0118 – Mathematical Pendulum

Description

The mathematical pendulum consists of a zero-weight rope and a mass point at its end. The pendulum is initially deflected by angle $\varphi(0) = \varphi_0$. Determine the angle $\varphi(t)$ of the rope at given test time t. The problem is shown in **Figure 1** and it is described by the following set of parameters.

System Properties	Mass	m	50.000	kg
	Cable Length	L	1.414	m
	Initial Angle	φ_0	$\pi/4$	rad
	Gravitational Acceleration	g	9.810	ms^{-2}

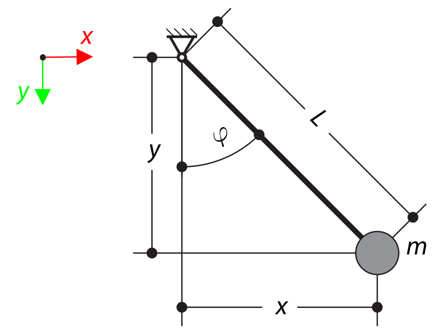

Figure 1: Problem Sketch

Analytical Solution

The problem can be solved by means of the Lagrange equations of the second kind

$$\frac{\partial}{\partial t}\frac{\partial E_k(q,\dot{q},t)}{\partial \dot{q}} - \frac{\partial E_k(q,\dot{q},t)}{\partial q} = Q, \tag{118 – 1}$$

where $E_k = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2\right)$ is the kinetic energy, $q = (x, y)$ is the generalized coordinate, and Q is the sum of the generalized forces. The dot denotes the time derivative. In this case, the kinetic energy is defined for the mass point in the directions x and y. It is convenient to choose the polar angle $\varphi(t)$ for the generalized coordinate q, so there is only one variable. Considering the following relations for the velocities $\dot{x}$ and $\dot{y}$,

$$x = L\sin\varphi \quad \Rightarrow \quad \dot{x} = L\dot{\varphi}\cos\varphi, \tag{118 – 2}$$

$$y = L\cos\varphi \quad \Rightarrow \quad \dot{y} = -L\dot{\varphi}\sin\varphi, \tag{118 – 3}$$

the kinetic energy can be expressed as

$$E_k = \frac{1}{2} m L^2 \dot{\varphi}^2. \tag{118 – 4}$$

The generalized force Q can be determined by the principle of virtual works. The variation δy is defined analogously to the above calculated velocity $\dot{y}$,

$$\delta y = -L \delta\varphi \sin\varphi, \tag{118 – 5}$$

that is,

$$Q\delta\varphi = -mg\delta y = -mgL \sin\varphi\delta\varphi. \tag{118 – 6}$$

The generalized force Q is then

$$Q = -mgL \sin\varphi. \tag{118 – 7}$$

Substituting **(118 – 4)** and **(118 – 7)** into the Lagrange equation **(118 – 1)**, the following motion equation is obtained

$$\ddot{\varphi} = \frac{g}{L} \sin\varphi. \tag{118 – 8}$$

This is a non–linear second–order differential equation, which is further solved numerically, for example, by the Runge–Kutta method.

For the small deflections, it could be linearized as follows

$$\sin\varphi \approx \varphi \quad \Rightarrow \quad \ddot{\varphi} = \frac{g}{L}\varphi. \tag{118 – 9}$$

RFEM 5 and RSTAB 8 Settings

- Modeled in RFEM 5.17.01 and RSTAB 8.17.01

Results

Structure Files	Program	Solution Method
0118.01	RFEM 5 – RF-DYNAM Pro	Explicit analysis
0118.02	RFEM 5 – RF-DYNAM Pro	Nonlinear implicit Newmark analysis
0118.03	RSTAB 8 – DYNAM Pro	Explicit analysis, Large Deformation Analysis

The comparison of the analytical solution with RFEM 5 and RSTAB 8 solutions can be seen in **Figure 2**. The results at test time $t_1 = 0.5$ s and $t_2 = 2$ s follows.

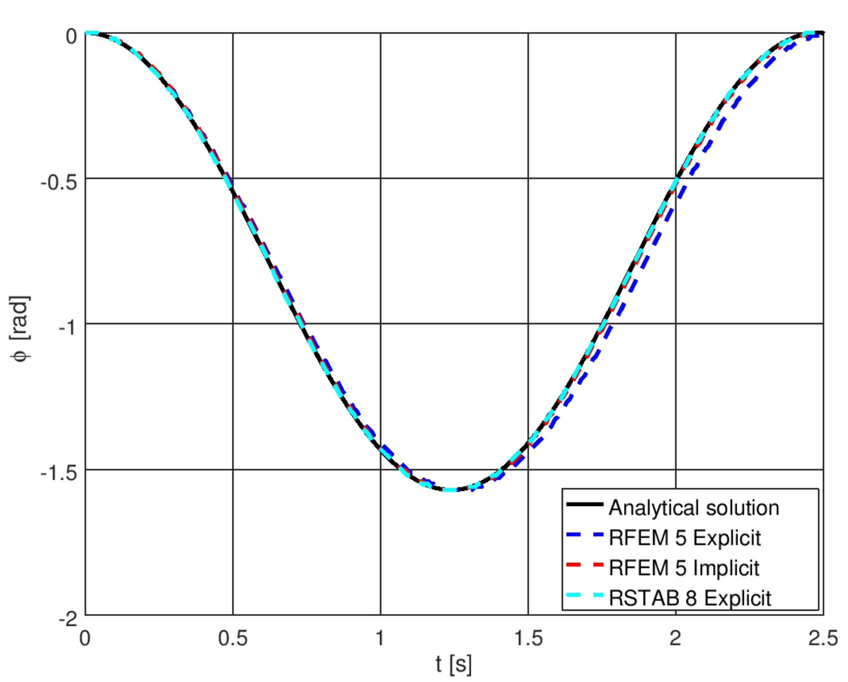

Figure 2: Analytical and RFEM 5 / RSTAB 8 solution

Model	Analytical Solution	RFEM 5 / RSTAB 8	
	$\varphi(0.5)$ [rad]	$\varphi(0.5)$ [rad]	Ratio [-]
RFEM 5, Explicit analysis	-0.547	-0.529	0.967
RFEM 5, Nonlinear implicit Newmark analysis		-0.537	0.982
RSTAB 8, Explicit analysis, Large Deformation Analysis		-0.547	1.000

Model	Analytical Solution	RFEM 5 / RSTAB 8	
	$\varphi(2)$ [rad]	$\varphi(2)$ [rad]	Ratio [-]
RFEM 5, Explicit analysis	-0.510	-0.585	1.147
RFEM 5, Nonlinear implicit Newmark analysis		-0.521	1.022
RSTAB 8, Explicit analysis , Large Deformation Analysis		-0.511	1.002