$M_{z,max}$ [kNm]	-28.0	-29.9	1.068	-29.9	1.068
$M_{T,max}$ [kNm]	2.38	2.54	1.068	2.54	1.068
$M_{Tpri,max}$ [kNm]	1.48	1.57	1.061	1.57	1.061
$M_{Tsec,max}$ [kNm]	1.03	1.19	1.155	1.19	1.155
$M_{\omega,max}$ [kNm ²]	2.61	2.77	1.061	2.78	1.061

Quantity	S3D [2]	RFEM 6	Ratio	RSTAB 9	Ratio [-]
$u_{y,max}$ [mm]	25.0	26.6	1.064	27.1	1.084
$u_{z,max}$ [mm]	19.8	20.8	1.051	18.9	0.955
$\varphi_{x,max}$ [mrad]	81.0	87.7	1.083	84.1	1.038
$M_{y,max}$ [kNm]	290.0	289.5	0.998	289.1	0.997
$M_{z,max}$ [kNm]	-28.0	-30.0	1.071	-28.7	1.025
$M_{T,max}$ [kNm]	2.38	2.56	1.034	2.45	1.029
$M_{Tpri,max}$ [kNm]	1.48	1.57	1.061	1.50	1.013
$M_{Tsec,max}$ [kNm]	1.03	1.21	1.175	1.15	1.117
$M_{\omega,max}$ [kNm ²]	2.61	2.84	1.088	2.70	1.034

References

- [1] LUMPE, G. and GENSICHEN, V. *Evaluierung der linearen und nichtlinearen Stabstatik in Theorie und Software: Prüfbeispiele, Fehlerursachen, genaue Theorie*. Ernst, 2014.
- [2] LUMPE, G. *S3D (Vers. 25.09.2011)*. Hochschule Biberach, 2011.