Program: RFEM 5, RSTAB 8, RF-DYNAM Pro, DYNAM Pro

Category: Second-Order Analysis, Isotropic Linear Elasticity, Dynamics, Member

Verification Example: 0115 – Bending Vibrations with Axial Force

0115 – Bending Vibrations with Axial Force

Description

A cantilever of rectangular cross-section $a \times b$ has a mass m at its end. Furthermore, it is loaded by an axial force F_x according to **Figure 1**. Calculate the natural frequency of the structure. Neglect the self-weight of the cantilever and consider the influence of axial force for the stiffness modification. The problem is described by the following set of parameters.

Material	Steel	Modulus of Elasticity	E	210000.0	MPa
		Poisson's Ratio	ν	0.300	–
Geometry		Width	a	0.050	m
		Height	b	0.010	m
		Length	L	0.500	m
Load		Axial Force	F_x	1.000	kN
		Mass	m	25.000	kg

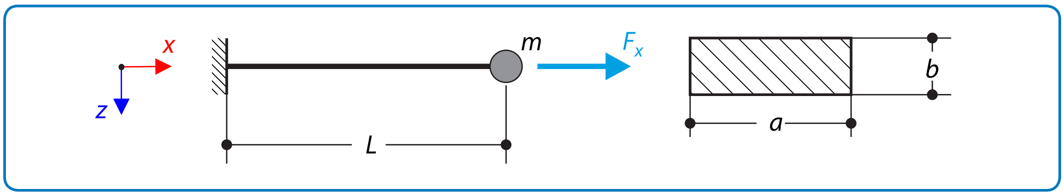

Figure 1: Problem Sketch

Analytical Solution

The axial force F_x influences the bending stiffness of the cantilever. Thus the natural bending frequency is also modified. The analytical solution of the bending stiffness is based on the solution presented in Verification Examples 0042 and 0048, [1], [2]. The deflection of the cantilever considering the effect of axial force is defined as follows

$$u_z = \frac{F_z \left(L\alpha e^{\alpha L} + L\alpha e^{-\alpha L} - e^{\alpha L} + e^{-\alpha L}\right)}{F_x \alpha \left(e^{\alpha L} + e^{-\alpha L}\right)} \qquad (115-1)$$

where F_z is the general transversal force, and α is defined as

$$\alpha = \sqrt{\frac{F_x}{EI_y}} \qquad (115-2)$$

The bending stiffness k evaluates then as

$$k = \frac{F_z}{u_z} = \frac{F_x \alpha \left(e^{\alpha L} + e^{-\alpha L}\right)}{L\alpha e^{\alpha L} + L\alpha e^{-\alpha L} - e^{\alpha L} + e^{-\alpha L}} \tag{115 – 3}$$

The natural frequency *f* of the single-mass oscillator is defined according to the well-known formula

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \approx 4.869 \text{ Hz} \tag{115 – 4}$$

RFEM 5 and RSTAB 8 Settings

- Modeled in RFEM 5.09.01 and RSTAB 8.09.01
- The element size is $l_{FE} = 0.010$ m
- The number of increments is 10
- Isotropic linear elastic model is used
- Root of the characteristic polynomial method is used

Results

Structure Files	Program	Entity
0115.01	RFEM 5 – RF-DYNAM Pro	Member
0115.02	RSTAB 8 – DYNAM Pro	Member
0115.03	RFEM 5 – RF-DYNAM Pro	Plate
0115.04	RFEM 5 – RF-DYNAM Pro	Solid

Model	Analytical Solution	RFEM 5 / RSTAB 8	
	f [Hz]	*f* [Hz]	Ratio [-]
RFEM 5, Beam	4.869	4.868	1.000
RSTAB 8, Beam		4.868	1.000
RFEM 5, Plate		4.887	1.004
RFEM 5, Solid		4.889	1.004

References

[1] DLUBAL SOFTWARE GMBH, *Verification Example 0042 – Bending Cantilever with Axial Force*. 2015.

[2] DLUBAL SOFTWARE GMBH, *Verification Example 0048 – Uniaxial Bending with Pressure*. 2016.