0054 – Influence of the Normal Force on the Torsion

Description

Member with the given boundary conditions is loaded with the moment M_T and the axial force F_x (**Figure 1**). Neglecting it's self-weight, determine beam's maximum torsional deformation max φ as well as its inner torsional moment M_T defined as a sum of a primary torsional moment M_{Tpri} and torsional moment caused by the normal force M_{TN}. Provide a comparison of those values while assuming or neglecting the influence of the normal force.

Material	Steel	Modulus of Elasticity	E	210.000	GPa
		Shear Modulus	G	81.000	GPa
Geometry	Beam	Length	L	3.000	m
		Height	h	0.400	m
		Width	b	0.180	m
		Web Thickness	s	0.010	m
		Flange Thickness	t	0.014	m
Load		Force	F_x	500.000	kN
		Moment	M_x	1.200	kNm

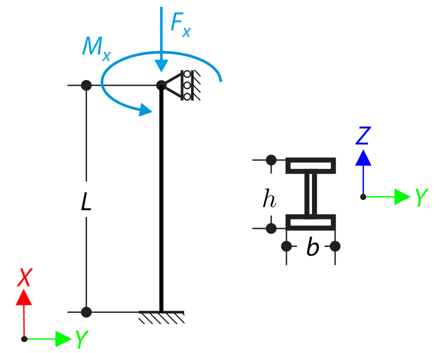

Figure 1: Problem sketch [1]

Analytical Solution

Assuming that the relative torsion φ' is constant and no secondary torsional moment acts on the structure, beam's torsional moment M_T can be obtained as a sum of a primary torsional moment M_{Tpri} and torsional moment caused by the normal force M_{TN}:

$$M_T = M_{Tpri} + M_{TN} \tag{54 – 1}$$

$$M_{Tpri} = GI_T\varphi' \tag{54 – 2}$$

$$M_{TN} = N(i_p)^2\varphi' \tag{54 – 3}$$

Knowing that beam's torsional moment equals to the acting moment ($M_x = M_T$) and the normal force has the opposite value of the acting force ($N = -F_x$), beam's relative torsion φ' can be expressed from the previous system of equations as follows:

$$\varphi' = \frac{M_T}{GI_T + N(i_p)^2} \tag{54 – 4}$$

where I_T is a torsional moment stiffness:

$$I_T = \frac{2}{3}\left[bt^3\left(1 - 0.63\frac{t}{b}\right)\right] + \frac{1}{3}\left[(h-t)s^3\left(1 - 0.63\frac{s}{h-t}\right)\right] \tag{54 – 5}$$

and i_p is a polar radius of inertia:

$$i_p = \sqrt{\frac{I_p}{A}} = \sqrt{\frac{I_y + I_z}{A}} = \sqrt{\frac{bh^3 - (b-s)(h-2t)^3 + (2tb^3) + (h-2t)s^3}{12[hb - (h-2t)(b-s)]}} \tag{54 – 6}$$

While neglecting the influence of the normal force, beam's relative torsion can be evaluated using the equation **(54 – 4)** by setting $N = 0$:

$$\varphi' = \frac{M_T}{GI_T} \tag{54 – 7}$$

Knowing the expression for the relative torsion φ', primary torsional moment M_{Tpri} can be evaluated using formula **(54 – 2)**, moment M_{TN} using formula **(54 – 3)** and maximum torsional deformation φ_{max} as follows:

$$\varphi_{max} = \varphi(x = L) = \varphi' L \tag{54 – 8}$$

RFEM Settings

- Modeled in RFEM 5.05.0029 and RFEM 6.01
- The element size is $l_{FE} = 0.300$ m
- The number of increments is 1
- The element type is member
- Isotropic linear elastic material model is used
- Shear stiffness of members is activated
- Torsional Warping add-on is used in RFEM 6

Results

Structure File	Program	Description
0054.01	RFEM 5 – RF-FE-LTB, RFEM 6	$N = 0$ kN
0054.02	RFEM 5 – RF-FE-LTB, RFEM 6	$N = -500$ kN

As can be seen from the following comparisons, good agreements of the analytical results with the numerical outputs were achieved. In RFEM there is torsional moment caused by the normal force M_{TN} included in the secondary torsional moment M_{Tsec}, however relative torsion φ' is constant and no other secondary torsional moment is present in the calculation so it can be set (for this example only) $M_{TN} = M_{Tsec}$ and comparison with the analytical solution can be done.

Axial Force Effect	Analytical Solution	RFEM 5 RF-FE-LTB		RFEM 6	
	φ_{max} [rad]	φ_{max} [rad]	Ratio [-]	φ_{max} [rad]	Ratio [-]
$N = 0$ kN	0.101	0.101	1.000	0.101	1.000
$N = -500$ kN	0.166	0.165	0.994	0.165	0.994

Axial Force Effect	Analytical Solution	RFEM 5 RF-FE-LTB		RFEM 6	
	M_{Tpri} [kNm]	M_{Tpri} [kNm]	Ratio [-]	M_{Tpri} [kNm]	Ratio [-]
$N = 0$ kN	1.200	1.200	1.000	1.200	1.000
$N = -500$ kN	1.972	1.966	0.997	1.966	0.997

Axial Force Effect	Analytical Solution	RFEM 5 RF-FE-LTB		RFEM 6	
	M_{TN} [kNm]	M_{TN} [kNm]	Ratio [-]	M_{TN} [kNm]	Ratio [-]
$N = 0$ kN	0.000	0.000	-	0.000	-
$N = -500$ kN	-0.772	-0.766	0.992	-0.766	0.992

Axial Force Effect	Analytical Solution	RFEM 5 RF-FE-LTB		RFEM 6	
	M_T [kNm]	M_T [kNm]	Ratio [-]	M_T [kNm]	Ratio [-]
$N = 0$ kN	1.200	1.200	1.000	1.200	1.000
$N = -500$ kN	1.200	1.200	1.000	1.200	1.000

References

[1] LUMPE, G. and GENSICHEN, V. *Evaluierung der linearen und nichtlinearen Stabstatik in Theorie und Software: Prüfbeispiele, Fehlerursachen, genaue Theorie*. Ernst, 2014.