Verification Example

Program: RFEM 5, RSTAB 8

Category: Geometrically Linear Analysis, Second-Order Analysis, Isotropic Linear Elasticity, Member

Verification Example: 0209 – Eccentric Axial Force

0209 – Eccentric Axial Force

Description

A console made of round bar of diameter d is loaded by means of eccentric axial force F_x according to **Figure 1**. Determine the maximal vertical deflection of the console $u_{z,\max}$ using geometrically linear and second-order analysis. The problem is described by the following set of parameters.

Material	Steel	Modulus of Elasticity	E	210000.000	MPa
		Poisson's Ratio	ν	0.300	–
Geometry		Length	L	1.000	m
		Diameter	d	20.000	mm
		Eccentricity	e_z	0.250	m
Load		Axial Force	F_x	1.000	kN

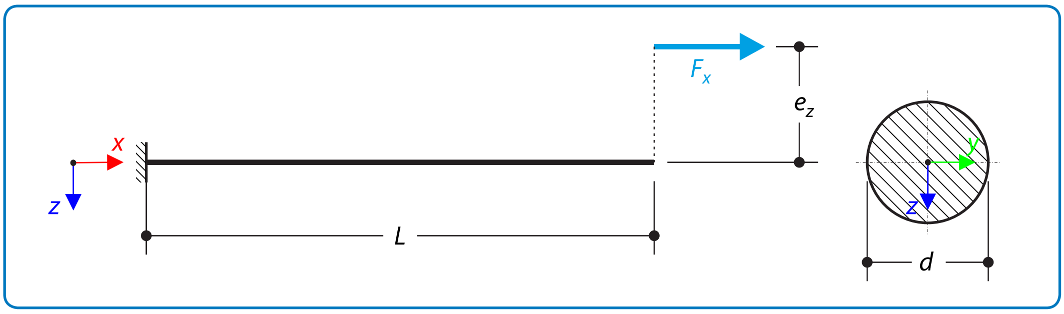

Figure 1: Problem sketch

Analytical Solution

Geometrically Linear Analysis

Considering geometrically linear analysis, the console is loaded by means of constant bending moment

$$M_y = F_x e_z. \tag{209 – 1}$$

The deflection of the console tip is in this case defined by the following simple formula

$$u_{z,\max} = \frac{F_x e_z L^2}{2EI_y} \approx 75.788 \text{ mm}, \tag{209 – 2}$$

where I_y is the moment of inertia of the circular cross-section.

Second-Order Analysis

In case of second-order analysis, the deflection of the console has to be taken into account. Thus, the bending moment has the following form

$$M_y = F_x \left(u_{z,\text{max}} - u_z(x) - e_z \right), \tag{209 – 3}$$

where $u_{z,\text{max}} = u_z(L)$. The deflection $u_z(x)$ can be determined by means of the Euler-Bernoulli differential equation

$$\frac{\mathrm{d}^2 u_z(x)}{\mathrm{d}x^2} = -\frac{M_y(x)}{EI_y}. \tag{209 – 4}$$

Considering the bending moment from **(209 – 3)** and substituting

$$\alpha = \sqrt{\frac{F_x}{EI_y}}, \tag{209 – 5}$$

(209 – 4) can be written as

$$\frac{\mathrm{d}^2 u_z(x)}{\mathrm{d}x^2} - \alpha^2 u_z(x) = \alpha^2 (e_z - u_{z,\text{max}}). \tag{209 – 6}$$

This equation is completed by the boundary conditions for the fixed end of the console

$$u_z(0) = 0, \tag{209 – 7}$$

$$\frac{\mathrm{d}u_z(0)}{\mathrm{d}x} = 0. \tag{209 – 8}$$

The general solution of **(209 – 6)** is then

$$u_z(x) = (e_z - u_{z,\text{max}}) \left(\frac{e^{\alpha x} + e^{-\alpha x}}{2} + 1 \right). \tag{209 – 9}$$

Hence, the deflection of the tip (at $x = L$) reads as

$$u_{z,\text{max}} = \frac{e_z \left(e^{\alpha x} + e^{-\alpha x} - 2 \right)}{e^{\alpha x} + e^{-\alpha x}} \approx 60.431 \text{ mm}. \tag{209 – 10}$$

RFEM 5 and RSTAB 8 Settings

- Modeled in RFEM 5.19.01 and RSTAB 8.19.01
- The element size is $l_{FE} = 0.010$ m
- Isotropic linear elastic material model is used

Results

Structure Files	Program	Analysis	Details
0209.01	RFEM 5	Geometrically Linear	Eccentric Member Load
0209.02	RSTAB 8	Geometrically Linear	Rigid Member
0209.03	RFEM 5	Second-Order	Eccentric Member Load
0209.04	RSTAB 8	Second-Order	Rigid Member

Geometrically Linear Analysis

Model	Analytical Solution	RFEM 5 / RSTAB 8	
	$u_{z,\mathrm{max}}$ [mm]	$u_{z,\mathrm{max}}$ [mm]	Ratio [-]
RFEM 5, Eccentric Member Load	75.788	75.788	1.000
RSTAB 8, Rigid Member		75.788	1.000

Second-Order Analysis

Model	Analytical Solution	RFEM 5 / RSTAB 8	
	$u_{z,\mathrm{max}}$ [mm]	$u_{z,\mathrm{max}}$ [mm]	Ratio [-]
RFEM 5, Eccentric Member Load	60.431	60.406	1.000
RSTAB 8, Rigid Member		60.431	1.000