Verification Example

Program: RFEM 5, RF-LAMINATE, RFEM 6

Category: Geometrically Linear Analysis, Orthotropic Linear Elasticity, Laminate, Plate

Verification Example: 0029 – Fiber Rotation Test in Laminated Plates

0029 – Fiber Rotation Test in Laminated Plates

Description

One layered square orthotropic plate of side length L and thickness t is fully fixed at its middle point and subjected to the pressure p according to the **Figure 1**. Problem is described by the following set of parameters.

Material	Laminate	Modulus of Elasticity	E_X	8000.000	MPa
			$E_Y = E_Z$	270.000	MPa
		Poisson's Ratio	ν_{YZ}	0.350	–
			$\nu_{XY} = \nu_{XZ}$	0.470	–
		Shear Modulus	$G_{XY} = G_{XZ}$	500.000	MPa
			G_{YZ}	100.000	MPa
Geometry		Side Length	L	10.000	m
		Thickness	t	0.100	m
		Fibers Angle	β	± 45	°
Load		Pressure	p	1.000	Pa

Compare the deflection u_z of the plate corners A $(0, 0, 0)$ and B $(L, 0, 0)$ for different fiber angles β to check the correctness of the transformation.

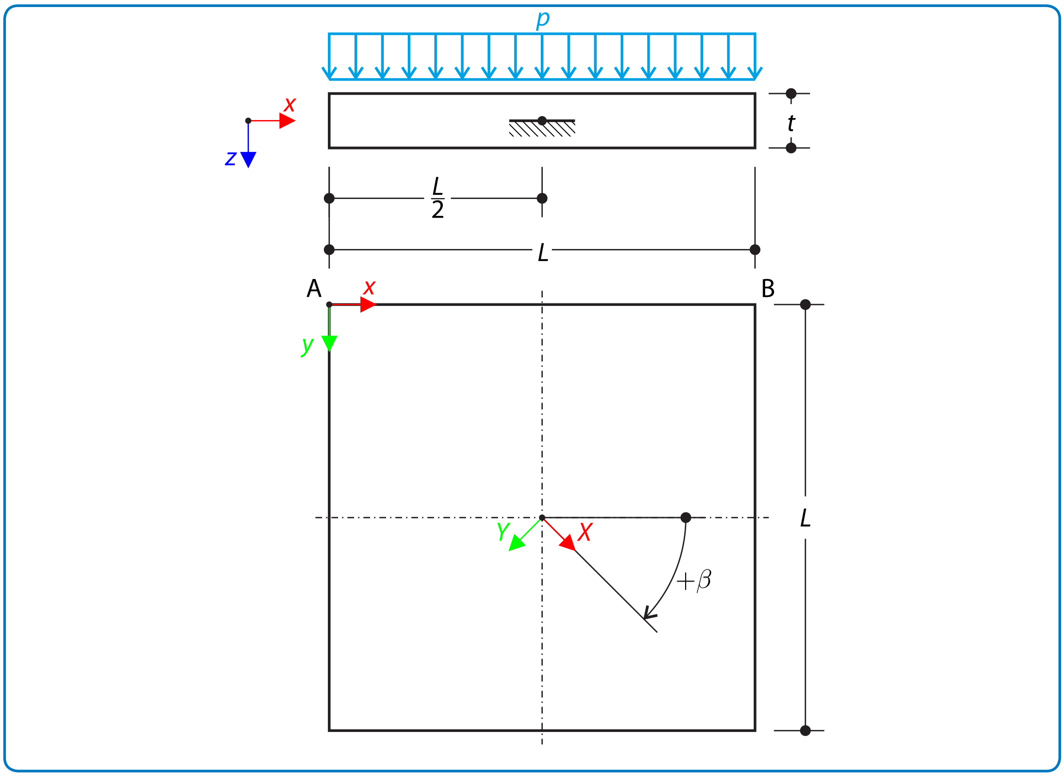

Figure 1: Problem sketch

Analytical Solution

The aim of this test is to prove the equality of the deflections in opposite corners for fibre orientation $\beta = \pm 45°$. Complete analytical solution is not available, because of the given boundary conditions. The reason of those boundary conditions is the suitable comparability of the results for fiber orientation angle $\beta = \pm 45°$. The stiffness of the plate is much grater in fibres direction (direction of X-axis). The different fiber orientation causes the different stiffness of the plate in directions of diagonals, see **Figure 2**. The plate is more stiffer in the direction of the diagonal which is parallel to the fibre direction. Thus it is possible to write

$$u_{z,\mathrm{A}} > u_{z,\mathrm{B}}, \quad \beta = 45° \tag{29 – 1}$$

$$u_{z,\mathrm{A}} < u_{z,\mathrm{B}}, \quad \beta = -45° \tag{29 – 2}$$

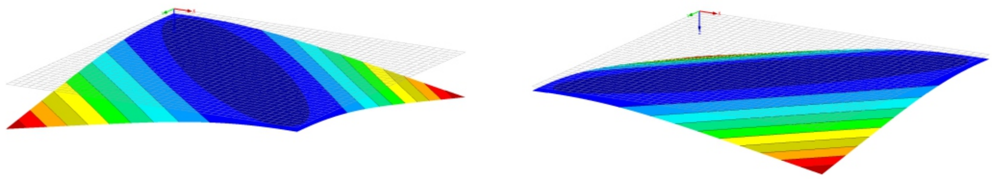

Figure 2: Different stiffness of the diagonals for fibre orientation $\beta = \pm 45°$

Stiffness Matrix

The stiffness matrix elements calculation follows. The global stiffness matrix can be calculated analytically. The stiffness matrix for one layer in local coordinates is defined as follows

$$\boldsymbol{d}' = \begin{bmatrix} \frac{E_X}{1-\nu_{XY}\nu_{YX}} & \frac{\nu_{XY}E_Y}{1-\nu_{XY}\nu_{YX}} & 0 \\ & \frac{E_Y}{1-\nu_{XY}\nu_{YX}} & 0 \\ \text{sym.} & & G_{XY} \end{bmatrix} \tag{29 – 3}$$

The relionship between moduli of elasticity and Poisson's ratios for the orthotropic materials is defined as follows:

$$\frac{\nu_{YX}}{E_Y} = \frac{\nu_{XY}}{E_X} \tag{29 – 4}$$

For the transformation of the local stiffness matrix $\boldsymbol{d}'$ to the global coordinate system by rotation by an angle β the transformation matrix $\boldsymbol{T}$ is used

$$\boldsymbol{d} = \boldsymbol{T}^{\mathrm{T}}\boldsymbol{d}'\boldsymbol{T} \tag{29 – 5}$$

The transformation matrix is defined as

$$\boldsymbol{T} = \begin{bmatrix} \cos^2\beta & \sin^2\beta & \cos\beta\sin\beta \\ \sin^2\beta & \cos^2\beta & -\cos\beta\sin\beta \\ -2\cos\beta\sin\beta & 2\cos\beta\sin\beta & \cos^2\beta - \sin^2\beta \end{bmatrix} \tag{29 – 6}$$

The stiffness matrix for one layer then results for $\beta = \pm 45°$:

$$\boldsymbol{d} = \begin{bmatrix} 2.647 & 1.647 & \pm 1.947 \\ 1.647 & 2.647 & \pm 1.947 \\ \pm 1.947 & \pm 1.947 & 2.019 \end{bmatrix} \text{GPa} \qquad (29-7)$$

Global stiffness matrix has the following form:

$$\boldsymbol{D} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & 0 & 0 & D_{16} & D_{17} & D_{18} \\ & D_{22} & D_{23} & 0 & 0 & & D_{27} & D_{28} \\ & & D_{33} & 0 & 0 & \text{sym.} & & D_{38} \\ & & & D_{44} & D_{45} & 0 & 0 & 0 \\ & & & & D_{55} & 0 & 0 & 0 \\ & \text{sym.} & & & & D_{66} & D_{67} & D_{68} \\ & & & & & & D_{77} & D_{78} \\ & & & & & & & D_{88} \end{bmatrix} \qquad (29-8)$$

Elements of the stiffness matrix $D_{11} - D_{33}$ define bending and torsion. General formula for these elements can be written as follows

$$D_{ij} = \sum_{k=1}^{n} \frac{z_{k,\max}^3 - z_{k,\min}^3}{3} d_{k,ij} \qquad (29-9)$$

where $i = 1, 2, 3; j = 1, 2, 3$ and n defines the number of the layers, $z_{k,\max}$ and $z_{k,\min}$ corresponds to the maximum and minimum distance of the appropriate layer surfaces from the zero layer ($z = 0$). In this special case with only one layer of the thickness t the elements can be calculated as follows.

$$D_{11} = \frac{t^3}{12} d_{11} = 220.580 \text{ kNm} \qquad (29-10)$$

$$D_{12} = \frac{t^3}{12} d_{12} = 137.246 \text{ kNm} \qquad (29-11)$$

$$D_{13} = \frac{t^3}{12} d_{13} = \pm 162.251 \text{ kNm} \qquad (29-12)$$

$$D_{22} = \frac{t^3}{12} d_{22} = 220.580 \text{ kNm} \qquad (29-13)$$

$$D_{23} = \frac{t^3}{12} d_{23} = \pm 162.251 \text{ kNm} \qquad (29-14)$$

$$D_{33} = \frac{t^3}{12} d_{33} = 168.259 \text{ kNm} \qquad (29-15)$$

Thanks to the symmetry (reference plane is in the middle of the layer) elements of the stiffness matrix $D_{16} - D_{38}$ are equal to zero. The elements $D_{44} - D_{55}$ are not taken into account due to the assumption of Kirchhoff plate bending theory (no shear effects). Elements of the stiffness matrix $D_{66} - D_{88}$, which define membrane loading can be calculated for angles $\beta = \pm 45°$ as follows

$$D_{i+5,j+5} = \sum_{k=1}^{n} (z_{k,\max} - z_{k,\min}) d_{k,ij} \qquad (29-16)$$

where $i = 1, 2, 3$; $j = 1, 2, 3$ and n defines the number of the layers, $z_{k,\max}$ and $z_{k,\min}$ corresponds to the maximum and minimum distance of the appropriate layer surfaces from the zero layer ($z = 0$). In this special case with only one layer of the thickness t the elements can be calculated as follows.

$$D_{66} = td_{11} = 264696 \text{ kNm} \tag{29 – 17}$$

$$D_{67} = td_{12} = 164696 \text{ kNm} \tag{29 – 18}$$

$$D_{68} = td_{13} = \pm 194702 \text{ kNm} \tag{29 – 19}$$

$$D_{77} = td_{22} = 264696 \text{ kNm} \tag{29 – 20}$$

$$D_{78} = td_{23} = \pm 194702 \text{ kNm} \tag{29 – 21}$$

$$D_{88} = td_{33} = 201910 \text{ kNm} \tag{29 – 22}$$

RFEM Settings

- Modeled in RFEM 5.26 and RFEM 6.01
- The element size is $l_{FE} = 0.250$ m
- Geometrically linear analysis is considered
- The number of increments is 5
- Kirchhoff plate bending theory is used
- Orthotropic Elastic 2D material model is used

Results

Structure File	Program	Fiber Orientation
0029.01	RFEM 5, RFEM 6	45°
0029.02	RFEM 5, RFEM 6	-45°
0029.03	RF-LAMINATE	45°
0029.04	RF-LAMINATE	-45°

Fiber orientation $\beta = +45°$	RFEM 5	RF-LAMINATE	RFEM 6
Test point	u_z [mm]	u_z [mm]	u_z [mm]
A	0.502	0.502	0.502
B	4.976	4.976	4.976

Fiber orientation $\beta = -45°$	RFEM 5	RF-LAMINATE	RFEM 6
Test point	u_z [mm]	u_z [mm]	u_z [mm]
A	4.976	4.976	4.976
B	0.502	0.502	0.502