0007 – Orthotropic Cantilever in Tension

Description

A vertical timber cantilever with fibers oriented at an angle β, with the square cross-section, is loaded at the top by the tensile pressure p. Base movement in the z-direction is restricted and always one edge of the base plane is fixed to move perpendicularly to its orientation. Assuming small deformation theory and neglecting cantilever's self-weight, determine its maximum deformation.

Material	Timber	Modulus of Elasticity	$E_x = E_y$	3.000	GPa
			E_z	11.000	GPa
		Poisson's Ratio	$\nu_{xy} = \nu_{yz} = \nu_{xz}$	0.000	–
		Shear Modulus	$G_{xy} = G_{yz} = G_{xz}$	5.500	GPa
		Fiber Angle	β	−60.000	°
Geometry	Cantilever	Height	h	1.000	m
		Width	b	0.050	m
		Depth	d	0.050	m
Load		Pressure	p	0.008	GPa

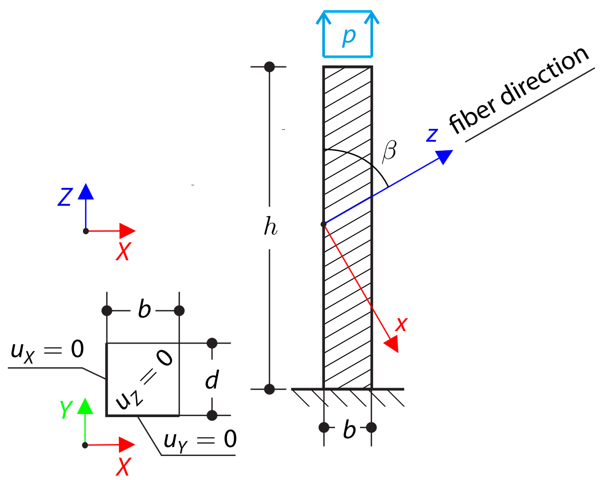

Figure 1: Problem sketch

Analytical Solution

The applied pressure p acts in a different direction than the timber fibres are oriented, therefore it is necessary to transform timber's stiffness matrix $\boldsymbol{D}_{xz}$ into the loading direction:

$$\boldsymbol{D}_{XZ} = \boldsymbol{T}^{\mathrm{T}} \boldsymbol{D}_{xz} \boldsymbol{T} \tag{7 – 1}$$

where $\boldsymbol{D}_{xz}$ is the stiffness 2D matrix acting in the material coordinate system xz, $\boldsymbol{D}_{XZ}$ is the corresponding stiffness 2D matrix in coordinate system XZ and $\boldsymbol{T}$ is the transformation matrix. The stiffness matrix in the material directions $\boldsymbol{D}_{xz}$ has the form

$$\boldsymbol{D}_{xz} = b \begin{bmatrix} E_x & 0 & 0 \\ 0 & E_z & 0 \\ 0 & 0 & G_{xz} \end{bmatrix} \tag{7 – 2}$$

The transformation matrix $\boldsymbol{T}$ has the form

$$\boldsymbol{T} = \begin{bmatrix} c^2 & s^2 & sc \\ s^2 & c^2 & -sc \\ -2sc & 2sc & c^2 - s^2 \end{bmatrix} \tag{7 – 3}$$

where $s = \sin\beta$ and $c = \cos\beta$ respectively. The formula (3 – 1) yields

$$\boldsymbol{D}_{XZ} = b \begin{bmatrix} c^4E_x + s^4E_z + 4s^2c^2G_{xz} & c^2s^2(E_x + E_z - 4G_{xz}) & cs[c^2E_x - s^2E_z - 2(c^2 - s^2)G_{xz}] \\ & s^4E_x + c^4E_z + 4s^2c^2G_{xz} & cs[s^2E_x - c^2E_z + 2(c^2 - s^2)G_{xz}] \\ \text{sym.} & & c^2s^2(E_x + E_z) + (c^2 - s^2)^2G_{xz} \end{bmatrix}$$

(7 – 4)

After that the strain 2D vector ε can be easily evaluated:

$$\varepsilon = \begin{bmatrix} \varepsilon_X \\ \varepsilon_Z \\ \gamma_{XZ} \end{bmatrix} = \boldsymbol{D}_{XZ}^{-1} \boldsymbol{p} \tag{7 – 5}$$

where $\boldsymbol{p}$ is the loading 2D vector:

$$\boldsymbol{p} = b \begin{bmatrix} 0 \\ p \\ 0 \end{bmatrix} \tag{7 – 6}$$

The maximum deflection $u_{\max}$ can be obtained according to deflections u_X and u_Z in the X and Z direction respectively.

$$u_X = h\gamma_{XZ} = -1.260 \text{ mm} \tag{7 – 7}$$

$$u_Z = h\varepsilon_Z = 1.818 \text{ mm} \tag{7 – 8}$$

$$u_{\max} = \sqrt{u_X^2 + u_Z^2} = h\sqrt{\gamma_{XZ}^2 + \varepsilon_Z^2} = 2.212 \text{ mm} \tag{7 – 9}$$

RFEM 5 Settings

- Modeled in version RFEM 5.03.0050
- The element size is $l_{FE} = 0.025$ m
- Geometrically linear analysis is considered
- The number of increments is 1
- The Mindlin plate theory is used

Results

Structure File	Program	Entity	Material Model
0007.01	RFEM 5	Solid	Orthotropic Elastic 3D
0007.02	RFEM 5	Plate	Orthotropic Elastic 2D
0007.03	RF-LAMINATE	Plate	-

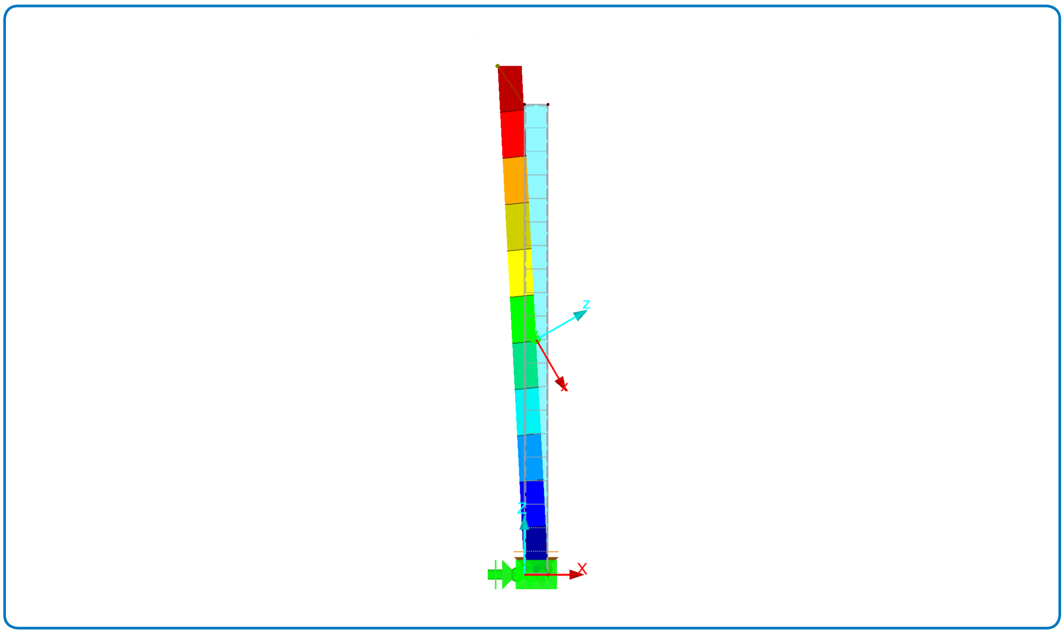

Figure 2: Deformation of a solid with the Orthotropic Elastic 3D material model

As can be seen from the following comparisons, an excellent agreement of analytical results with RFEM 5 outputs were achieved.

Quantity	Analytical Solution	RFEM 5 Solid		RFEM 5 Plate		RF-LAMINATE Plate	
	[mm]	[mm]	Ratio [-]	[mm]	Ratio [-]	[mm]	Ratio [-]
u_X	−1.260	−1.260	1.000	−1.260	1.000	−1.260	1.000
u_Z	1.818	1.819	1.000	1.818	1.000	1.818	1.000
u_{max}	2.212	2.213	1.000	2.212	1.000	2.212	1.000