Program: RFEM 5

Category: Member, Plate, Solid

Verification Example: 0081 – Center of Gravity

0081 – Center of Gravity

Description

Determine the y-position of the center of gravity Y_C [1] for the following bodies described in **Figure 1**, namely semicircle, half-disc, hemispherical shell and half-ball.

Geometry	Semicircle, Hemisphere	Radius	r	1.000	m

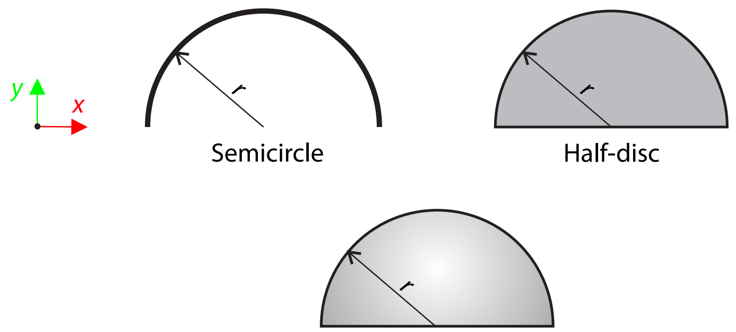

Figure 1: Problem Sketch

Analytical Solution

The calculation of the center of gravity position is based on the following equation, [1]

$$Y_C = \frac{1}{G} \int_G y \, dG. \qquad (81-1)$$

The gravitational force G can be further defined for the gravitational acceleration g and constant density ρ

$$dG = g \, dm = g\rho \, dV. \qquad (81-2)$$

Considering these relations, the calculation of the center of gravity position for the volumetric, flat and wire body can be determined.

[1] The remaining coordinates of the center of gravity X_C, Z_C coincide with the axis of symmetry for the corresponding body.

$$Y_C = \frac{1}{V} \int_V y \, dV \qquad (81-3)$$

$$Y_C = \frac{1}{A} \int_A y \, dA \qquad (81-4)$$

$$Y_C = \frac{1}{L} \int_L y \, dL \qquad (81-5)$$

The center of gravity of the semicircle (wire) can be calculated from the arc length element $ds = r\,d\varphi$, the y-coordinate is defined as $y = r \sin\varphi$.

$$Y_C = \frac{\int_0^\pi r \sin\varphi r \, d\varphi}{\int_0^\pi r \, d\varphi} = \frac{2r}{\pi} \approx 0.637 \text{ m} \qquad (81-6)$$

For the center of gravity of the semicircle (plate) the circular pattern area is used, $dA = \frac{1}{2} r^2 \, d\varphi$, $y = \frac{2}{3} r \sin\varphi$.

$$Y_C = \frac{\int_0^\pi \frac{2}{3} r \sin\varphi \frac{1}{2} r^2 \, d\varphi}{\int_0^\pi \frac{1}{2} r^2 \, d\varphi} = \frac{4r}{3\pi} \approx 0.424 \text{ m} \qquad (81-7)$$

For the hemisphere shell the area element is defined as $dA = 2\pi x \, ds$, $x = r\cos\varphi$, $y = r \sin\varphi$.

$$Y_C = \frac{\int_0^{\pi/2} 2\pi r^3 \sin\varphi \cos\varphi \, d\varphi}{\int_0^{\pi/2} 2\pi r^2 \sin\varphi \, d\varphi} = \frac{r}{2} = 0.500 \text{ m} \qquad (81-8)$$

The volumetric element of the hemisphere is defined as $dV = \pi x^2 \, dy$, where $x^2 = r^2 - y^2$.

$$Y_C = \frac{\int_0^r y\pi(r^2 - y^2) \, dy}{\int_0^r \pi(r^2 - y^2) \, dy} = \frac{3r}{8} = 0.375 \text{ m} \qquad (81-9)$$

RFEM 5 Settings

- Modeled in RFEM 5.09.01
- Element size is $l_{FE} = 0.050$ m

Results

Structure File	Program
0081.01	RFEM 5

Y_C [m]	Analytical Solution	RFEM 5	Ratio
Semicircle (wire)	0.637	0.636	0.998
Semicircle (plate)	0.424	0.424	1.000
Hemisphere (shell)	0.500	0.500	1.000
Hemisphere (volume)	0.375	0.375	1.000

References

[1] STEJSKAL, V., BŘEZINA, J. and KNĚZŮ, A. *Mechanika I*. Vydavatelství ČVUT v Praze, 2008.