Program: RFEM 5

Category: Geometrically Linear Analysis, Isotropic Linear Elasticity, Plate

Verification Example: 0072 – Bending of a Simply Supported Triangular Plate

0072 – Bending of a Simply Supported Triangular Plate

Description

A simply supported equilateral triangular plate is subjected to uniformly distributed transversal load p. Assuming small deformation theory and neglecting self-weight, determine the maximum out-of-plane deflection u_{max} of the plate.

Material	Linear Elastic	Modulus of Elasticity	E	50.000	GPa
		Poisson's Ratio	ν	0.200	–
Geometry	Triangle	Thickness	t	0.200	m
		Side Length	l	2.000	m
Load		Pressure	p	10.000	MPa

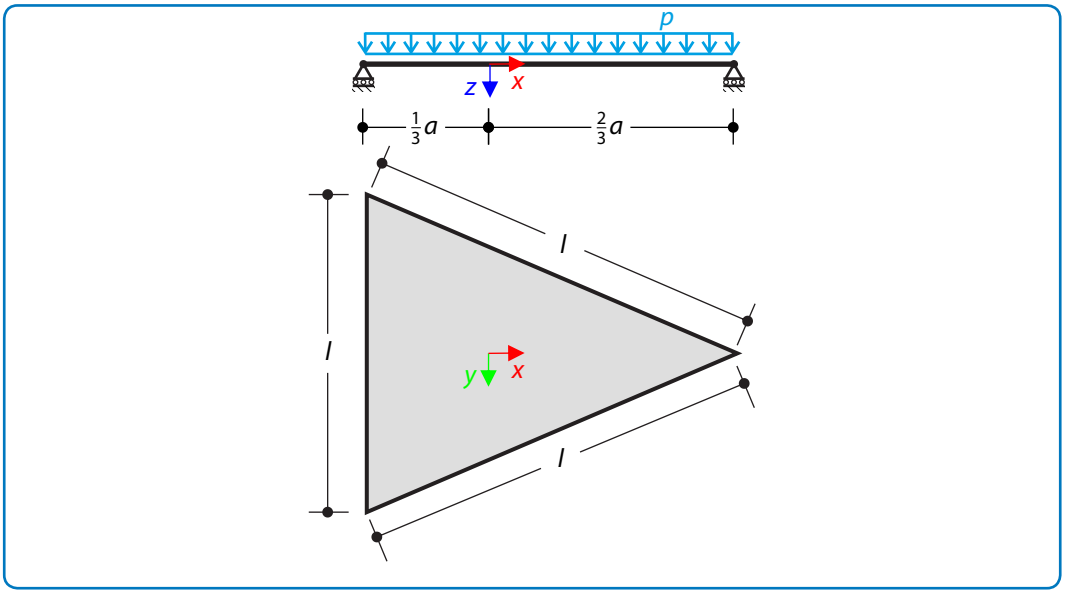

Figure 1: Problem sketch

Analytical Solution

For a simply supported equilateral triangular plate under uniform lateral loads, a closed-form solution has been proposed by Woinowsky & Krieger and can be found, e.g., in [1]

$$u(x,y) = \frac{p}{64Da}\left(x^3 - x^2a - 3xy^2 - y^2a + \frac{4}{27}a^3\right)\left(\frac{4}{9}a^2 - x^2 - y^2\right) \qquad (72-1)$$

where D is the flexural rigidity of the plate

$$D = \frac{Et^3}{12(1-\nu^2)} \qquad (72-2)$$

and a the length of the triangle altitude, namely

$$a = \frac{\sqrt{3}}{2} l \tag{72 – 3}$$

The plate will be deflected the most at its orthocenter where $x = 0$ and $y = 0$, as this simply supported equilateral triangle can be replaced by an "equivalent" circular plate centered at the orthocenter of the triangle with radius $r = 0.35a$, see [1] for the details. Hence the maximum deflection reads as

$$u_{max} = u(0,0) = \frac{p l^4 (1 - \nu^2)}{144 E t^3} \approx 2.667 \text{ mm} \tag{72 – 4}$$

RFEM 5 Settings

- Modeled in version RFEM 5.06.3039
- The element size is $l_{FE} = 0.01$ m
- Geometrically linear analysis is considered
- The number of increments is 1
- Kirchhoff plate theory is used

Results

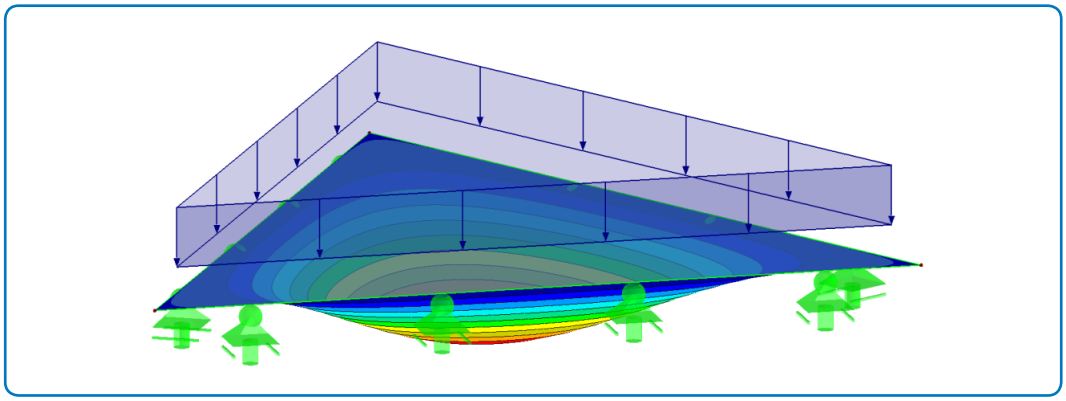

Figure 2: RFEM 5 Solution

Structure File	Program
0072.01	RFEM 5

As can be seen from the table below, excellent agreement of numerical output with the analytical result was achieved.

Analytical Solution	RFEM 5	
u_{max} [mm]	u_{max} [mm]	Ratio [-]
2.667	2.666	1.000

References

[1] SZILARD, R. *Theories and Application of Plate Analysis: Classical Numerical and Engineering Method*. Hoboken, New Jersey.