DESIGN AND ANALYSIS OF MEMBRANE STRUCTURES IN FEM-BASED SOFTWARE

MASTER THESIS

ARCHINEER® INSTITUTES FOR MEMBRANE AND SHELL TECHNOLOGIES, BUILDING AND REAL ESTATE e.V.
ANHALT UNIVERSITY OF APPLIED SCIENCES

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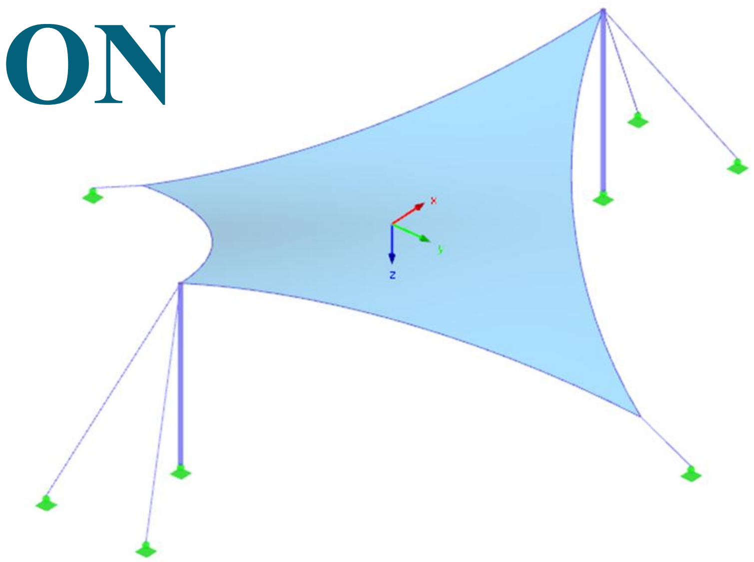
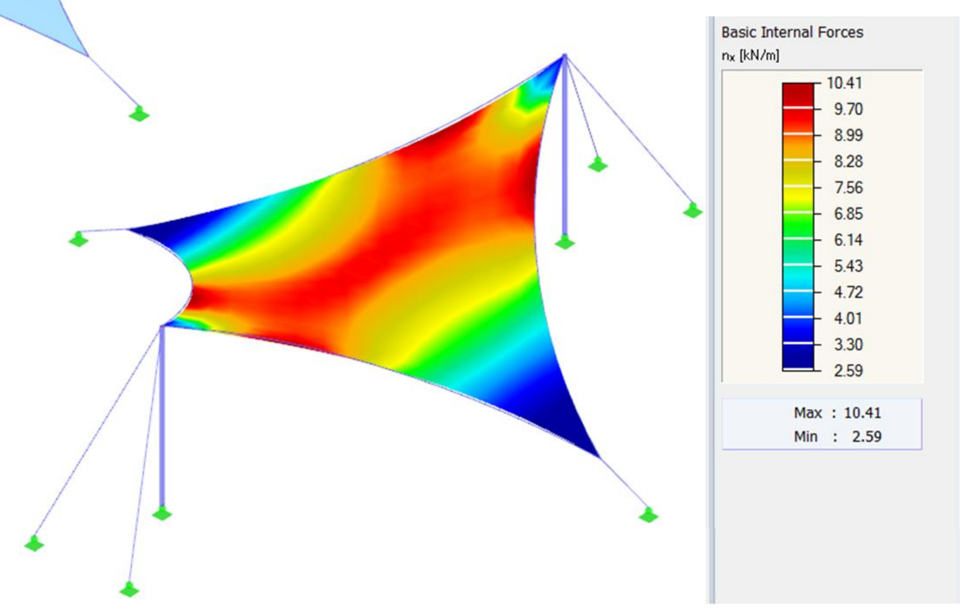
SEPTEMBER 2017
DESSAU-ROSSLAU
GERMANY

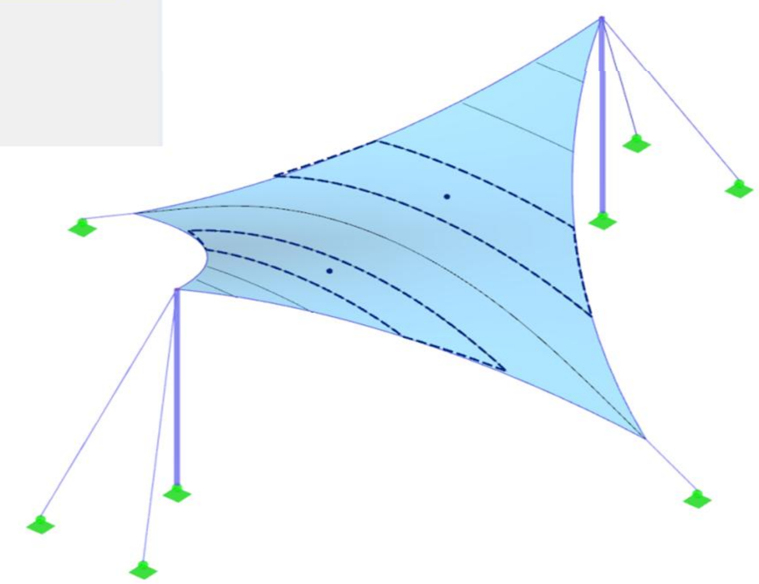
INTRODUCTION



FORM-FINDING



STRUCTURAL ANALYSIS



GENERATION OF CUTTING PATTERNS

Note: the presented work follows the software development focused on membrane structures performed by cooperating companies Dlubal Software s.r.o. and FEM Consulting s.r.o. The presented examples were created in the RFEM software.

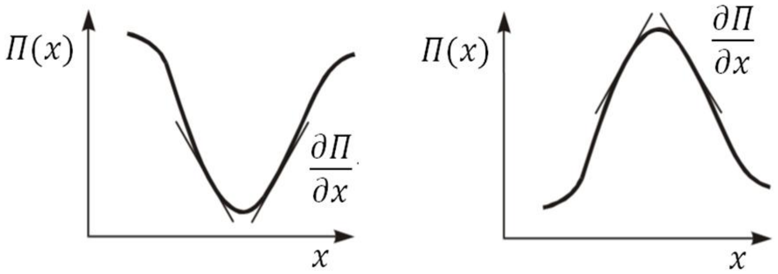
FORM-FINDING

Shape Analysis of Structures

- The shape is given by the equilibrium of forces and boundary conditions

Different Methods

- Force Density Method
- Dynamic Relaxation
- Updated Reference Strategy
- Natural Force Density Method
- Etc.

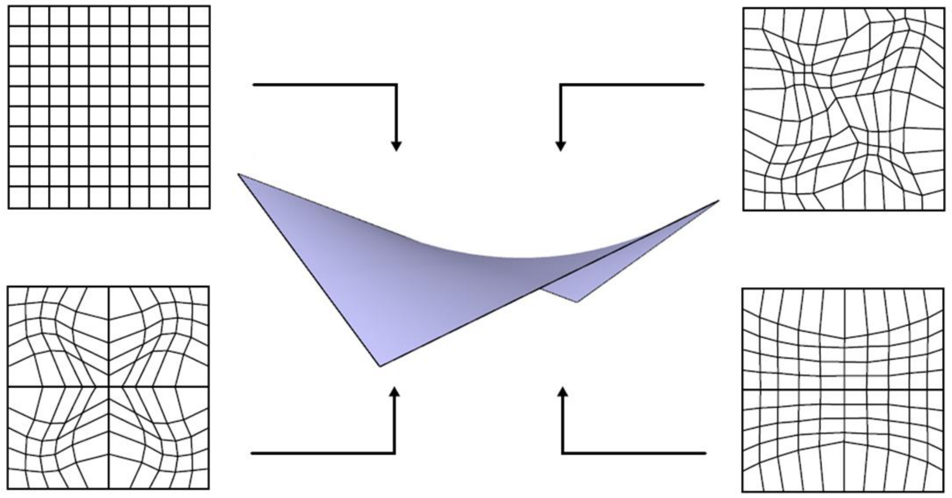


Stable and unstable equilibrium position

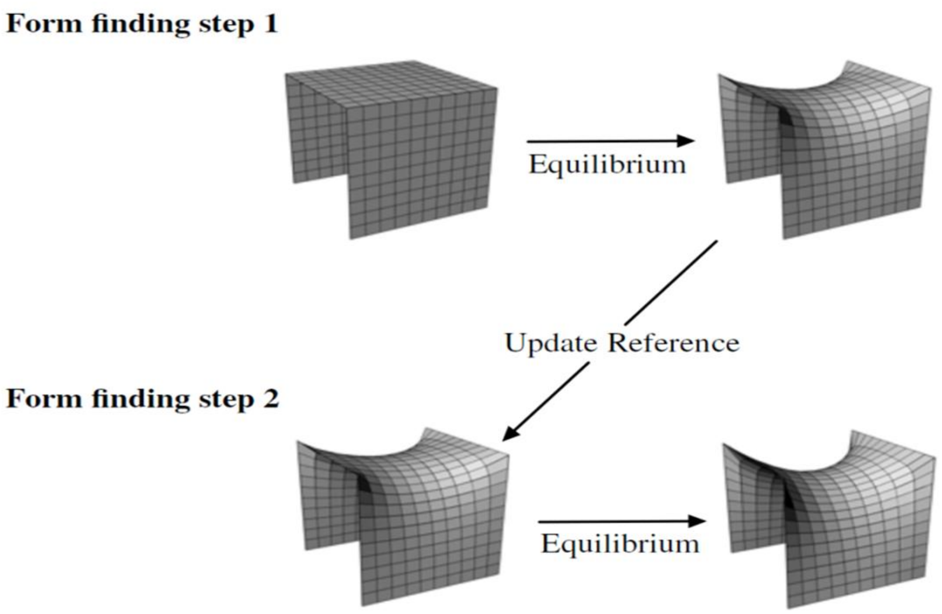
General Finite Element Approach

- Direct solution of the shape is not possible
- Nonlinear calculation is necessary for reaching the shape with given prestress
- Any load can be a part of the form-finding

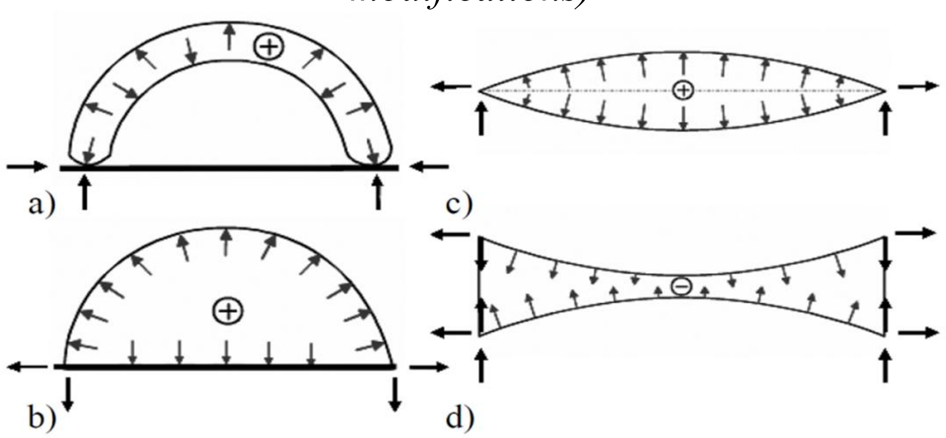
$$\frac{\partial \Pi}{\partial d} = \frac{\partial \Pi^{int}}{\partial d} + \frac{\partial \Pi^{ext}}{\partial d} = \int_{\Omega_0} S : \delta E \, d\Omega_0 - \int_{\Omega_0} q \cdot \delta d \, d\Omega_0 = \int_{\Omega} \sigma : \delta e \, d\Omega - \int_{\Omega} q \cdot \delta d \, d\Omega = 0$$



Arbitrarily deformed meshes for the same surface geometry ([6] with modification)



Iterations of the form-finding process ([20] with modifications)

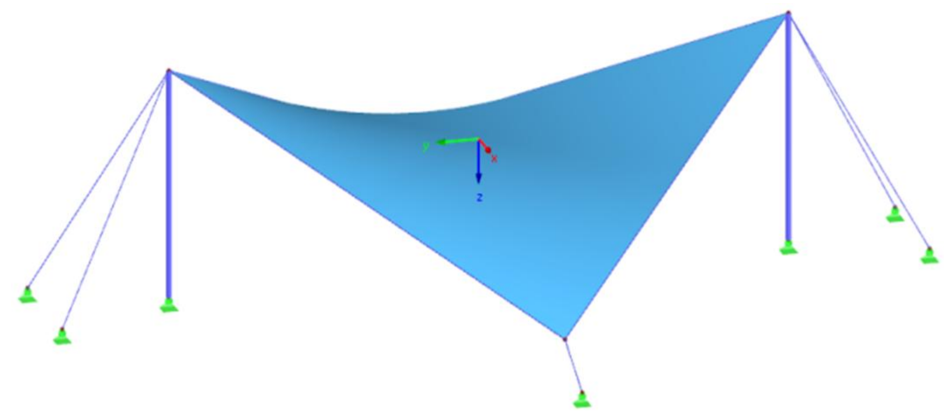


Pneumatic prestressed and stabilized structures (+ overpressure, - low pressure) [21]

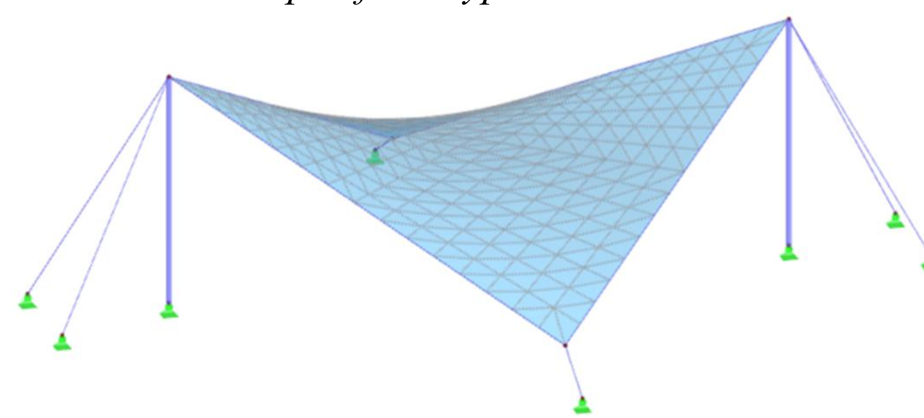
FORM-FINDING

Hypar Structure

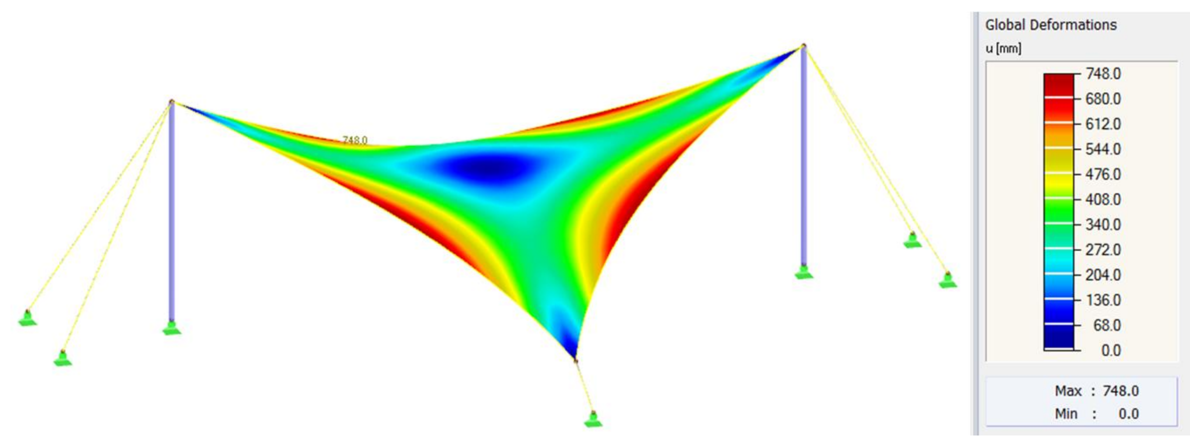
- The isotropic prestress
- The independence on the initial shape



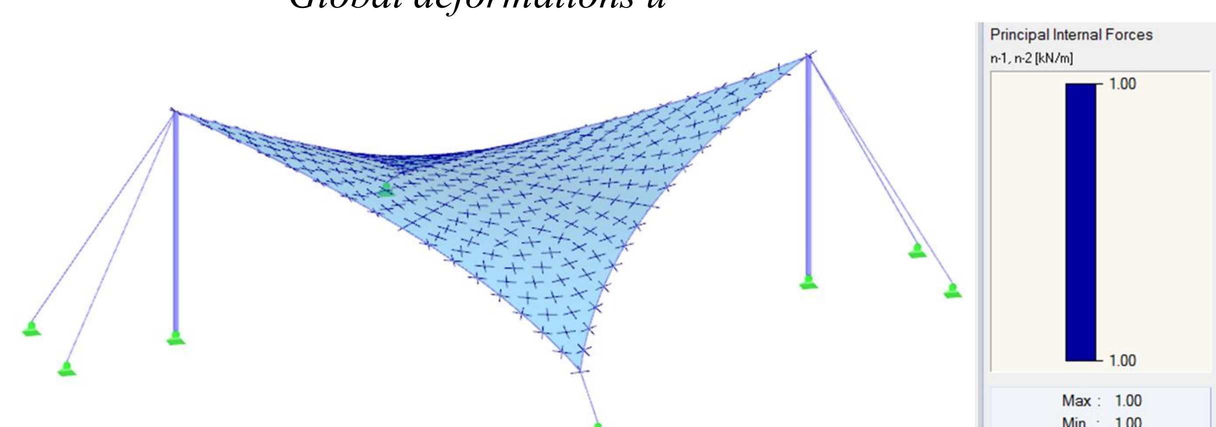
Initial shape of the hypar membrane structure



FE mesh

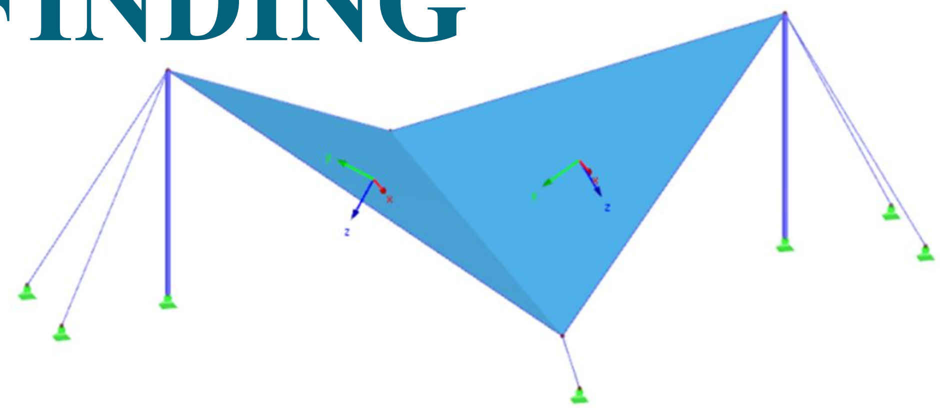


Global deformations u

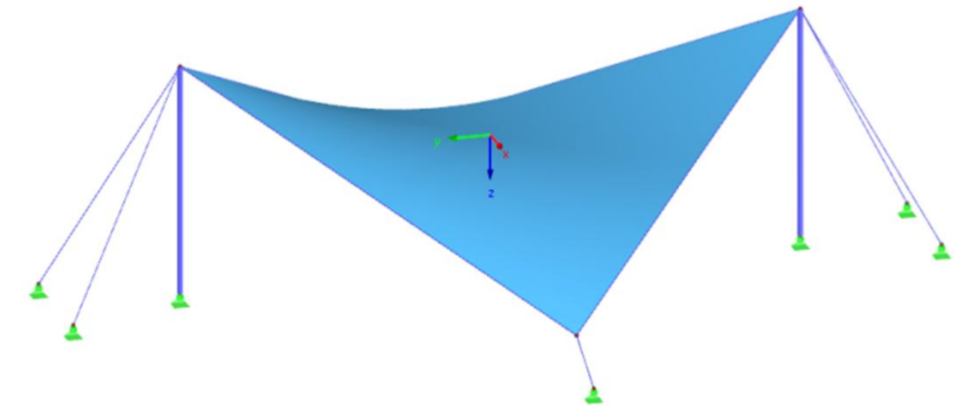


Vectors of the principal internal forces n_1 and n_2

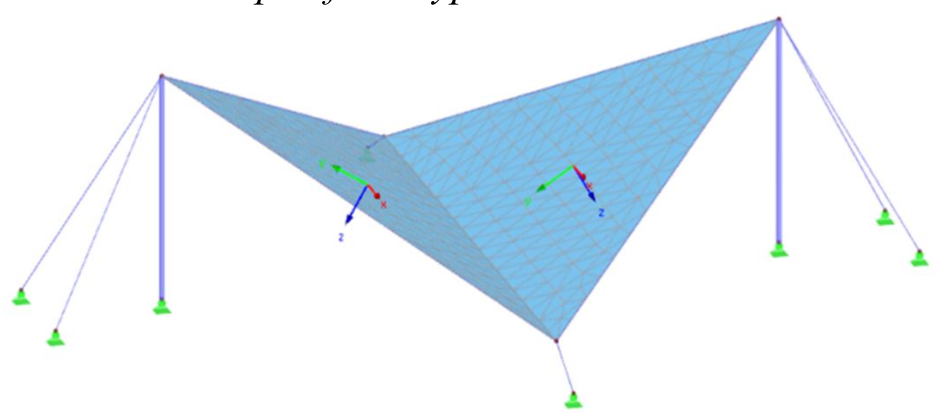
FORM-FINDING



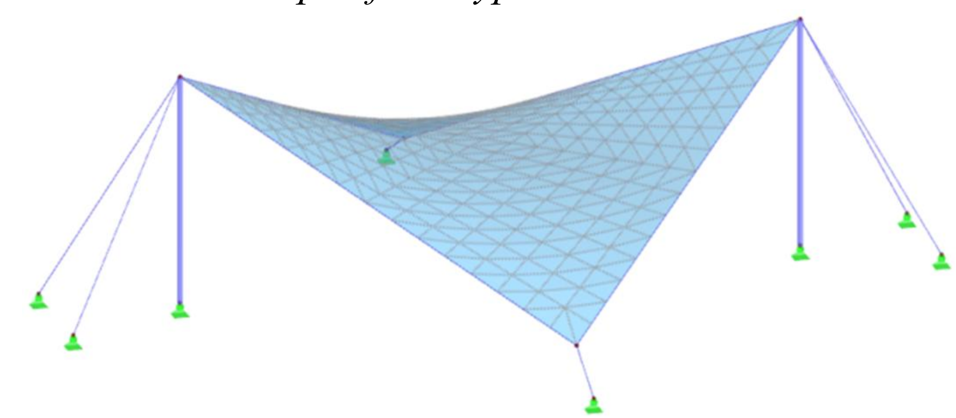
Initial shape of the hypar membrane structure



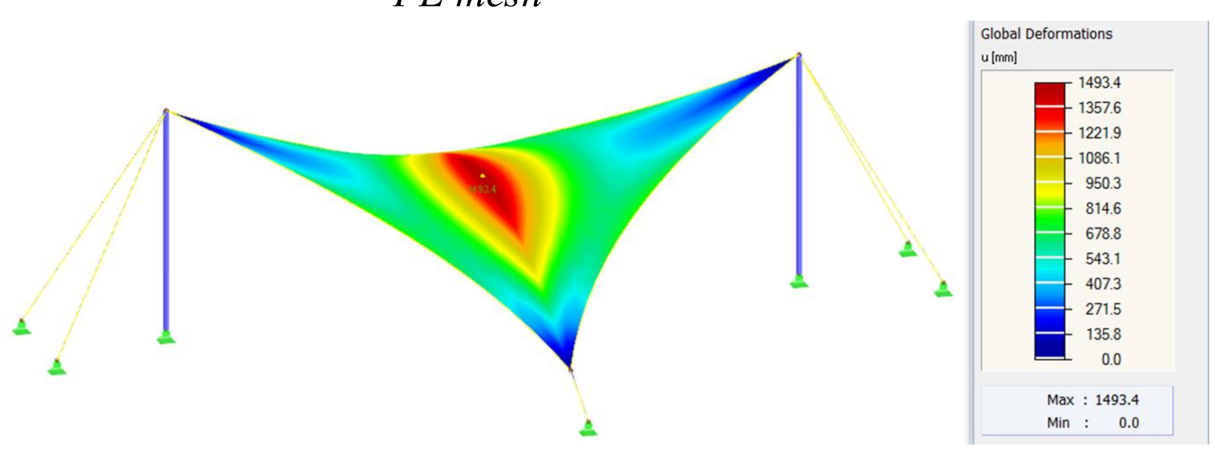
Initial shape of the hypar membrane structure



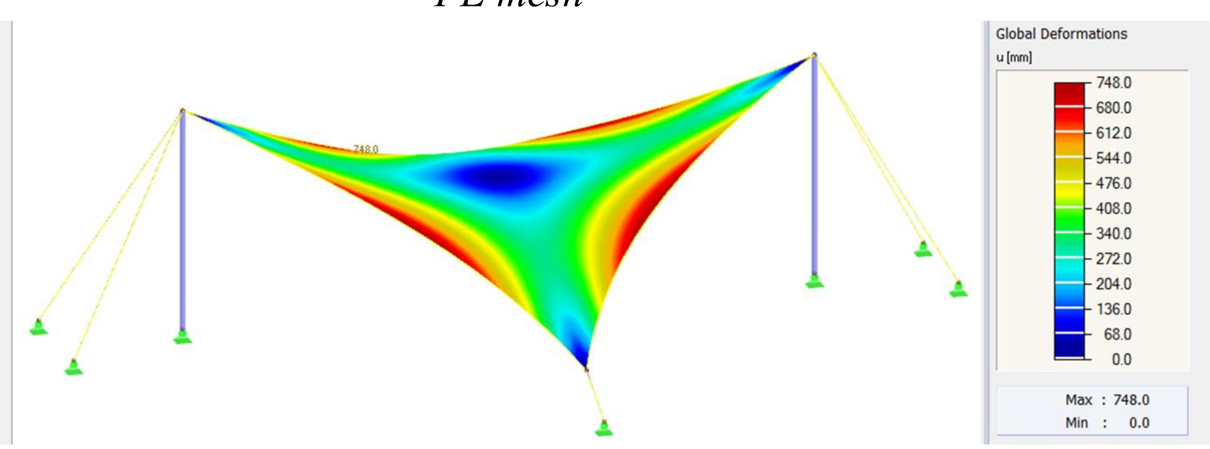
FE mesh



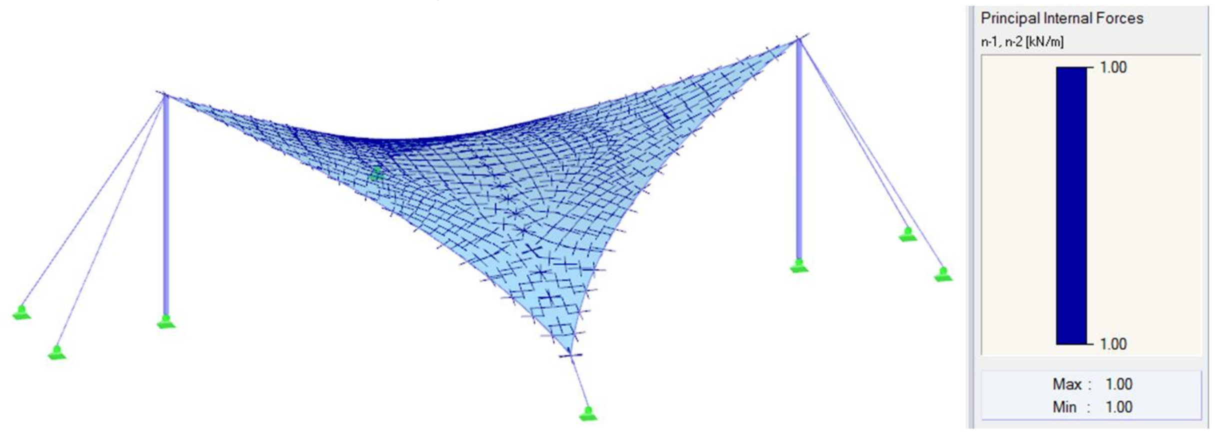
FE mesh



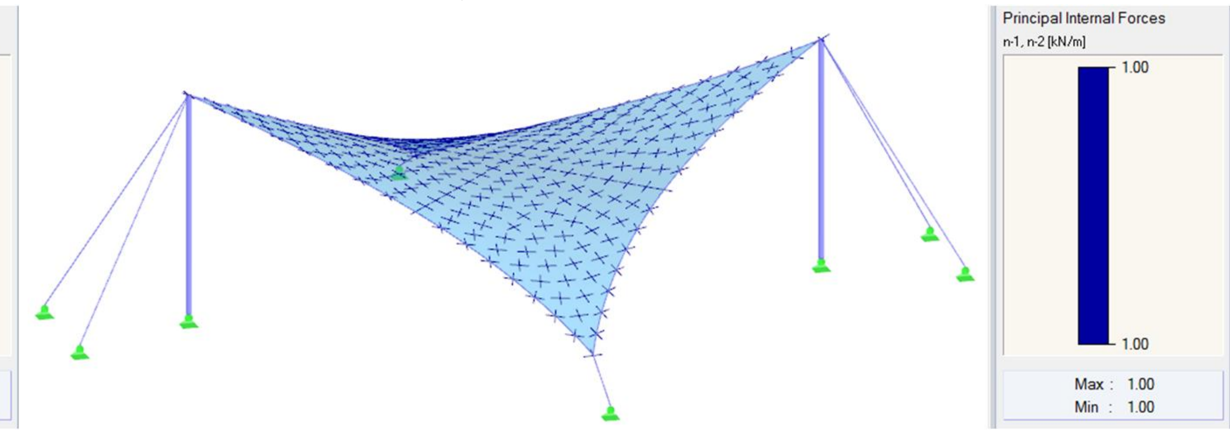
Global deformations u



Global deformations u



Vectors of the principal internal forces n_1 and n_2

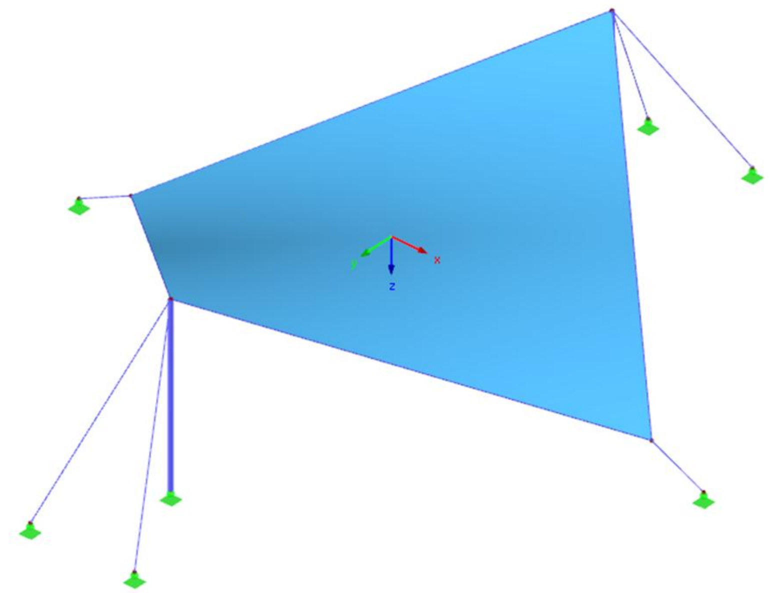


Vectors of the principal internal forces n_1 and n_2

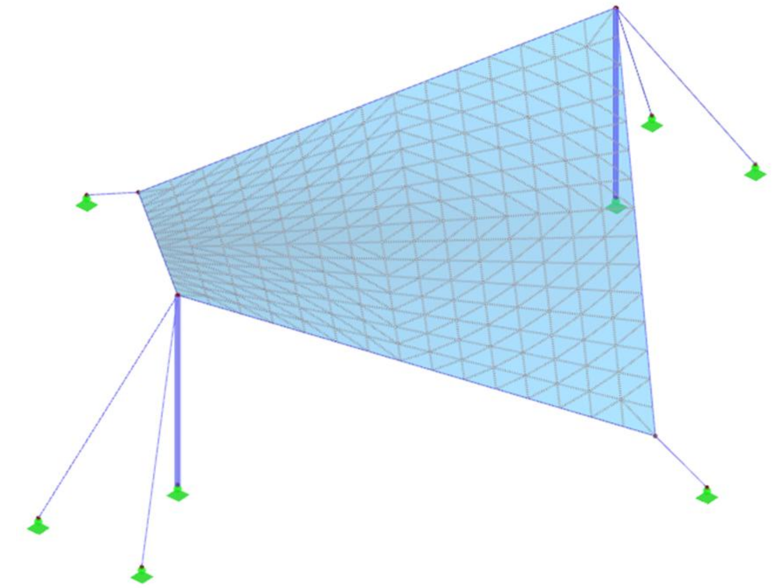
FORM-FINDING

Hypar Structure

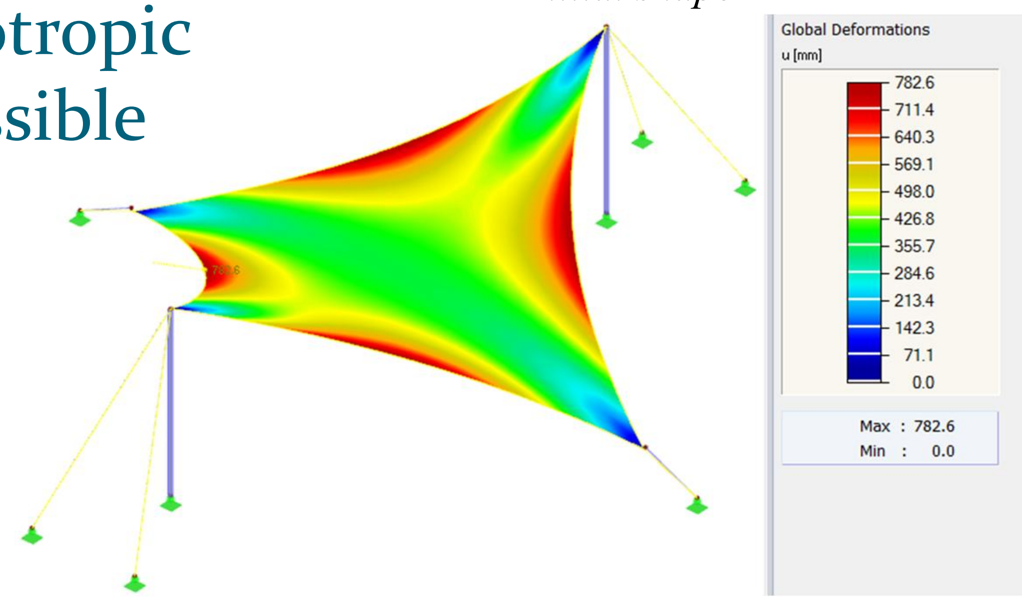
- The isotropic prestress
- The independence on the initial shape
- The constant orthotropic prestress is not possible



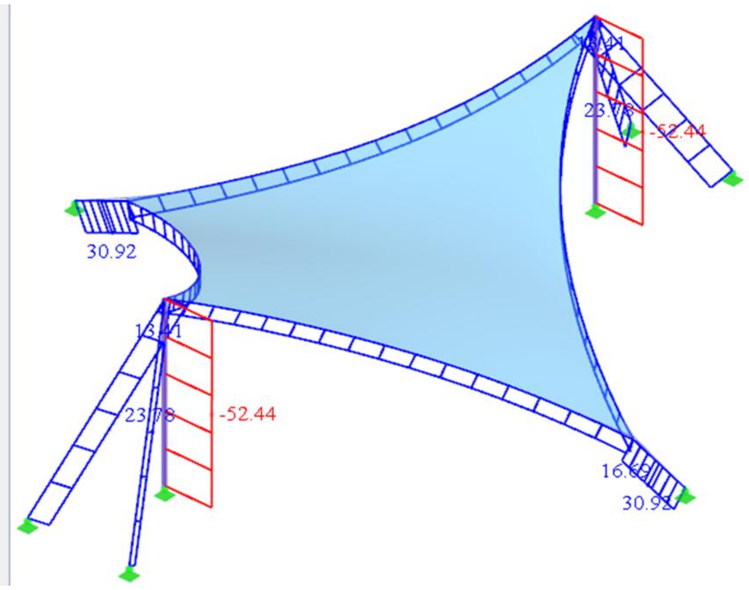
Initial shape



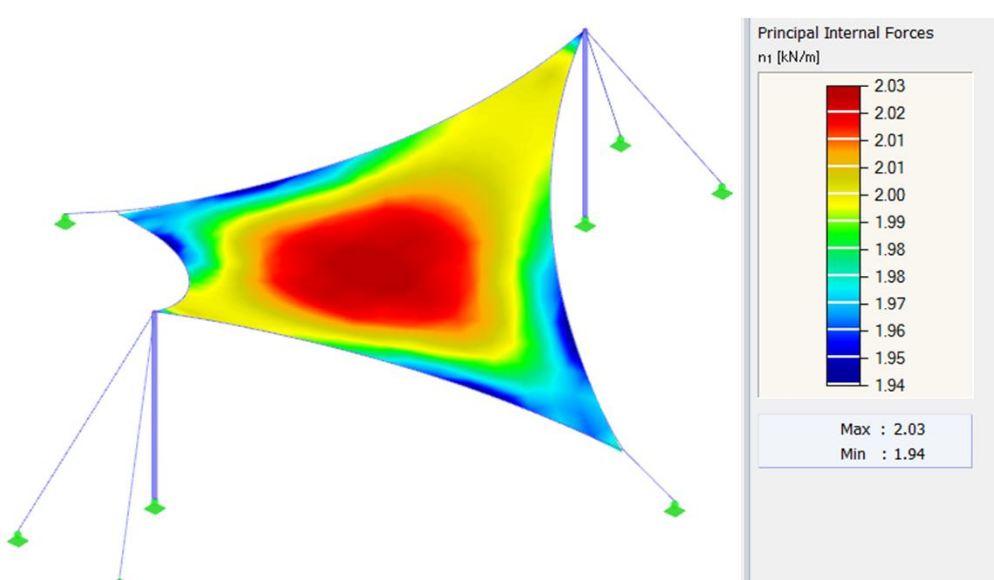
FE mesh (right)



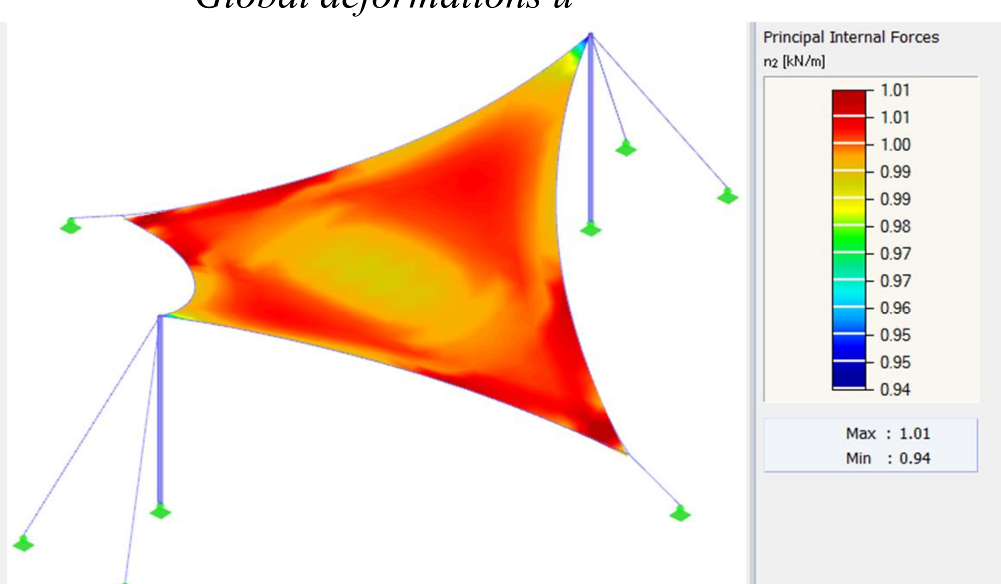
Global deformations u



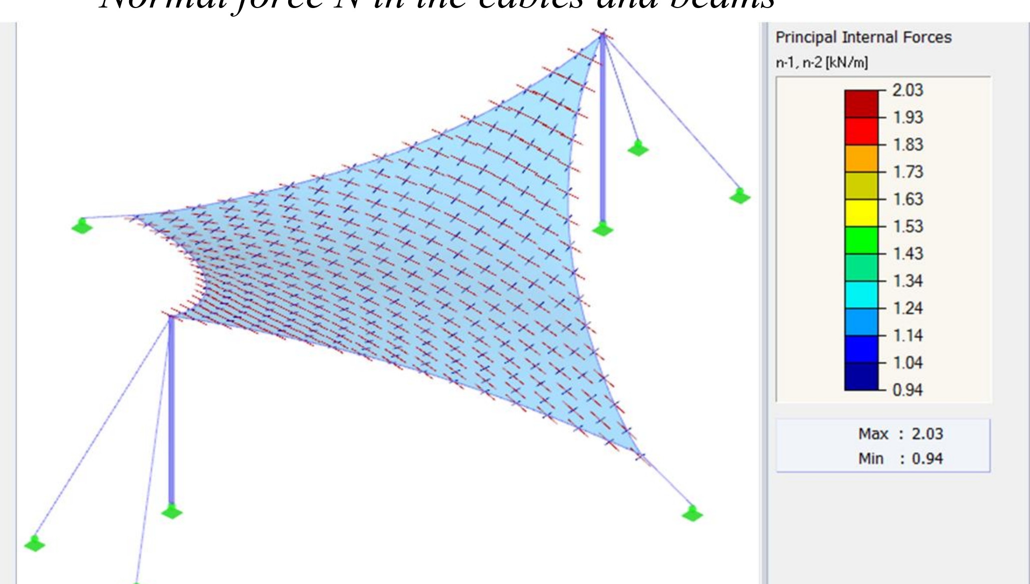
Normal force N in the cables and beams



Principal internal forces n_1



Principal internal forces n_2



Vectors of the principal internal forces n_1 and n_2

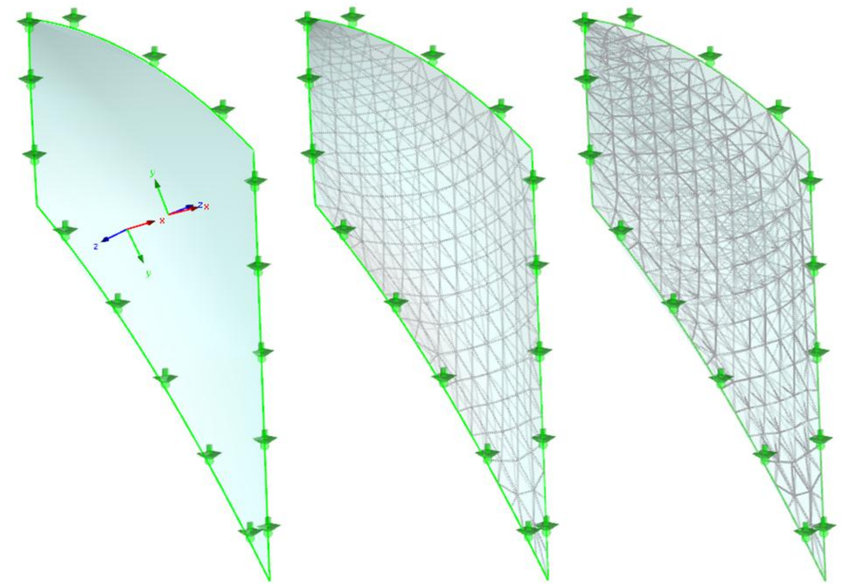
FORM-FINDING

Hypar Structure

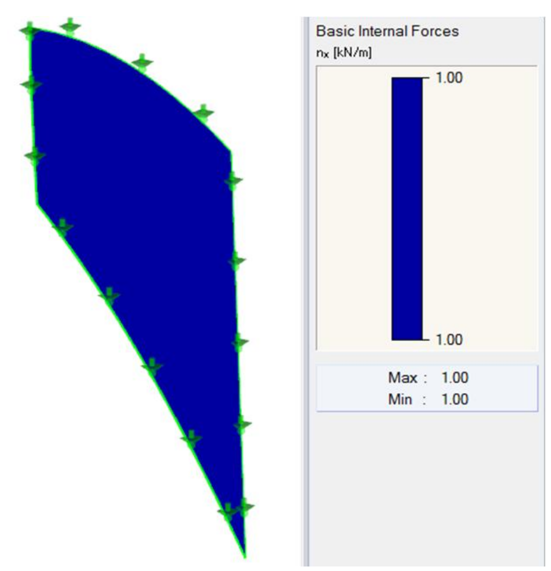
- The isotropic prestress
- The independence on the initial shape
- The constant orthotropic prestress is not possible

ETFE Cushion

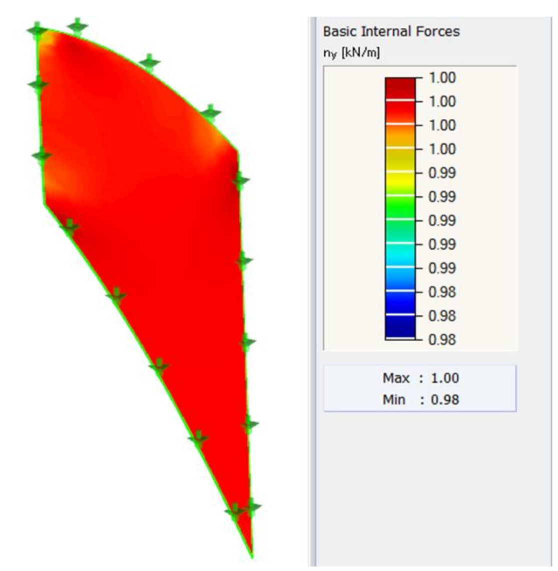
- Prestress, overpressure and boundary conditions



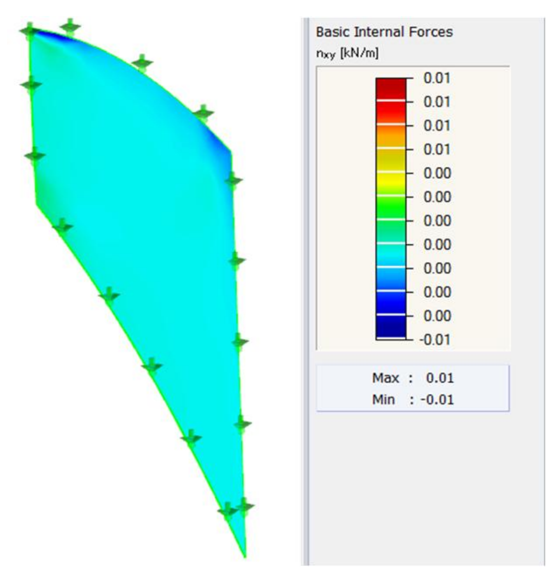
ETFE cushion (left), FE mesh of the layers (middle) and the air chamber (right)



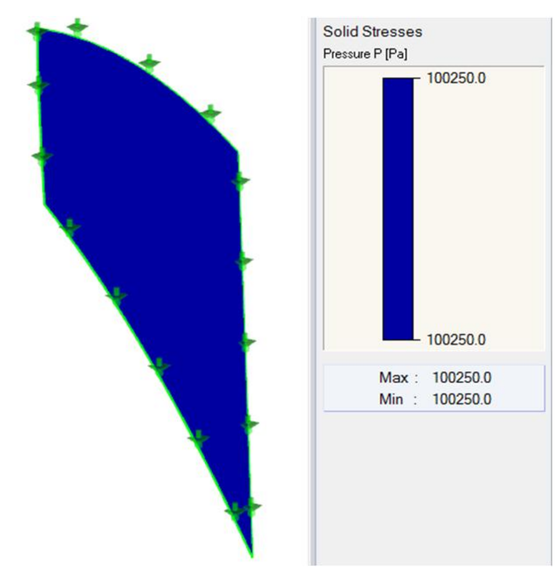
Basic internal forces n_x



Basic internal forces n_y



Basic internal forces n_{xy}



Total pressure $p = p_a + p_0$ (p_a...atmospheric pressure, p_0...overpressure)

FORM-FINDING

Hypar Structure

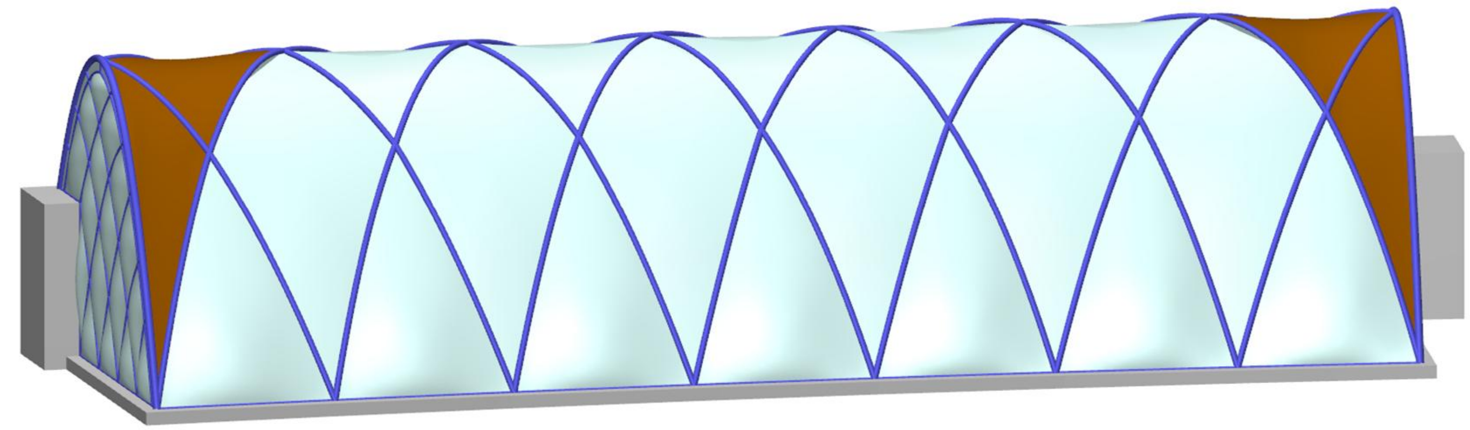
- The isotropic prestress
- The independence on the initial shape
- The constant orthotropic prestress is not possible

ETFE Cushion

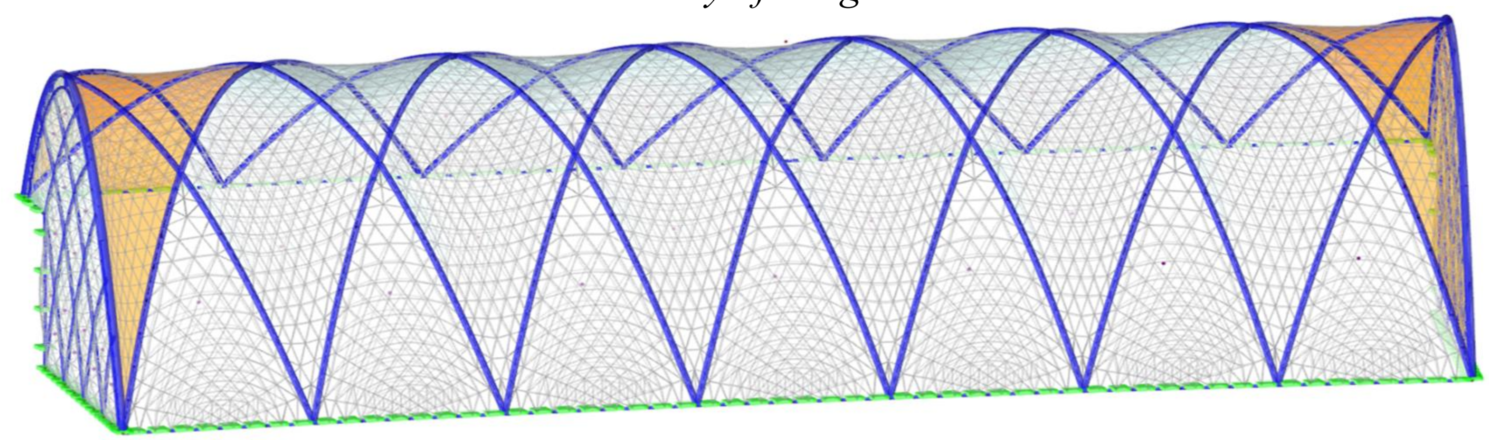
- Prestress, overpressure and boundary conditions

Pneumatic Structure

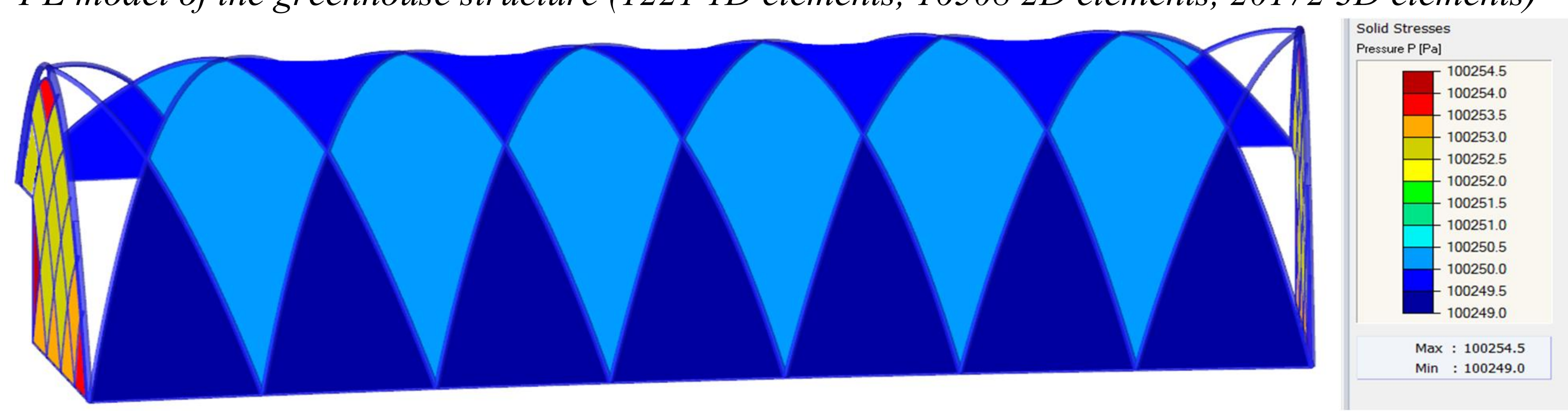
- Combination of 1D/2D/3D elements



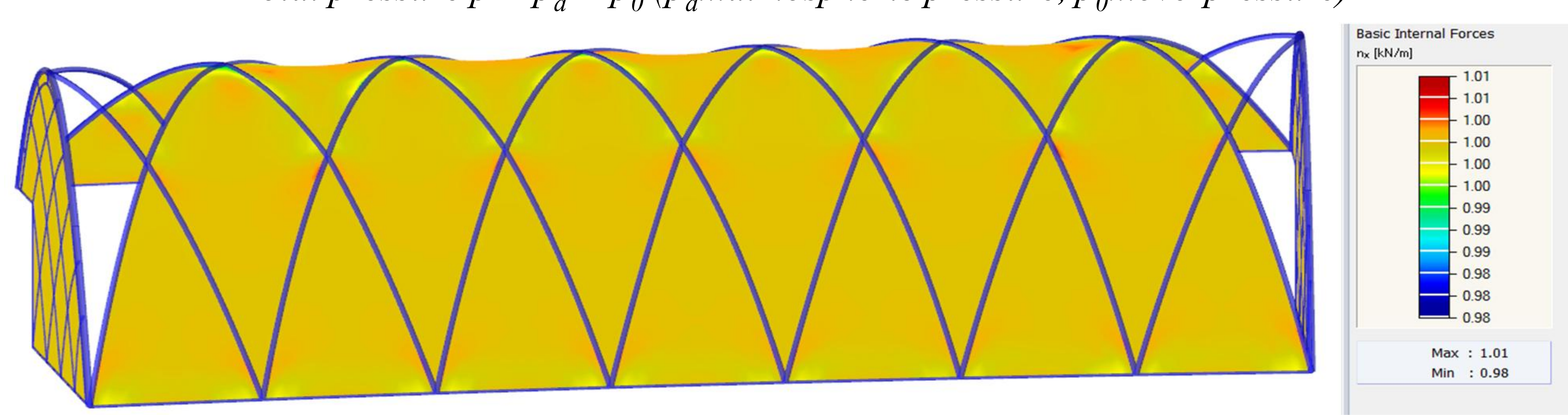
Geometry of the greenhouse structure



FE model of the greenhouse structure (1221 1D elements, 16508 2D elements, 20172 3D elements)



Total pressure $p = p_a + p_0$ (p_a...atmospheric pressure, p_0...overpressure)

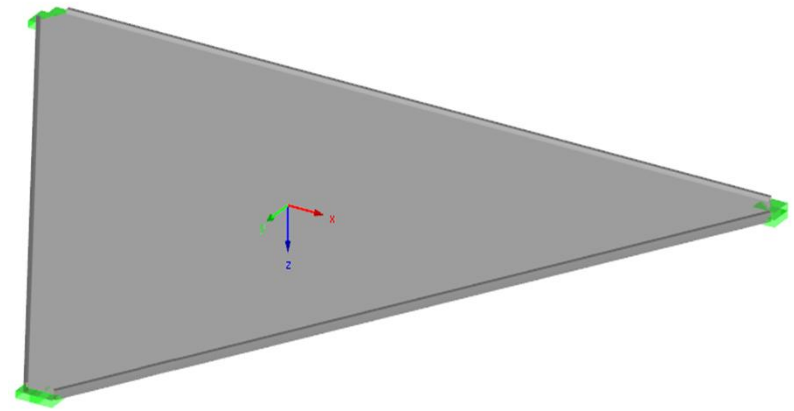


Basic internal forces n_x in the ETFE layers

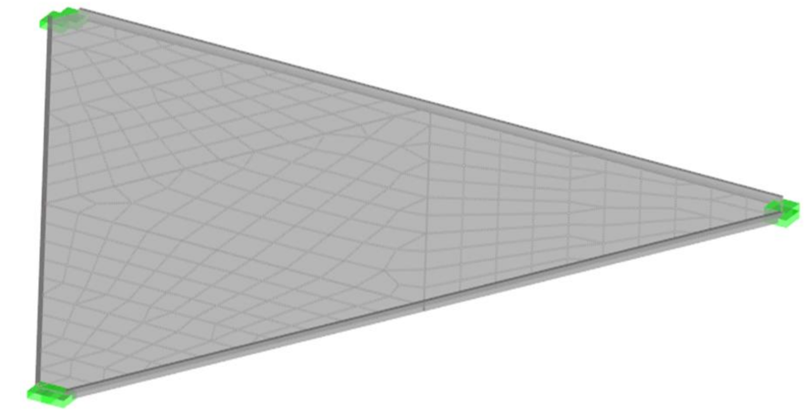
FORM-FINDING

Shell Structure

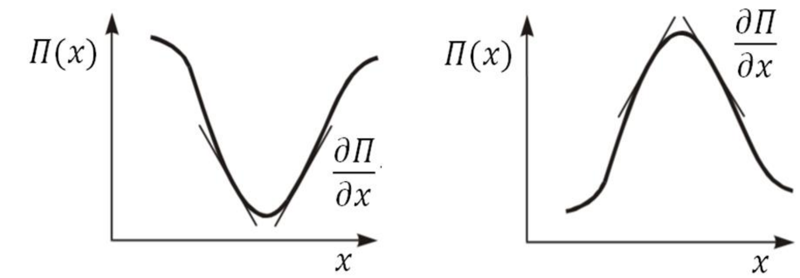
- Compression, self weight and boundary conditions



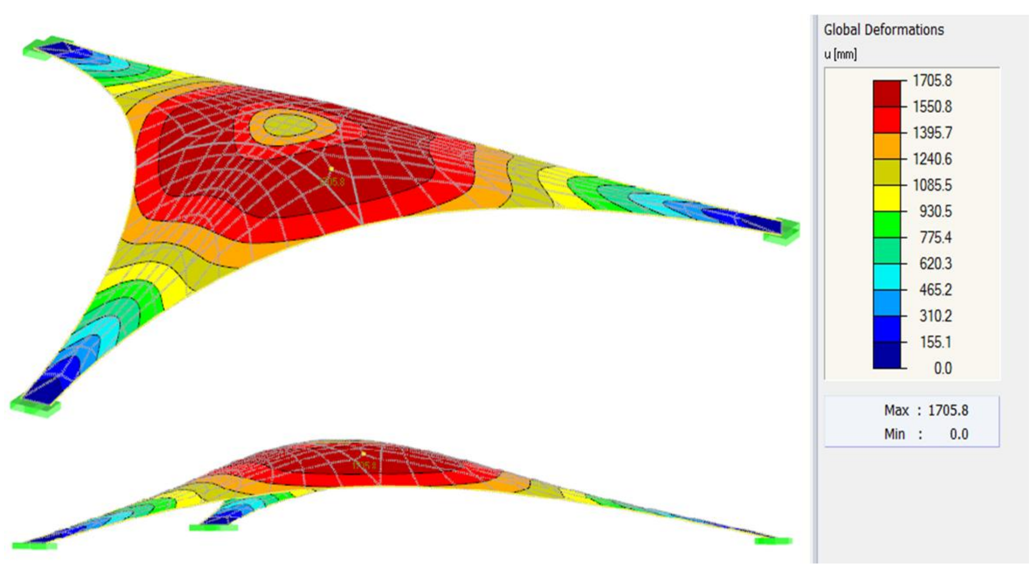
Initial shape of a shell



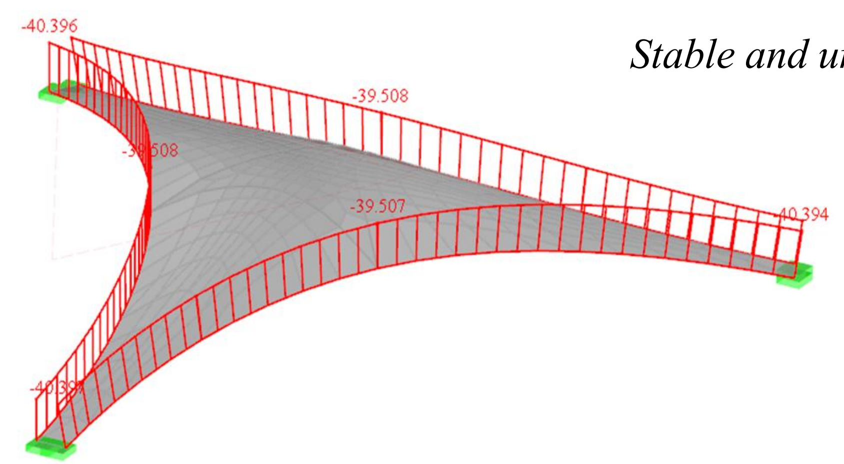
FE mesh



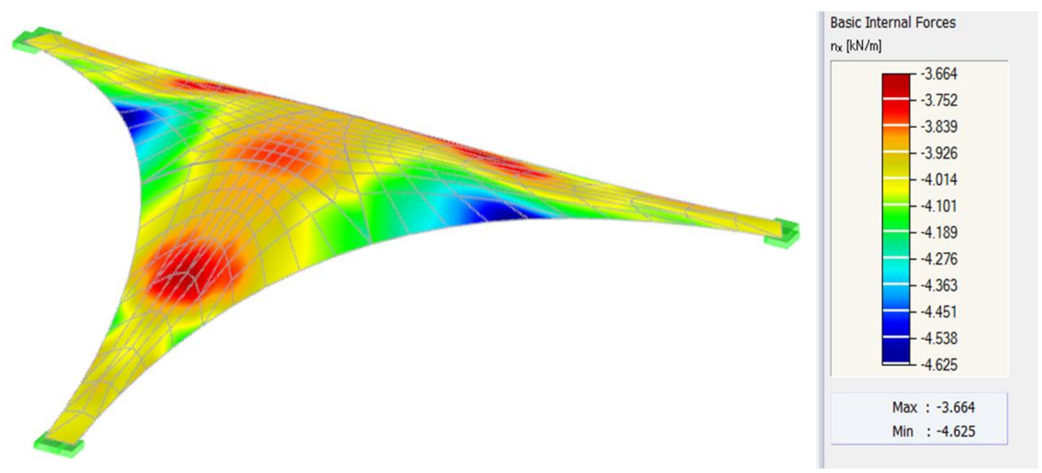
Stable and unstable equilibrium position



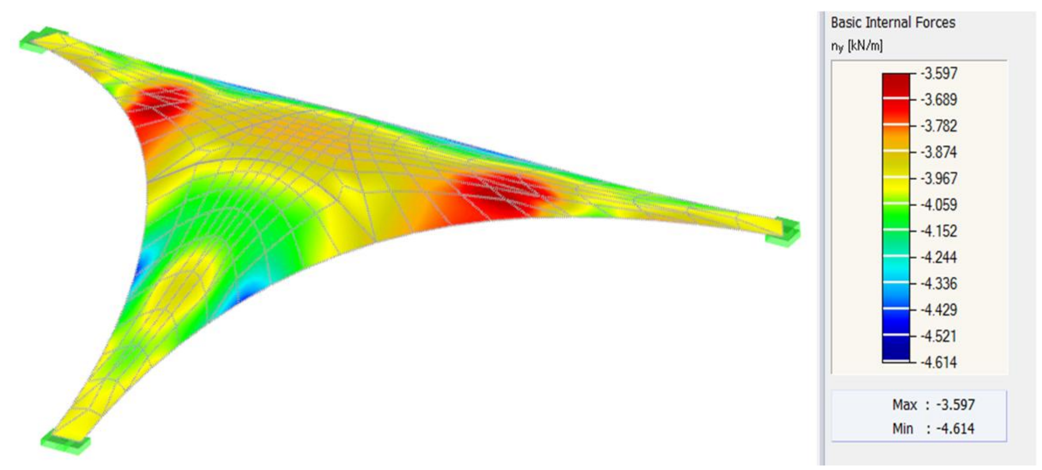
Two views of global deformations u in the form-finding



Normal forces N



Basic internal forces n_x



Basic internal forces n_y

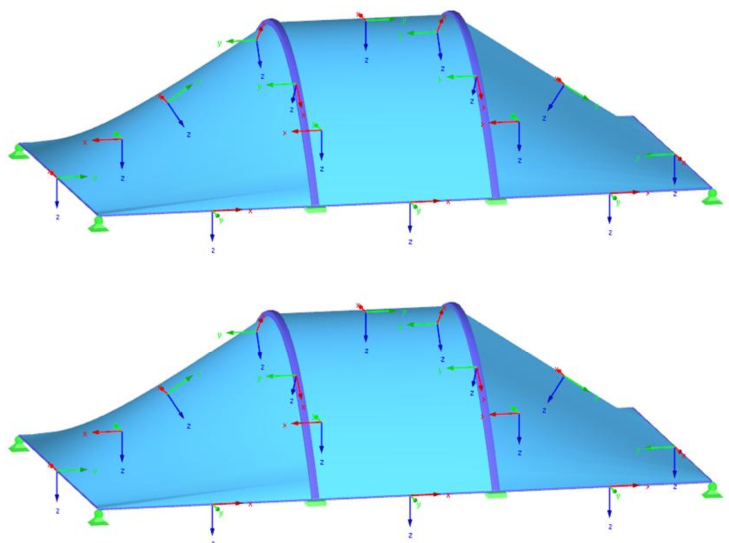
FORM-FINDING

Shell Structure

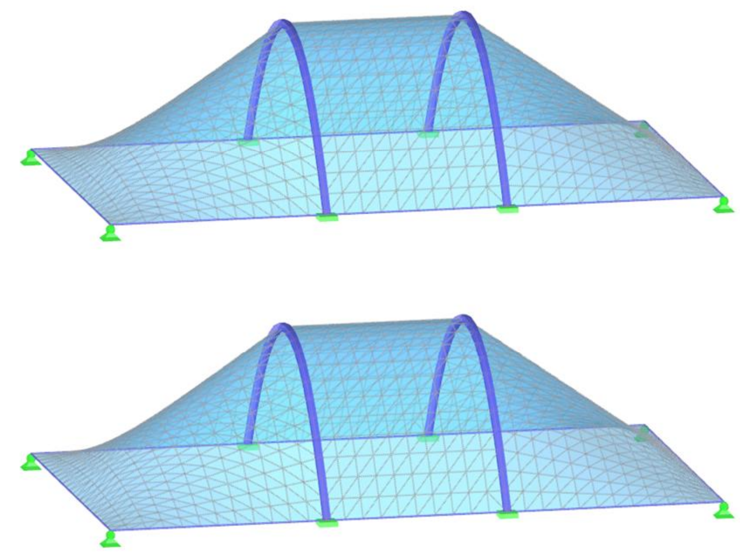
- Compression, self weight and boundary conditions

Combined Structure

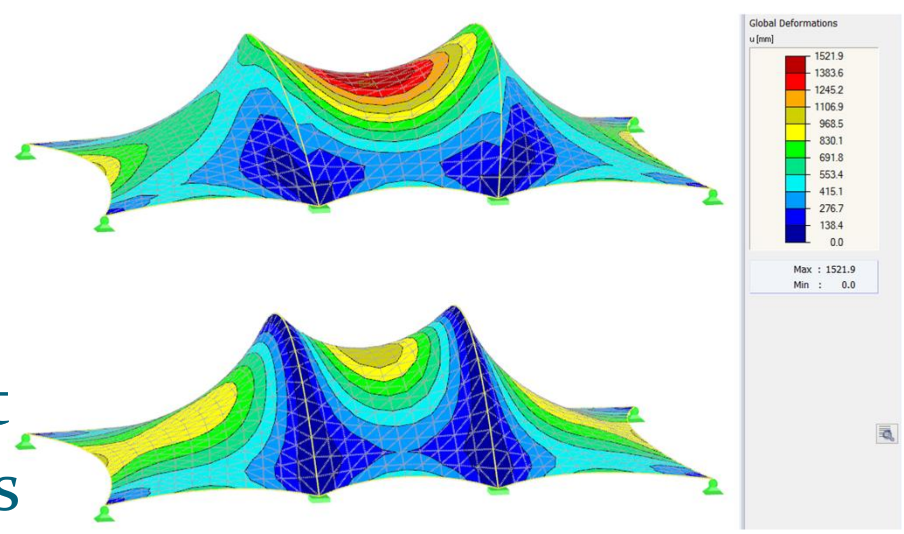
- Tension in membranes and cables, compression in beams
- Structure with and without shape analysis of the beams



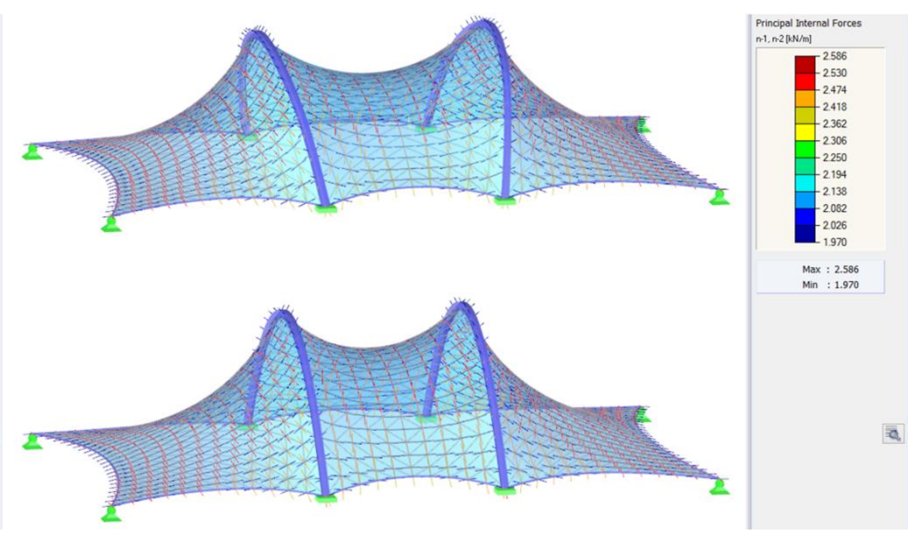
Initial shape of the membrane structures, the structure with (above) and without (bellow) the analysis of the shape of steel arches



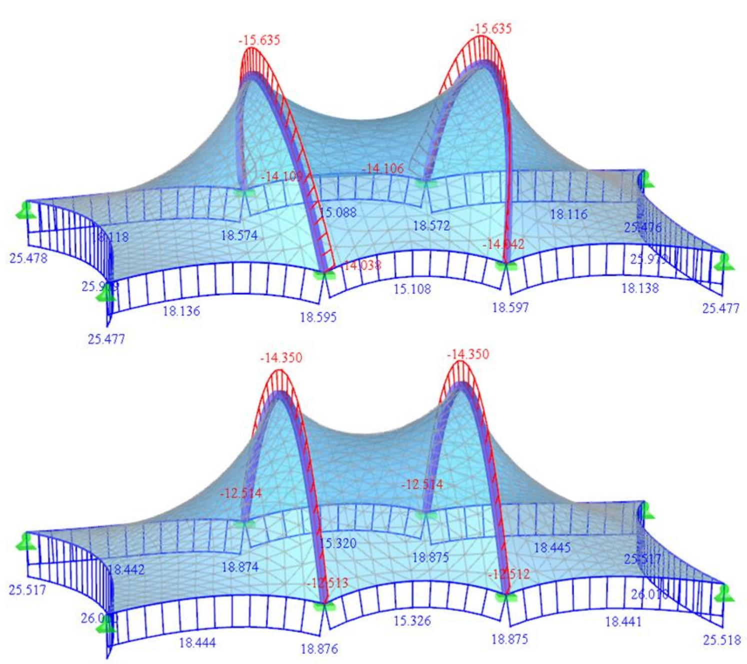
FE mesh in the initial position, the structure with (above) and without (bellow) the analysis of the shape of steel arches



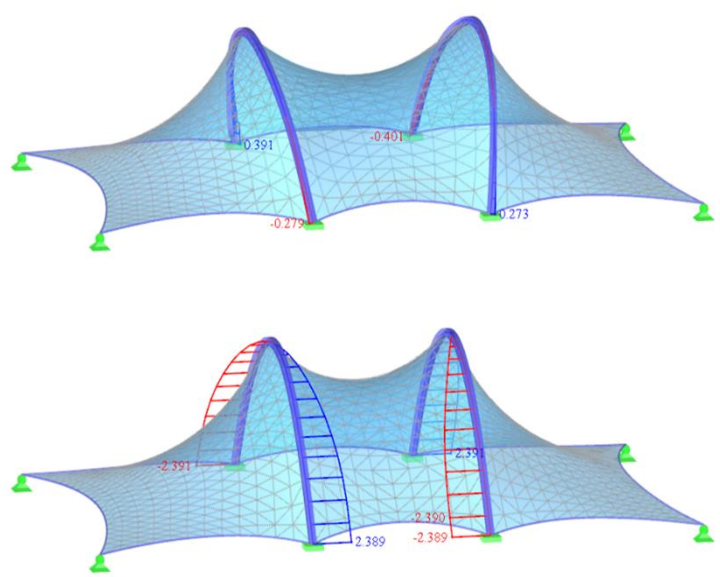
Global deformations u during the form-finding



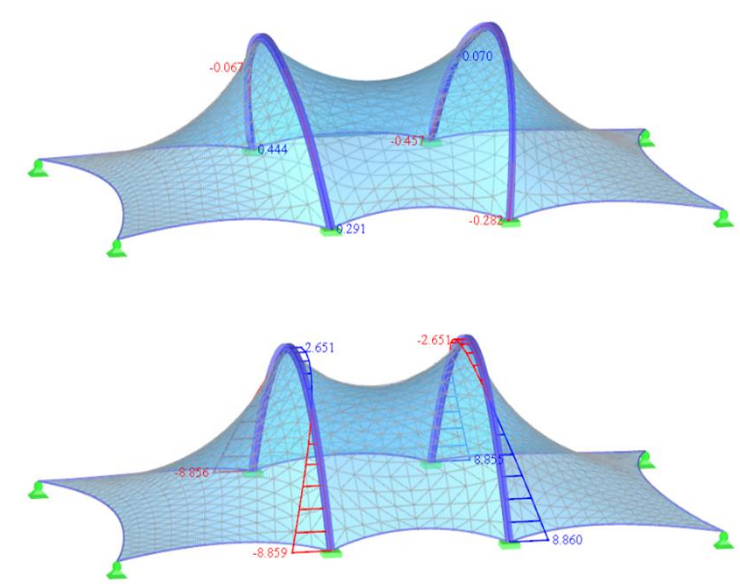
Vectors of the principal internal forces n_1 and n_2



Normal forces N



Shear forces V_y

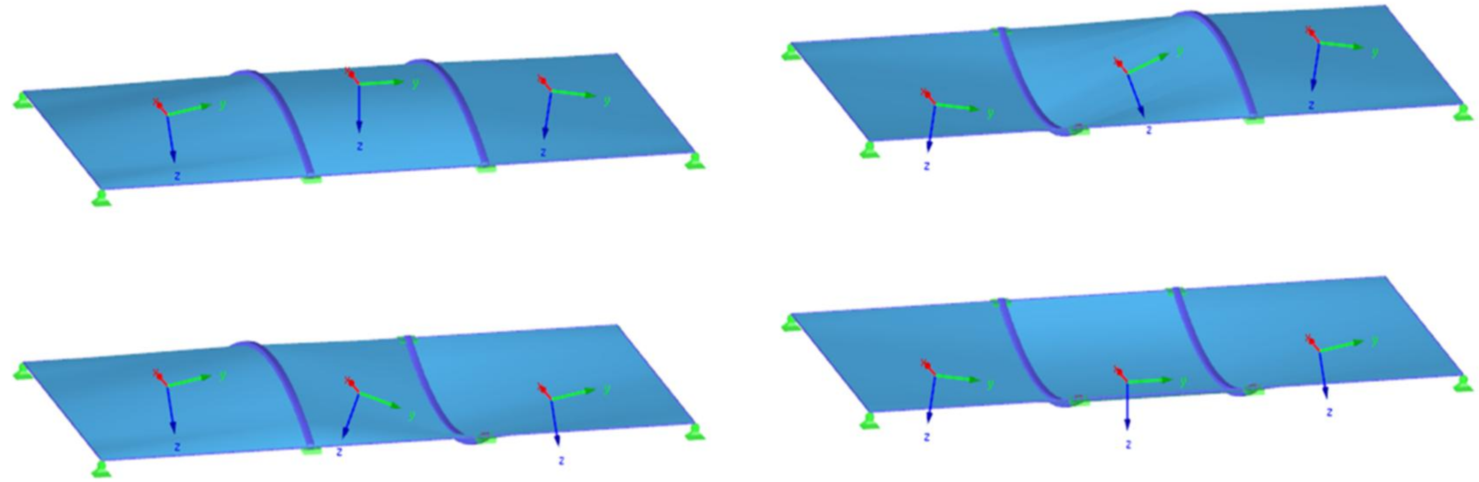


Bending moments M_z

FORM-FINDING

Shell Structure

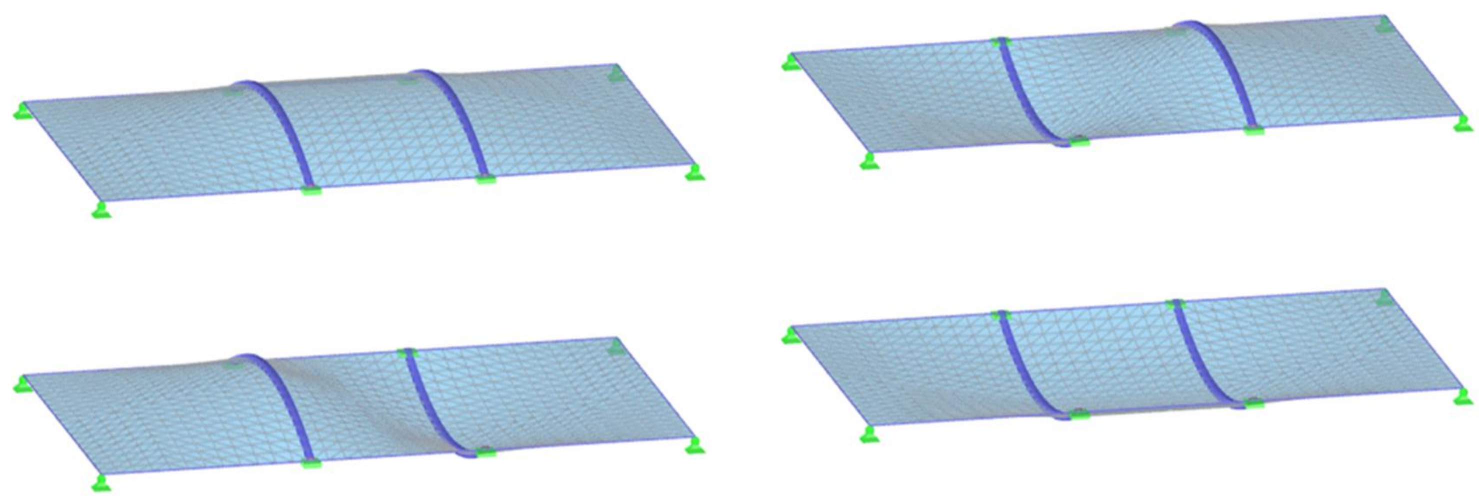
- Compression, self weight and boundary conditions



Different initial shapes of the membrane structures

Combined Structure

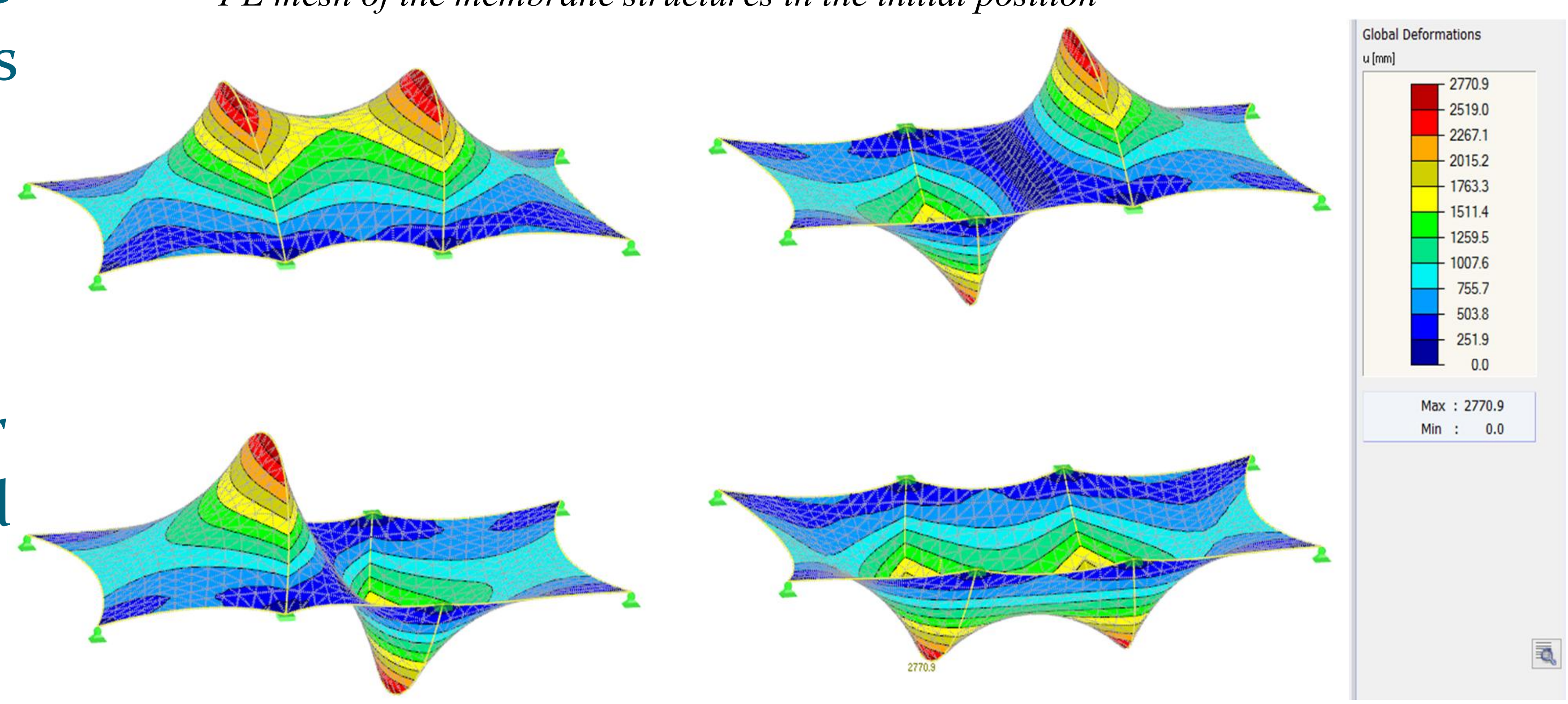
- Tension in membranes and cables, compression in beams
- Structure with and without shape analysis of the beams



FE mesh of the membrane structures in the initial position

Interesting Phenomenon

- Possible existence of different right solutions for shape analysis of combined structures



Global deformations u in the form-finding

STRUCTURAL ANALYSIS

Nonlinear Behavior

- Geometric nonlinearity
- Material nonlinearity

Geomatrical Nonlinearity

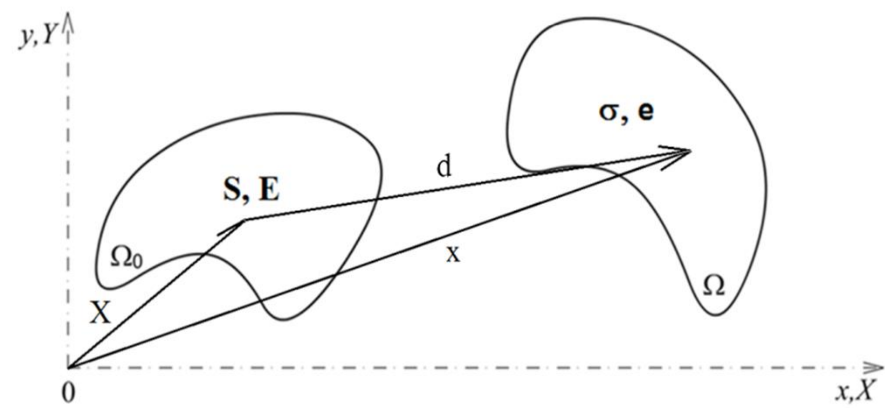
- The changes of the shape have significant impact on the structural response

Material Nonlinearity

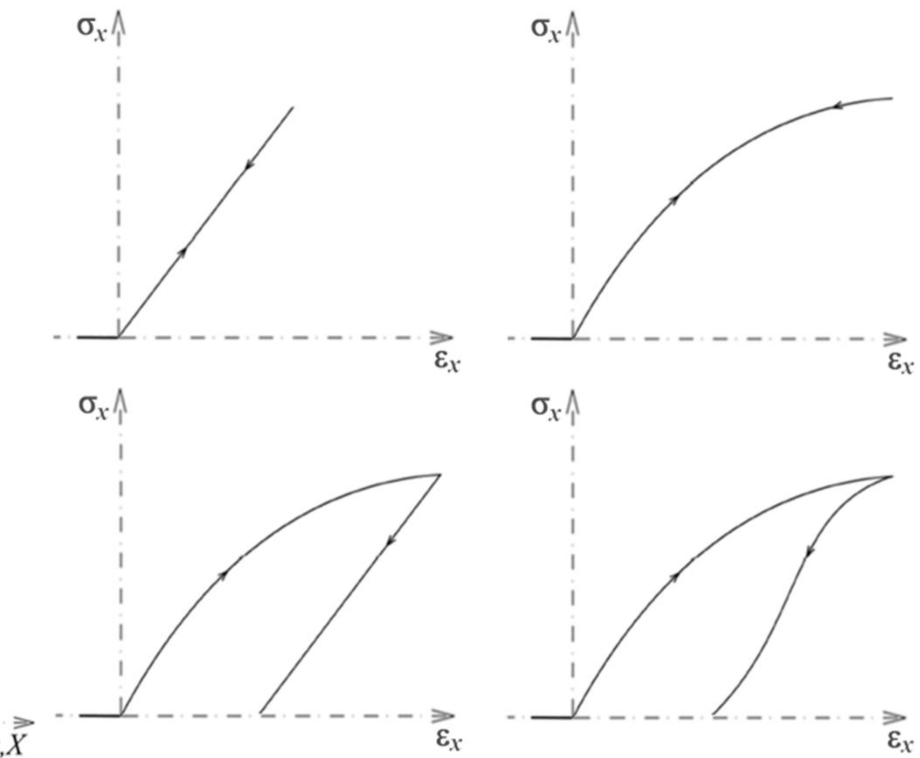
- Avoiding pressures
- Different material models

Methods of solving

- Implicit
- Explicit



Reference (X,Ω0) and the current (x,Ω) configuration of the body



Elastic, non-linear elastic, elastic-plastic and general material models

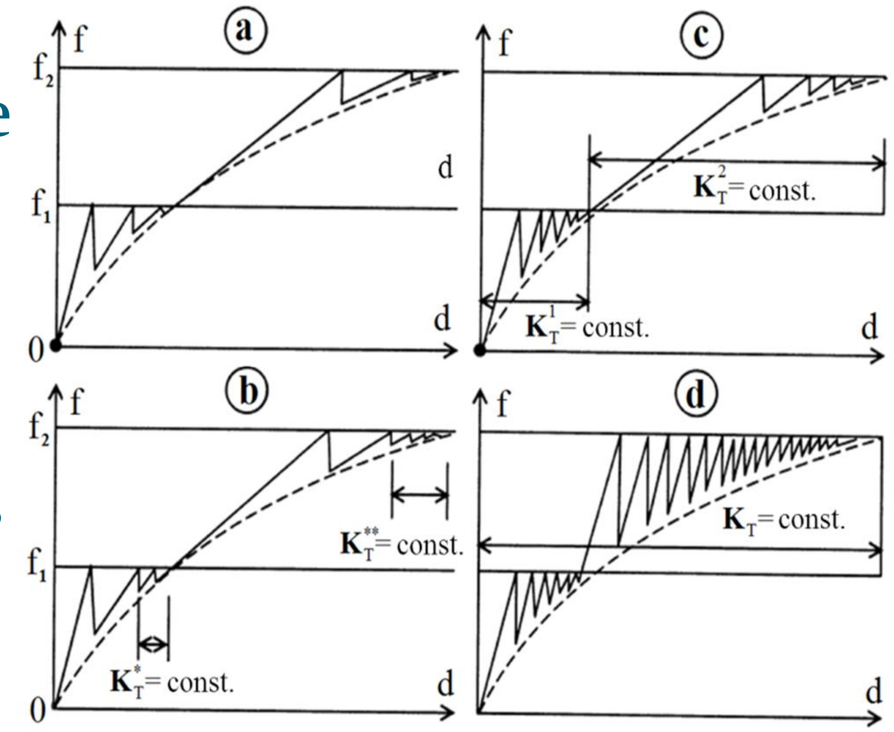
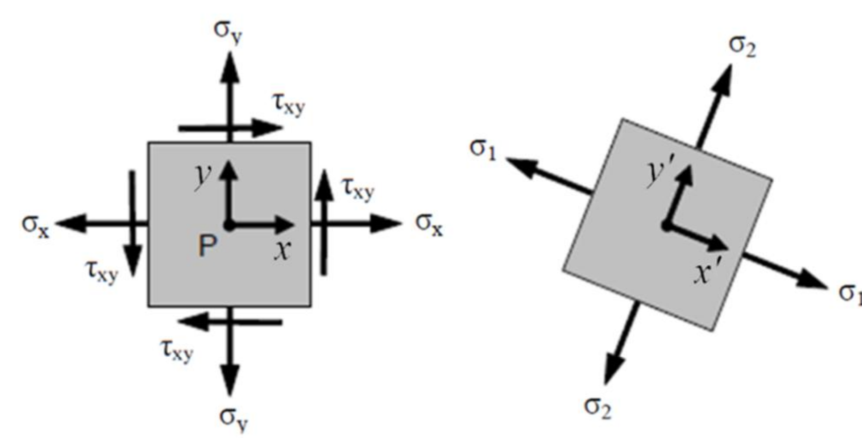


Diagram of the Newton-Raphson iterative method a) and its three modifications b),c),d)



Stress state: stresses in the planar axis direction, main stresses in the main directions ([19] with modifications)

$$K(d)d = f(d) \quad K(d) = K_M(d) + K_\sigma(d)$$

$$\frac{\partial \Pi}{\partial d} = \frac{\partial \Pi^{int}}{\partial d} + \frac{\partial \Pi^{ext}}{\partial d} = \int_{\Omega_0} S : \delta E \, d\Omega_0 - \int_{\Omega_0} q \cdot \delta d \, d\Omega_0 = \int_{\Omega} \sigma : \delta e \, d\Omega - \int_{\Omega} q \cdot \delta d \, d\Omega = 0$$

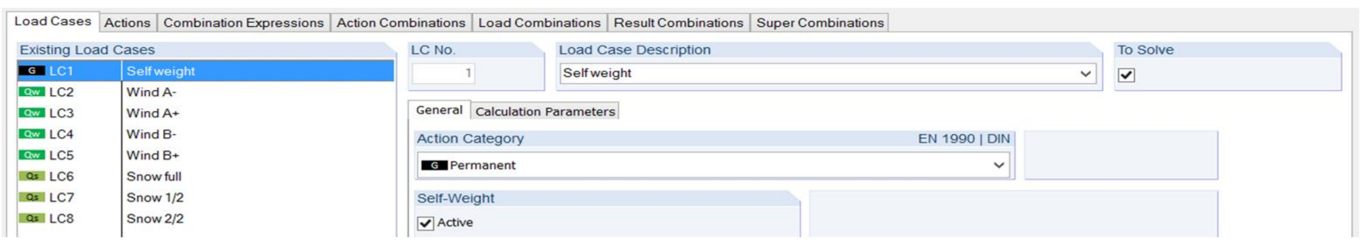
STRUCTURAL ANALYSIS

Hypar Structure

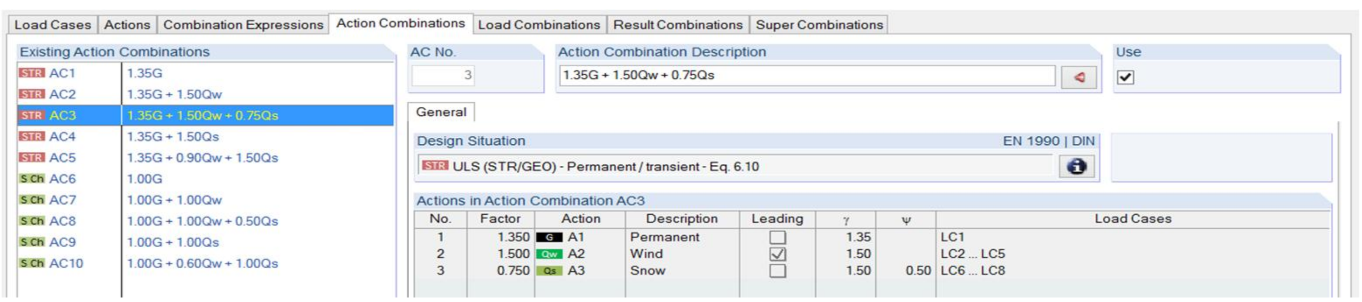
- Eight load cases
- Linear orthotropic material

$$E_x = 1000.0\frac{kN}{m}, E_y = 800.0\frac{kN}{m}, G_{xy} = 100.0\frac{kN}{m}, \nu_{xy} = 0.10, \nu_{yx} = 0.08$$

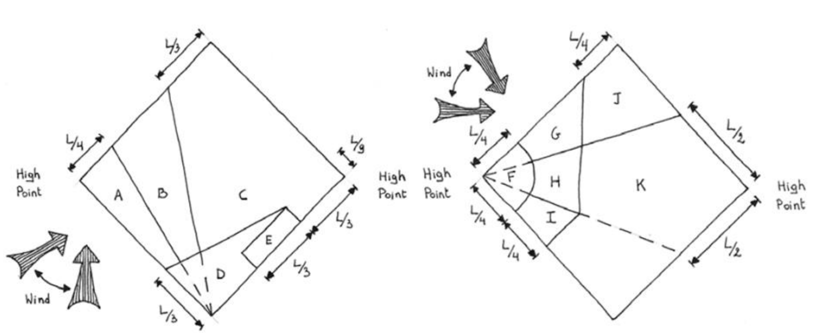
$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ & C_{22} & C_{23} \\ sym. & & C_{33} \end{bmatrix} = \begin{bmatrix} C_{xxxx} & C_{xxyy} & C_{xxxy} \\ & C_{yyyy} & C_{yyxy} \\ sym. & & C_{xyxy} \end{bmatrix} = \begin{bmatrix} 1008.06 & 80.65 & 0.00 \\ & 806.45 & 0.00 \\ sym. & & 100.00 \end{bmatrix} \frac{kN}{m}$$



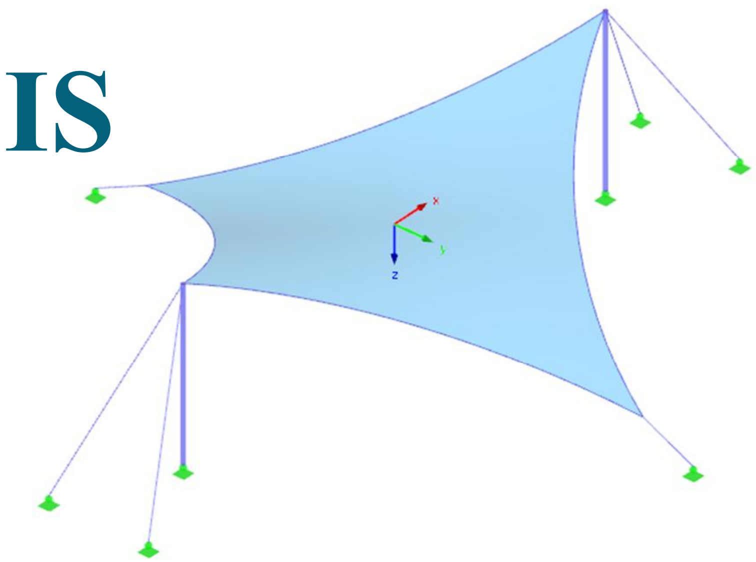
Definition of load cases



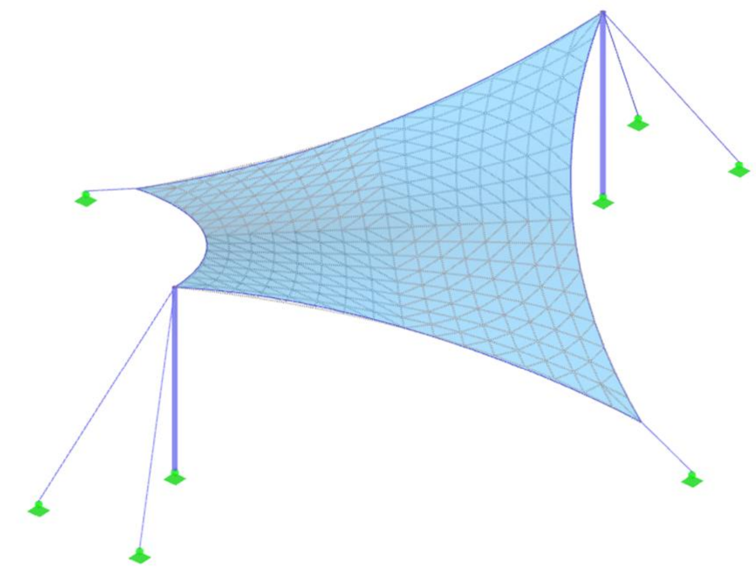
Load combinations (ULS and SLS)



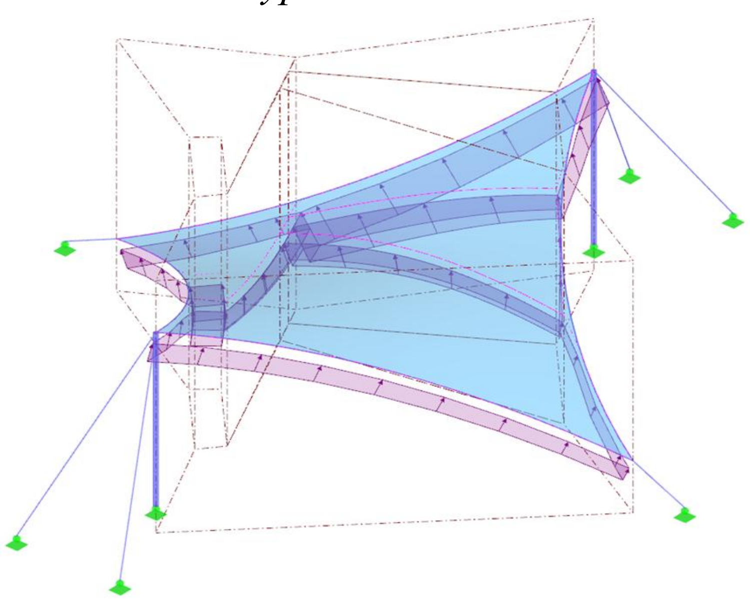
C_p zones definition on the hypar structure [35]



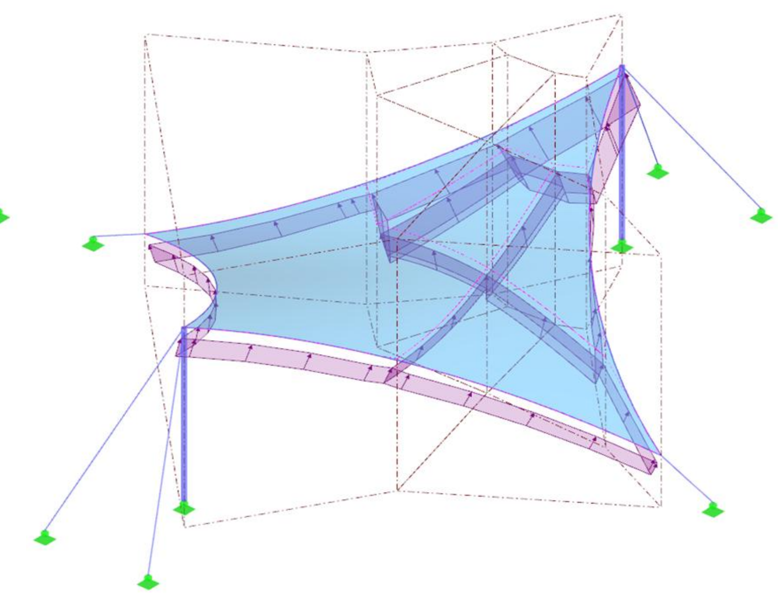
Hypar membrane



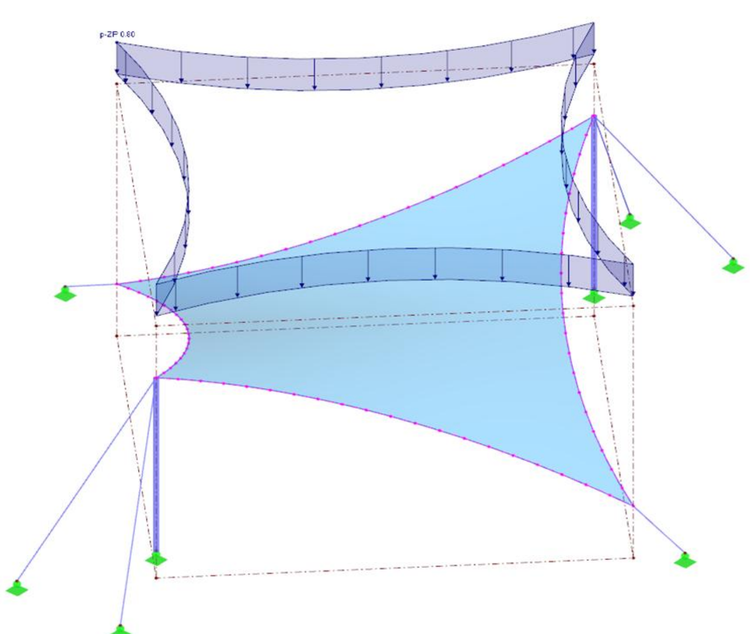
FE mesh



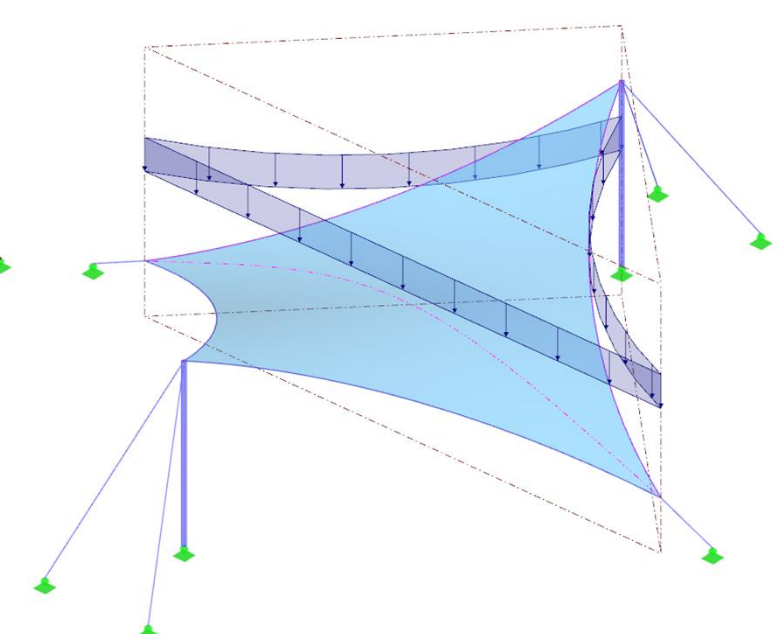
Load cases LC2 (Wind A-)



Load cases LC4 (Wind B-)



Load case LC6 (Snow full)

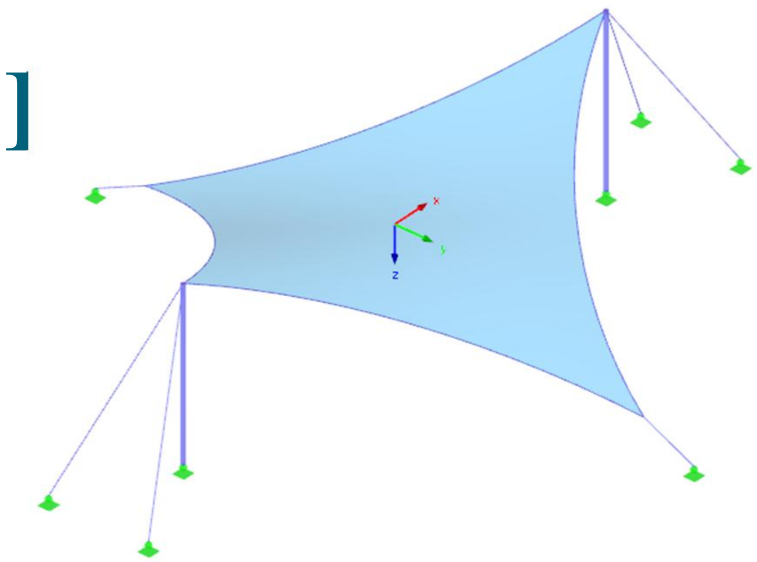


Load cases LC7 (Snow 1/2)

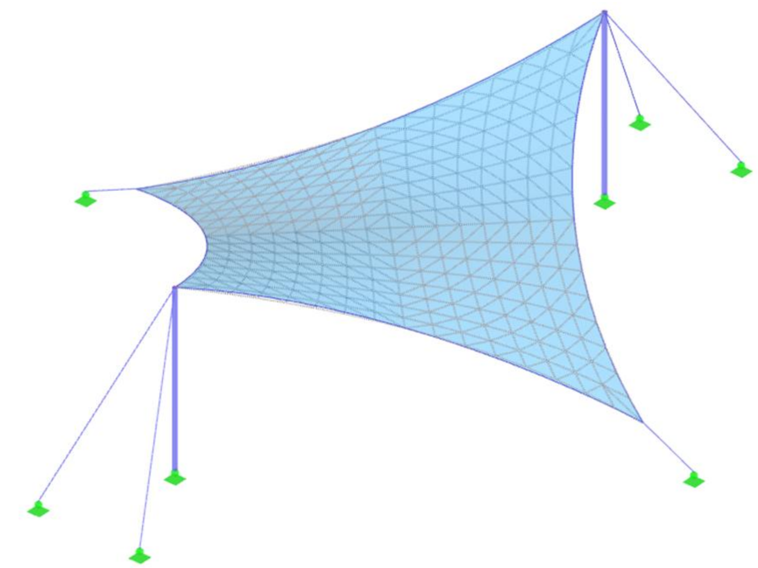
STRUCTURAL ANALYSI

Hypar Structure

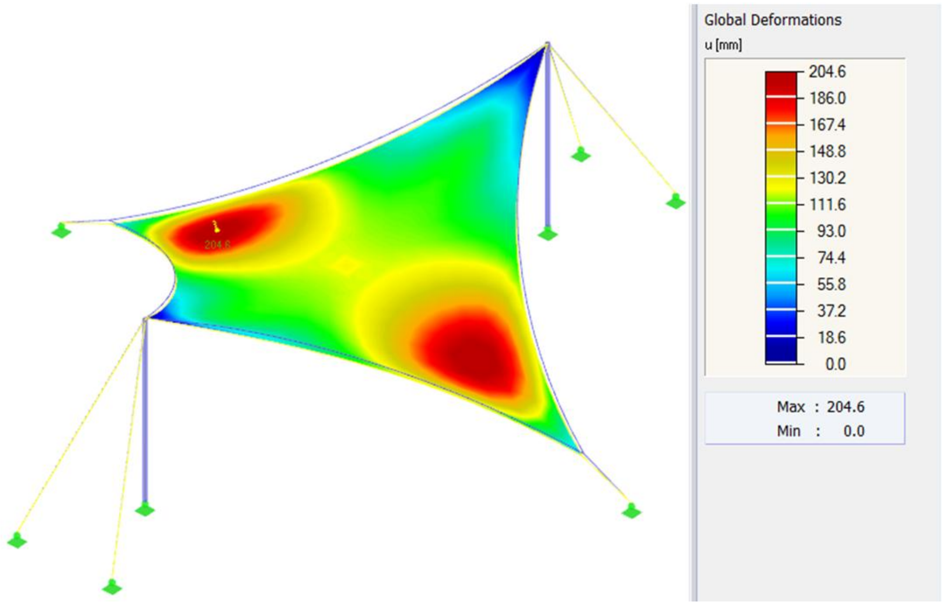
- Eight load cases
- Linear orthotropic material



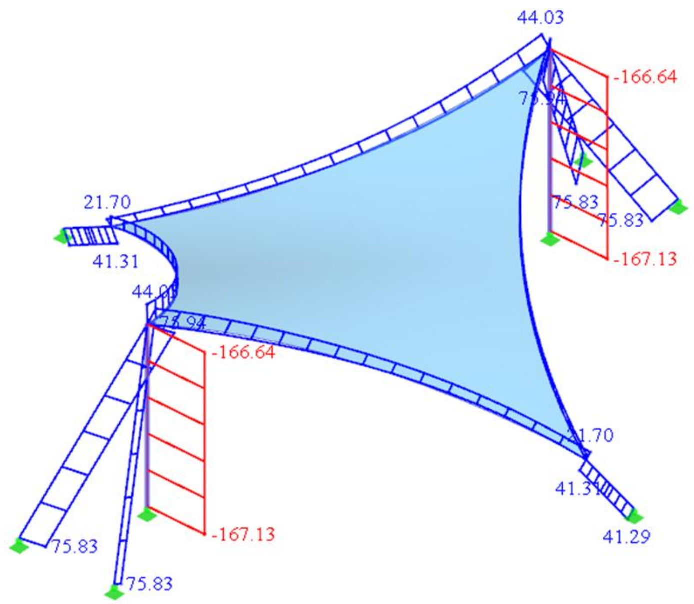
Hypar membrane



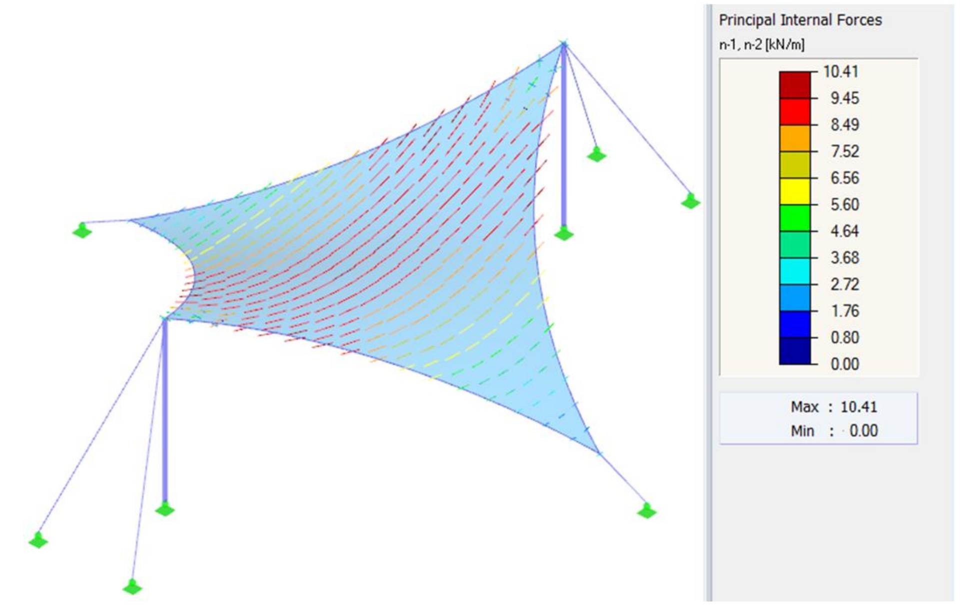
FE mesh



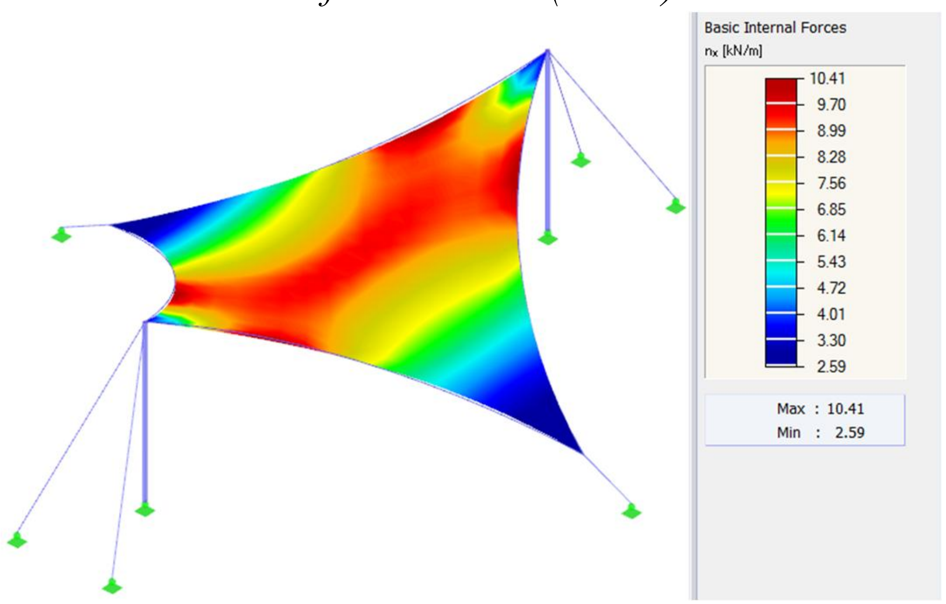
Global deformations u (CO18)



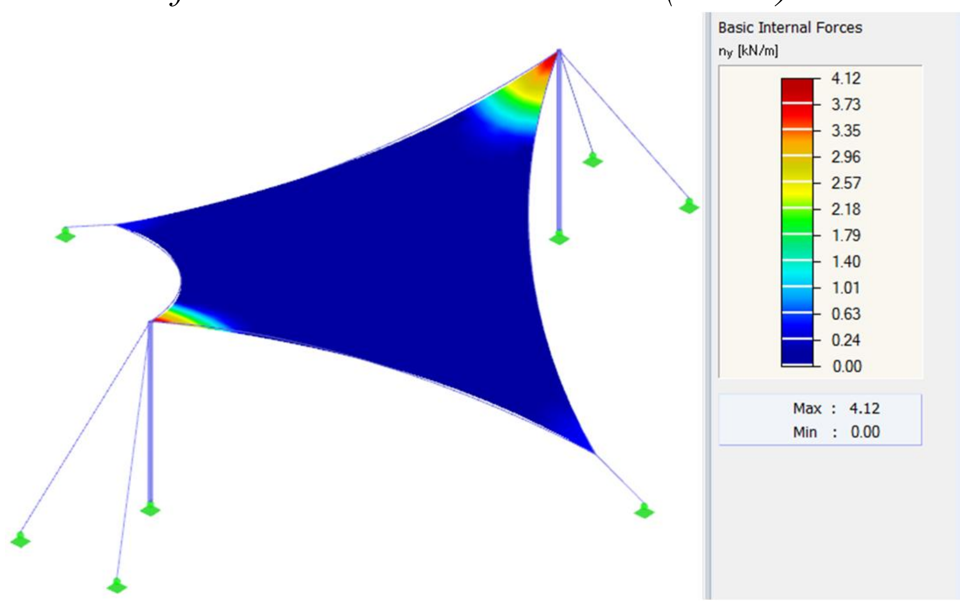
Normal force N in cables and beams (CO18)



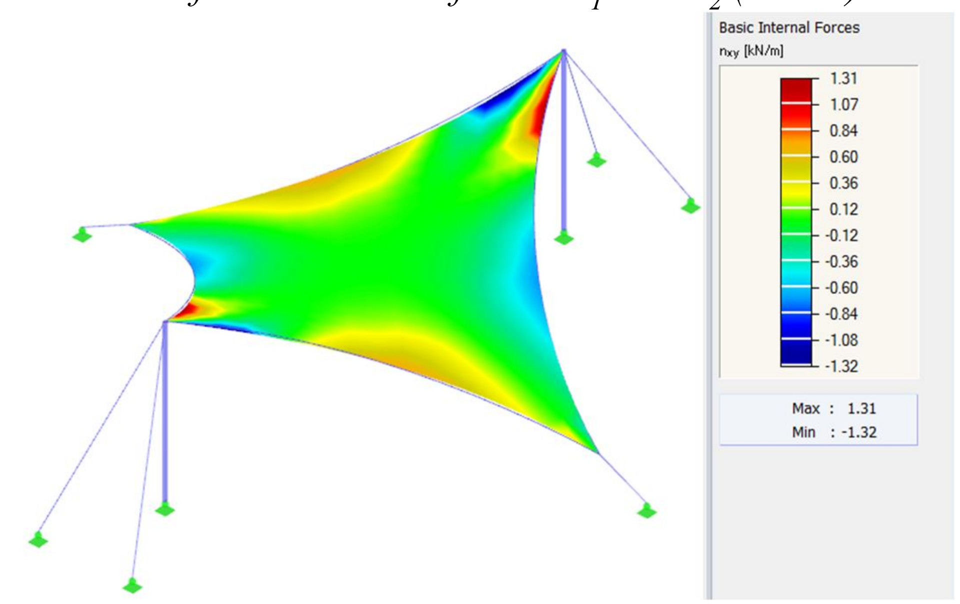
Vectors of main internal forces n_1 and n_2 (CO18)



Basic internal forces n_x in the membrane (CO18)



Basic internal forces n_y in the membrane (CO18)

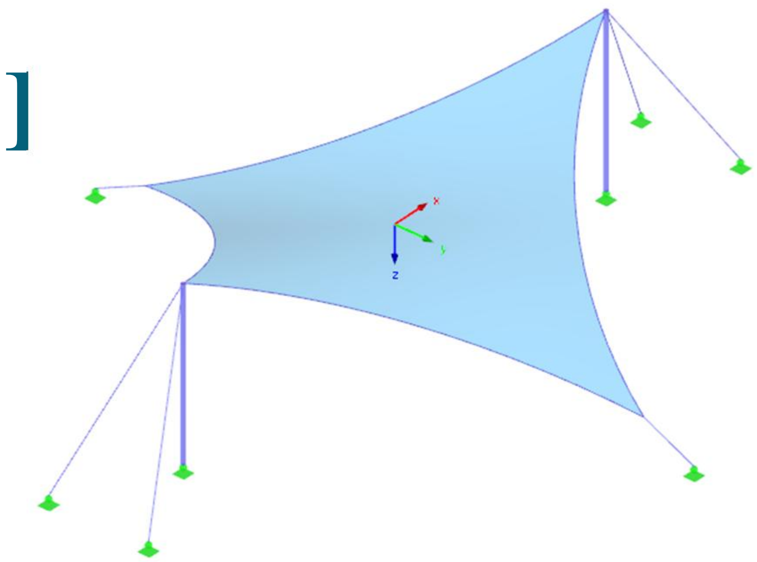


Basic internal forces n_{xy} in the membrane (CO18)

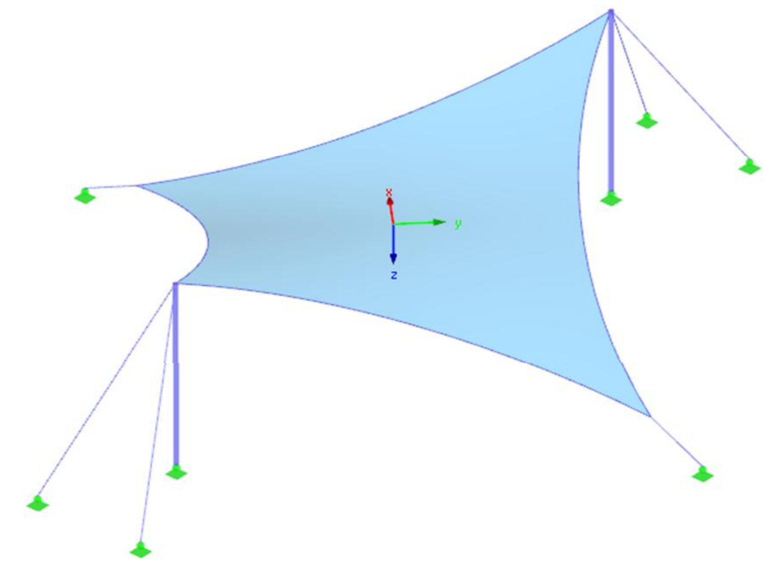
STRUCTURAL ANALYSI

Hypar Structure

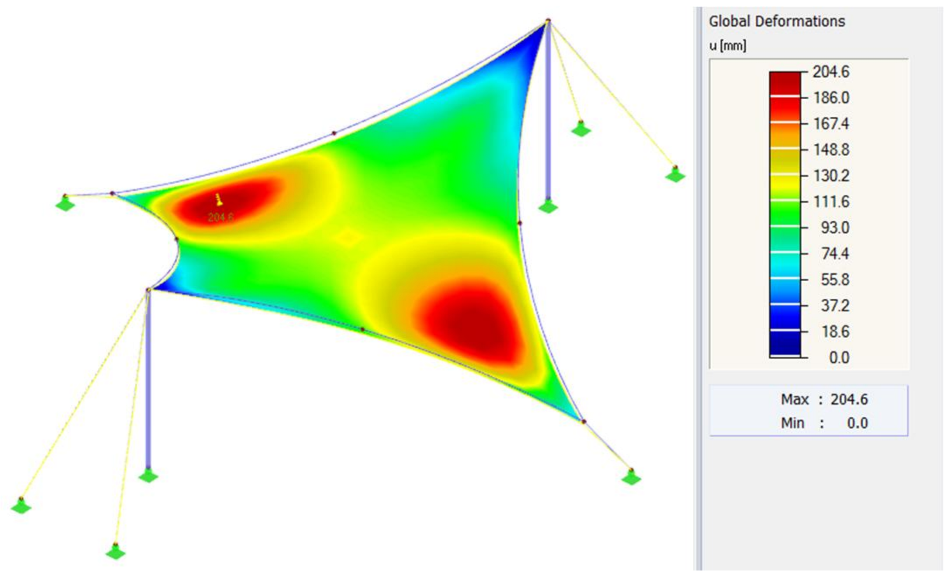
- Eight load cases
- Linear orthotropic material
- Impact of the warp/weft orientation



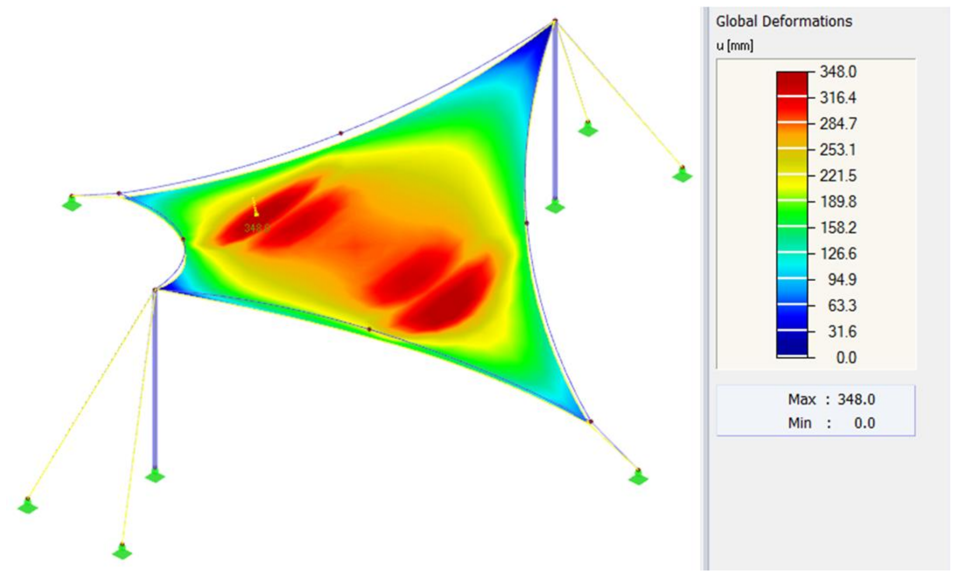
First axes orientation (Model 1)



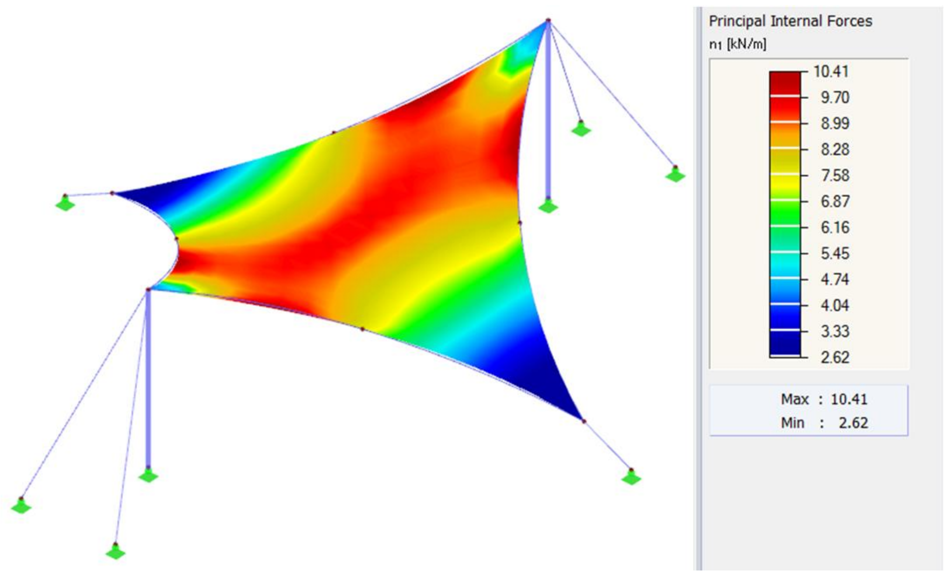
Second axes orientation (Model 2)



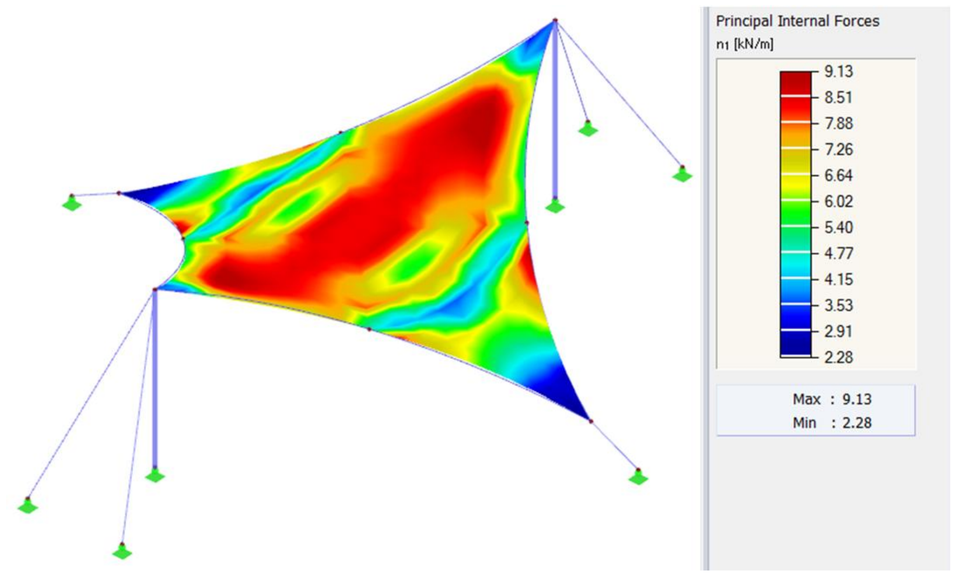
Global deformations u of Model 1 (CO18)



Global deformations u of Model 2 (CO18)



Main internal forces n_1 of Model 1 (CO18)



Main internal forces n_1 of Model 2 (CO18)

STRUCTURAL ANALYSIS

Pneumatic Structure

- Eight load cases
- Linear isotropic material
- Air management

$E = 900.0\ MPa, G = 310.0\ MPa, \nu = 0.452$

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ & C_{22} & C_{23} \\ sym. & & C_{33} \end{bmatrix} = \begin{bmatrix} C_{xxxx} & C_{xxyy} & C_{xxxy} \\ & C_{yyyy} & C_{yyxy} \\ sym. & & C_{xyxy} \end{bmatrix} = \begin{bmatrix} 339.18 & 153.18 & 0.00 \\ & 339.18 & 0.00 \\ sym. & & 93.00 \end{bmatrix} \frac{kN}{m}$$

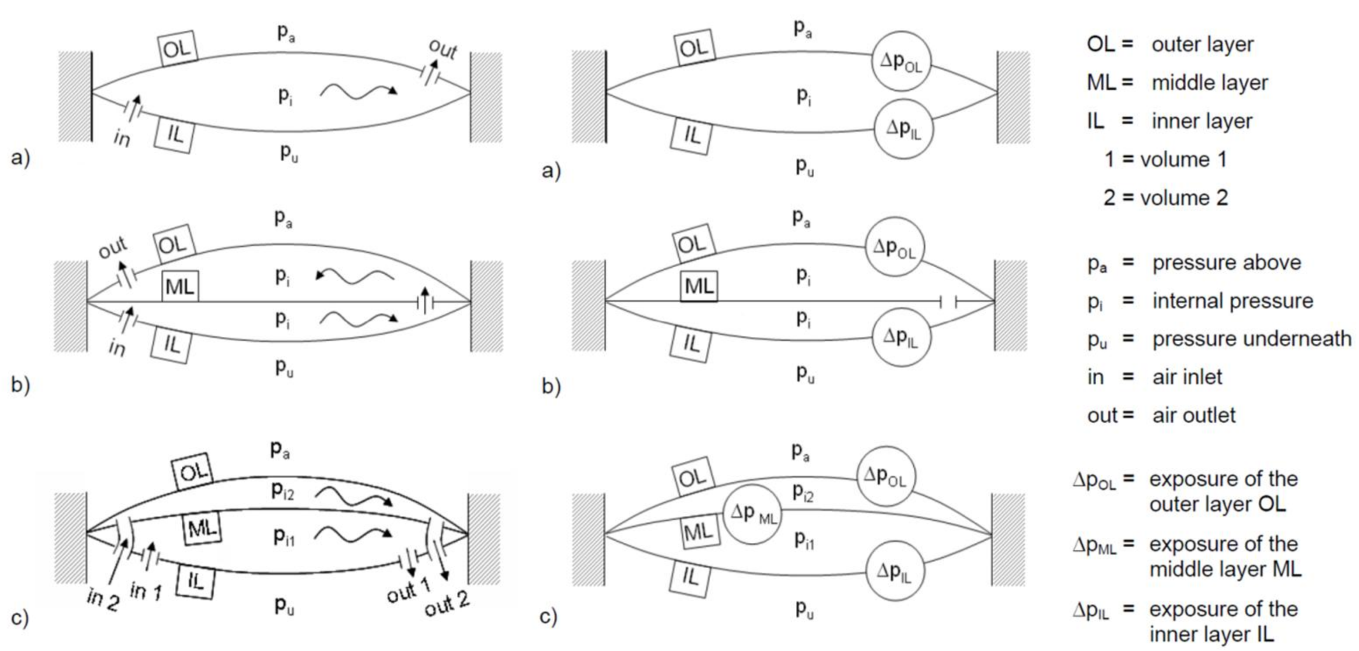
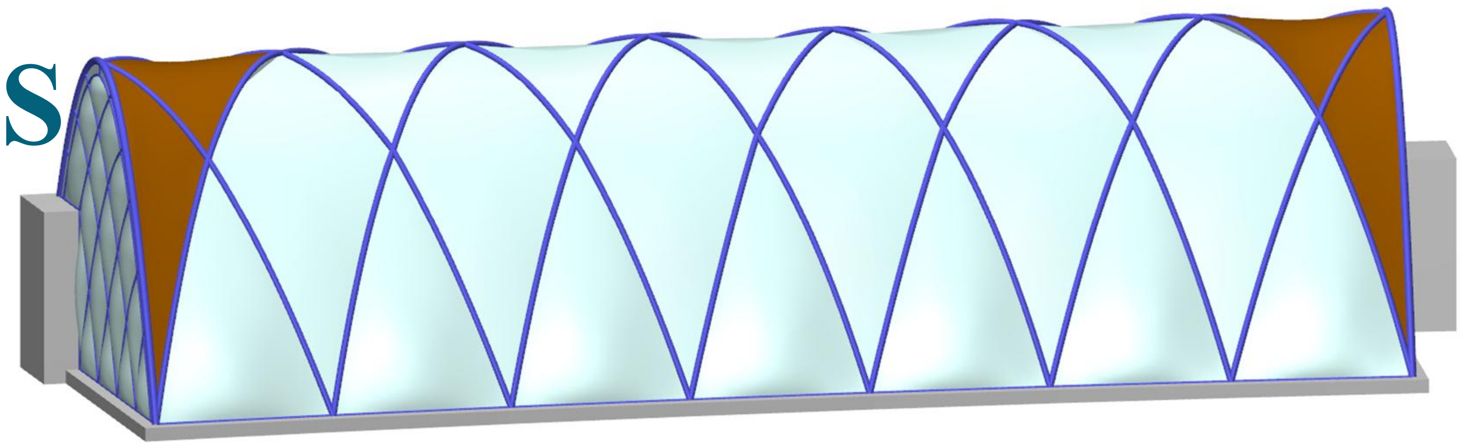
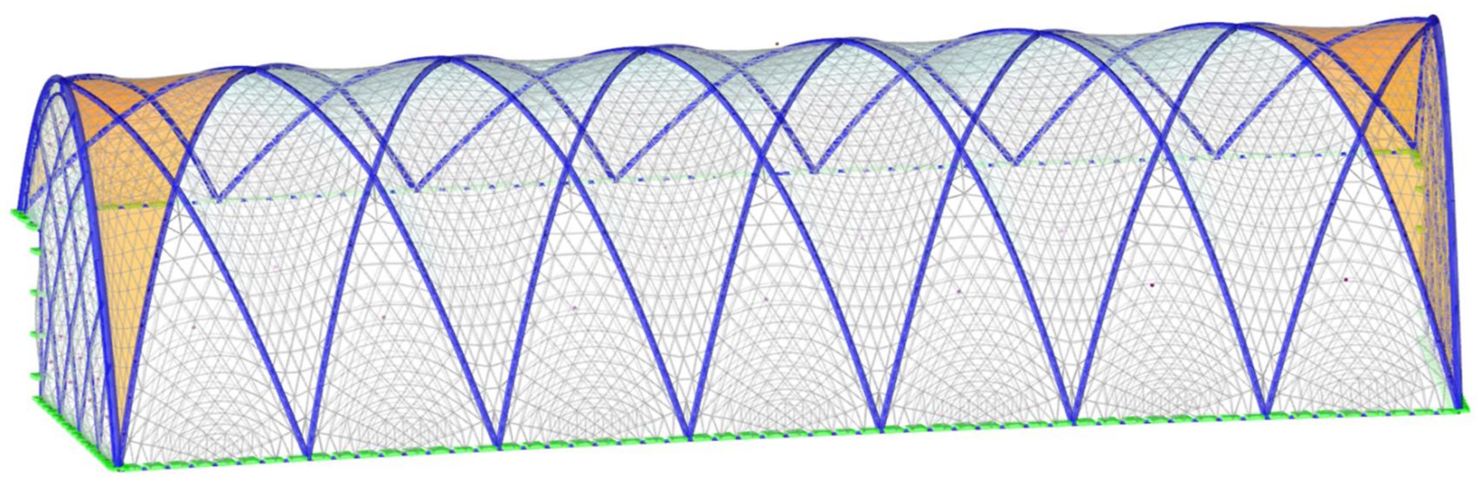


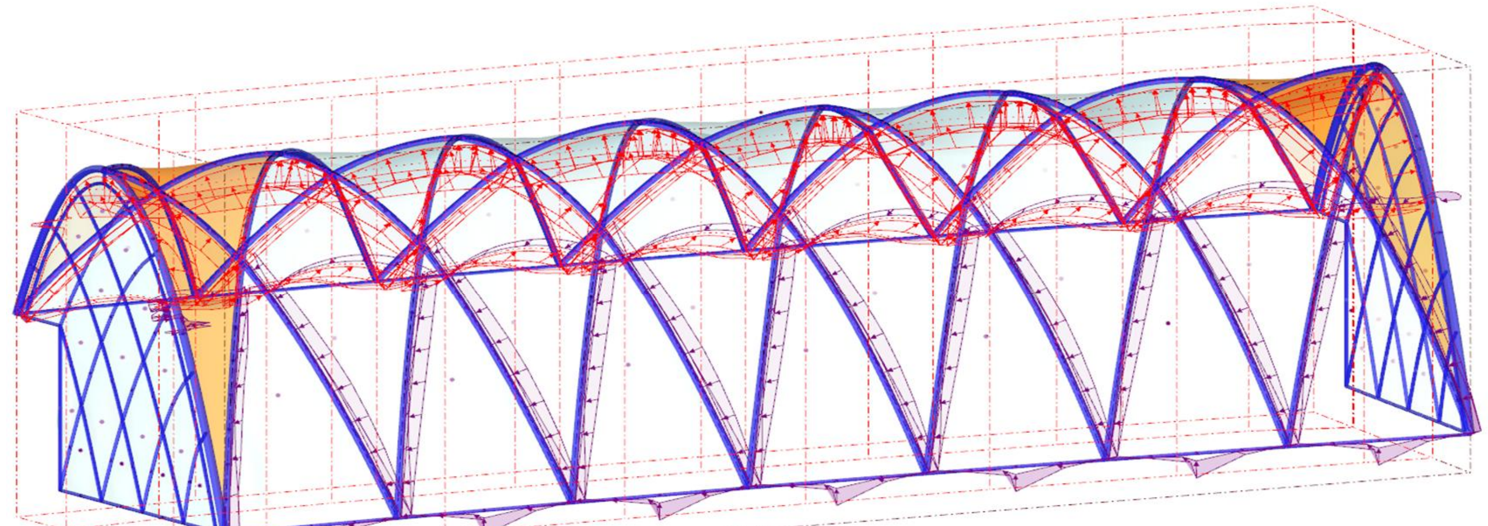
Fig. 114 – Air management of pneumatic stabilized and prestressed cushions (on the left) and air behaviour of enclosed cushions (on the right): a) 2-layer system, b) 3-layer system, flat middle layer (mechanically prestressed) and c) 3-layer system, curved middle layer (pneumatically prestressed) [21]



Geometry of the greenhouse structure



The FE model of the greenhouse structure (1221 1D elements, 16508 2D elements, 20172 3D elements)

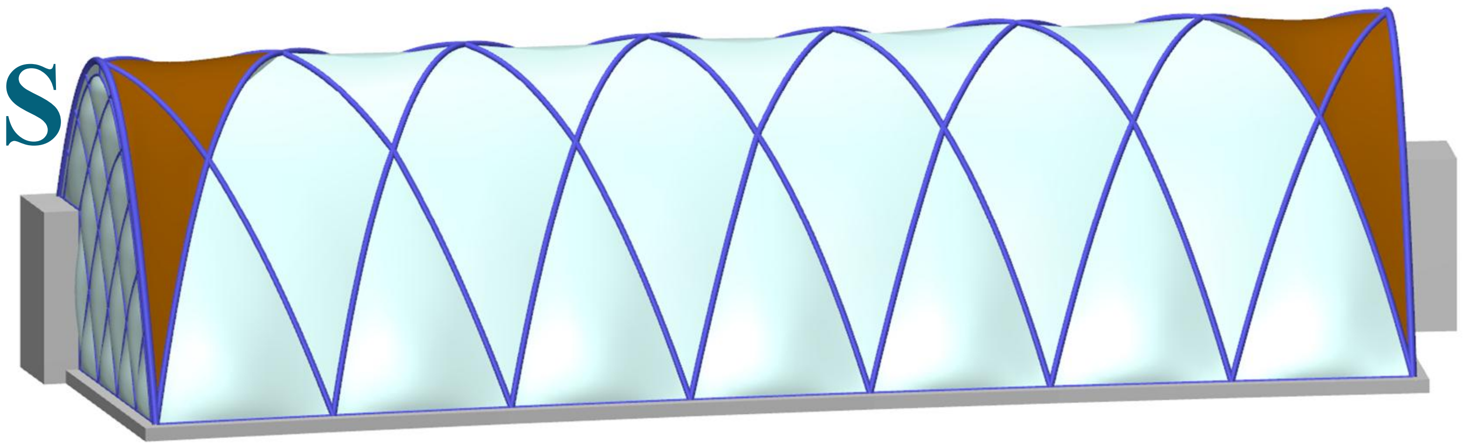


Load combination CO2 = 1.35LC1 + 1.50LC5

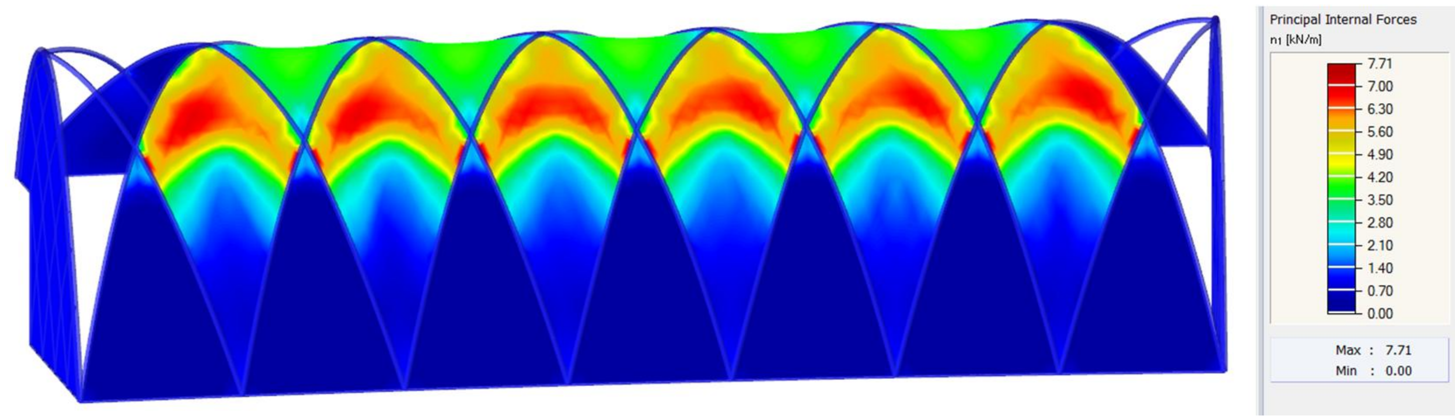
STRUCTURAL ANALYSIS

Pneumatic Structure

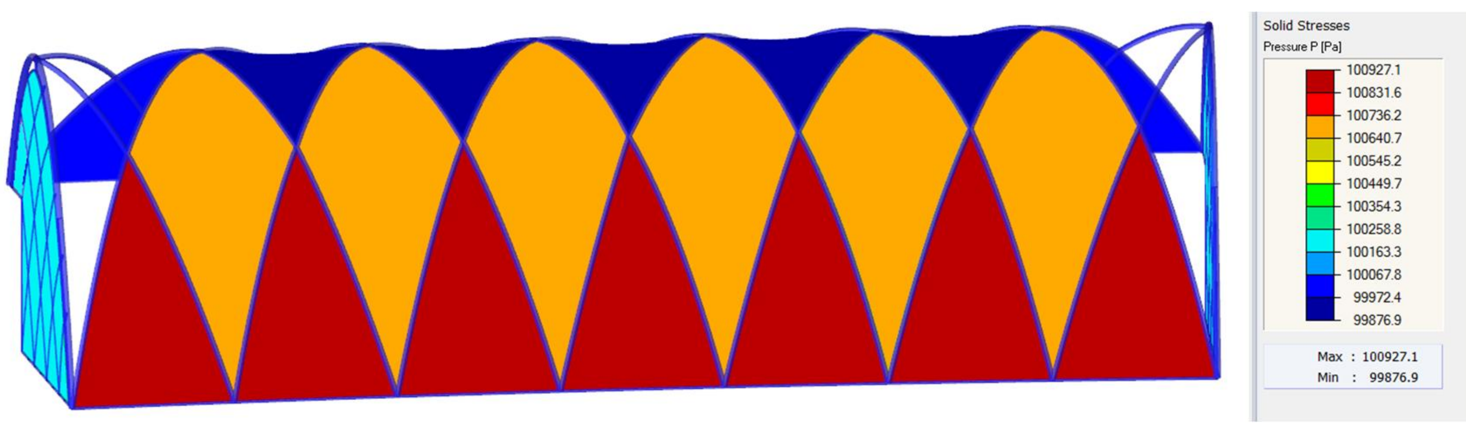
- Eight load cases
- Linear isotropic material
- Air management



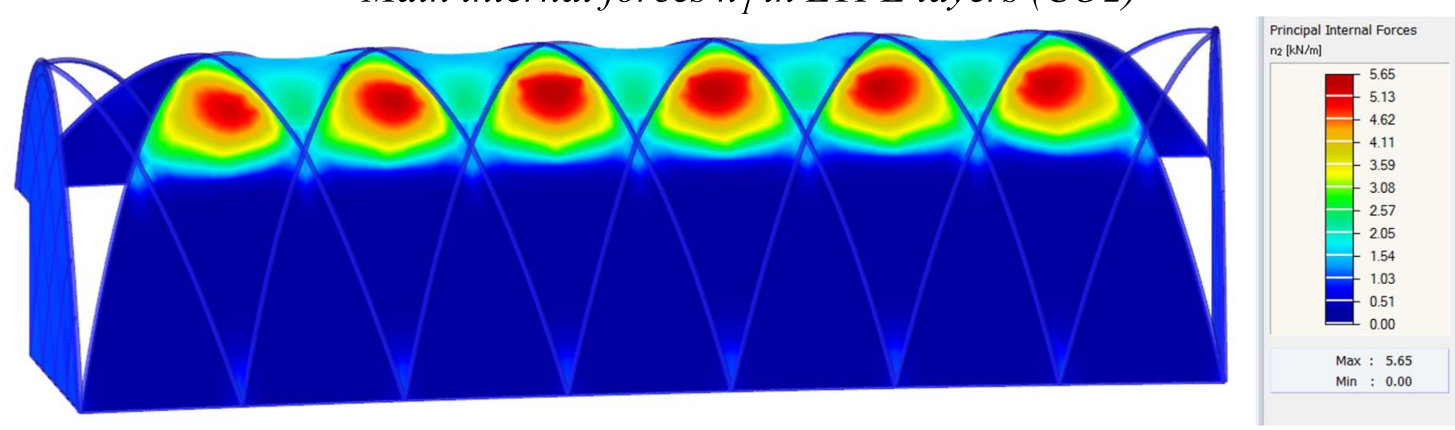
Geometry of the greenhouse structure



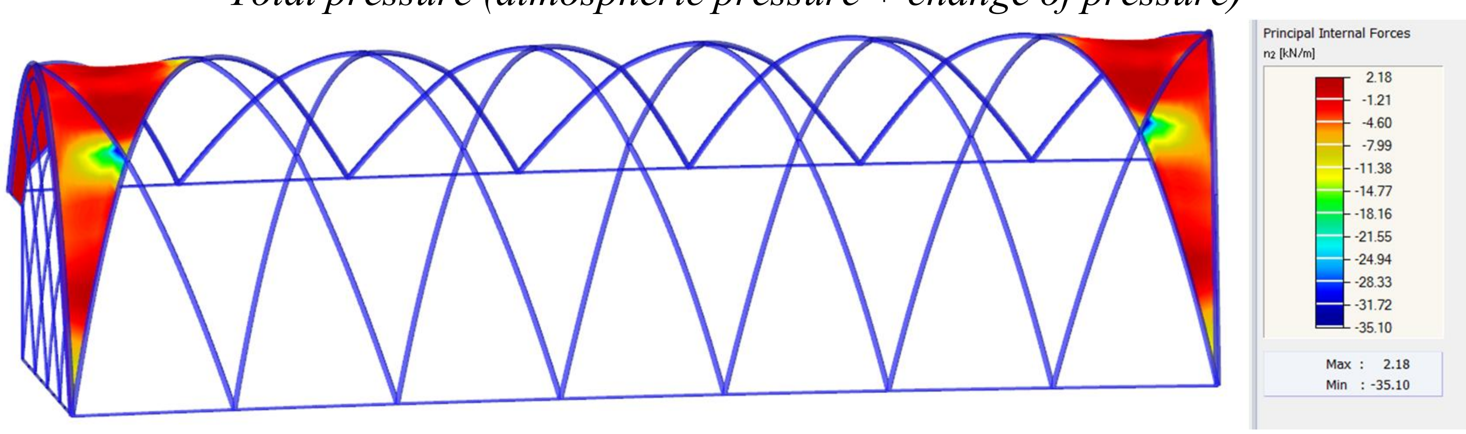
Main internal forces n_1 in ETFE layers (CO2)



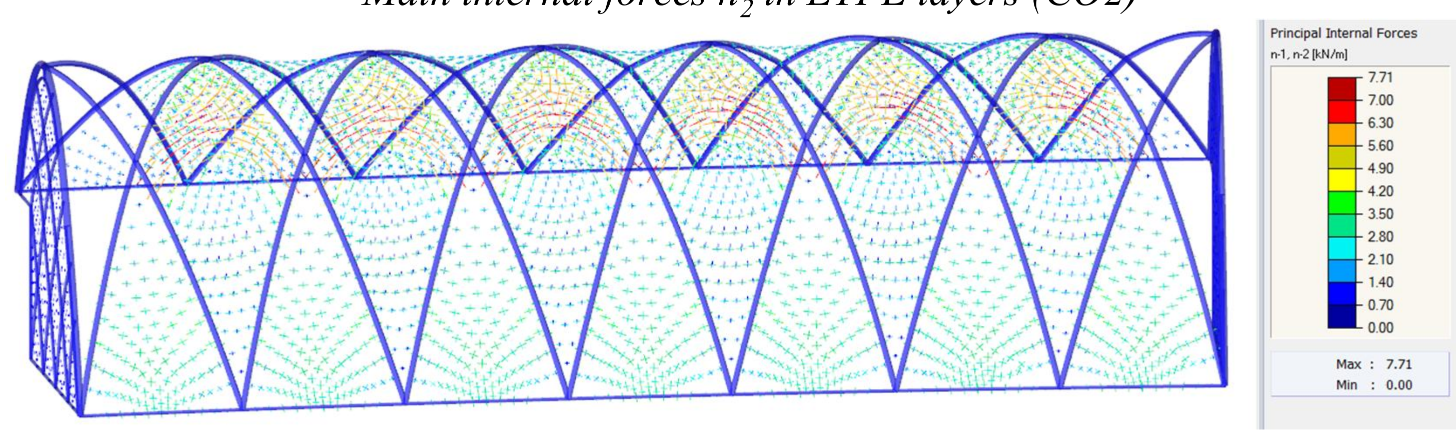
Total pressure (atmospheric pressure + change of pressure)



Main internal forces n_2 in ETFE layers (CO2)



Main internal forces n_2 in wooden shells (CO2)

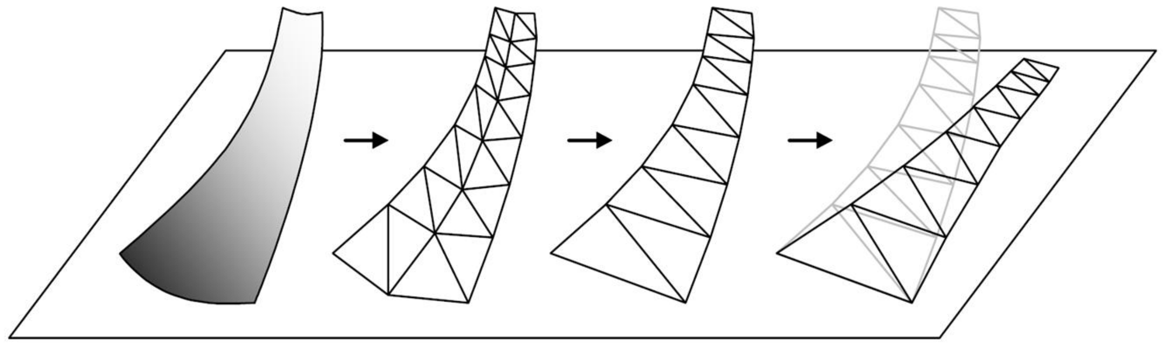


Vectors of the main internal forces n_1 and n_2 in ETFE layers (CO2)

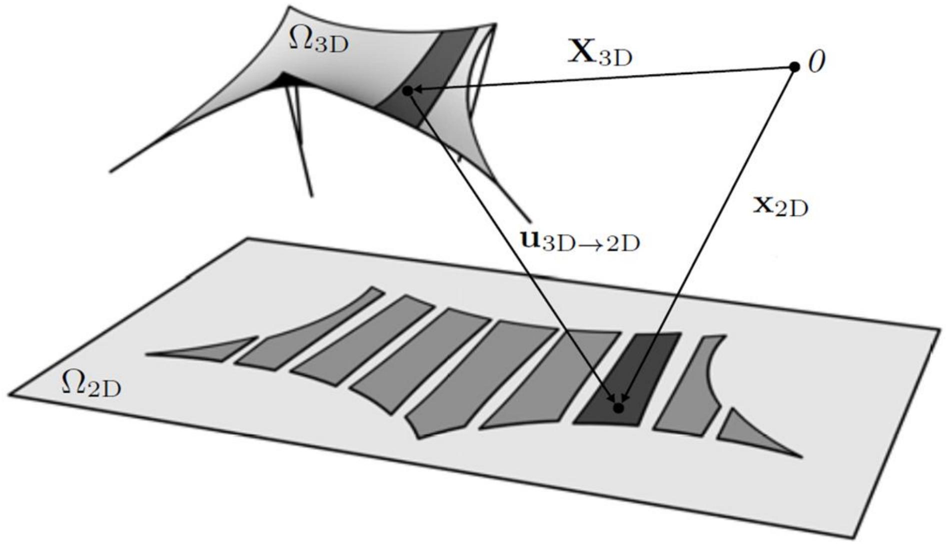
GENERATION OF CUTTING PATTERNS

Basic Steps

- Splitting the surface by cutting lines
- Flattening the spatial patterns into the plane



The basis of the simple triangulation method (from the left: the spatial shape, the FE mesh of the spatial model used for form-finding and structural analysis purposes, the modified mesh for flattening purposes, the flattened pattern) [19]



The flattening process ([20] with modifications)

Flattening Methods

- Simple Triangulation Method
- Mathematical Squashing by Least Square Approach

  $F(x,y) = \frac{1}{2} v^T P v \longrightarrow min.$

- Physical Squashing by Least Square Approach

  $F(x,y) = F(x_{2D}) = \frac{1}{2} \int_{\Omega_{2D}} (\sigma_{3D\to 2D} + \sigma_{pre}) : (\sigma_{3D\to 2D} + \sigma_{pre}) d\Omega_{2D} \longrightarrow min.$

- Physical Squashing with Energy Minimization

$$\frac{\partial \Pi}{\partial d} = \frac{\partial \Pi^{int}}{\partial d} = \frac{\partial (\Pi^{int}_{3D\to 2D} + \Pi^{int}_{pre})}{\partial d} = \int_{\Omega_{3D}} (S_{3D\to 2D} + S_{pre}) : \delta E_{3D\to 2D}\, d\Omega_{3D} = \int_{\Omega_{2D}} (\sigma_{3D\to 2D} + \sigma_{pre}) : \delta e_{3D\to 2D}\, d\Omega_{2D} = 0$$

GENERATION OF CUTTING PATTERNS

Construction Requirements

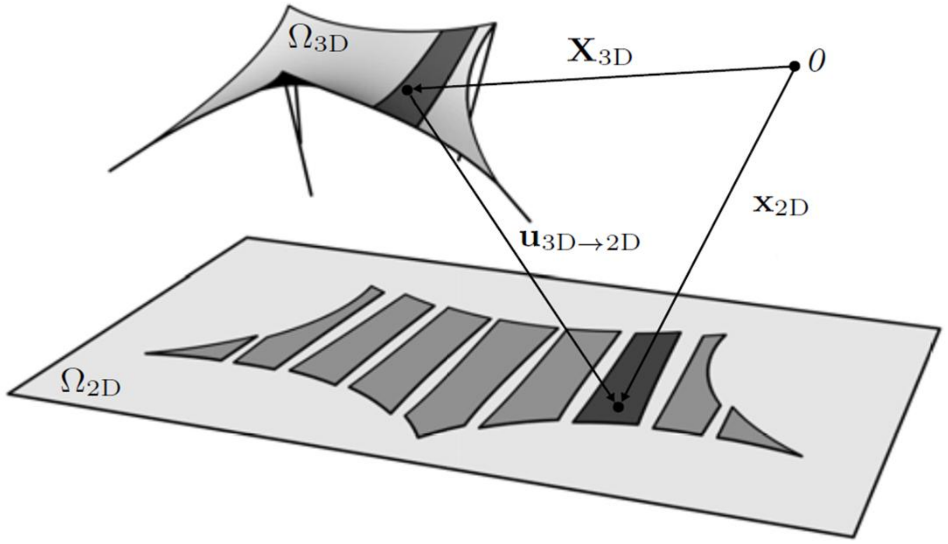
- Same lengths of the lines for welding
- Specific compensation for boundary lines

Compensation

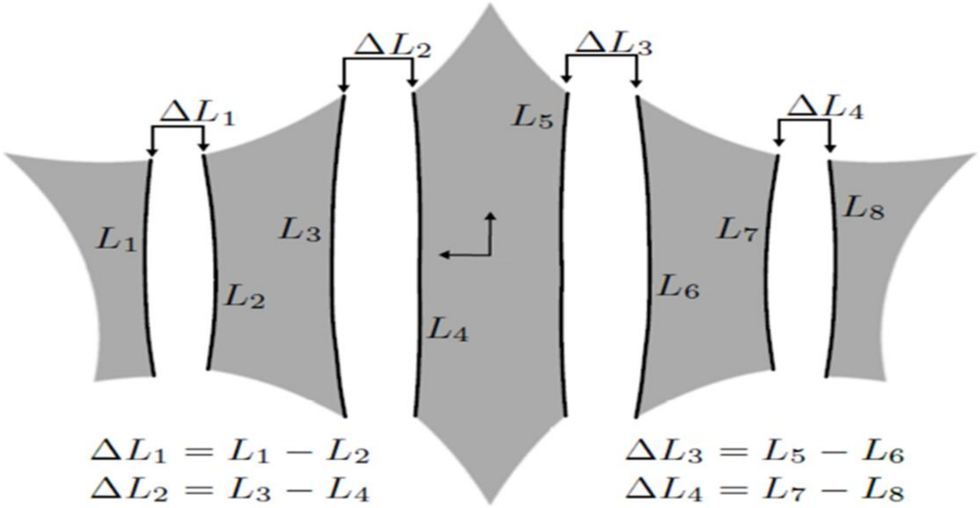
- According can be a part of the flattenging process

The influence of material on flattening

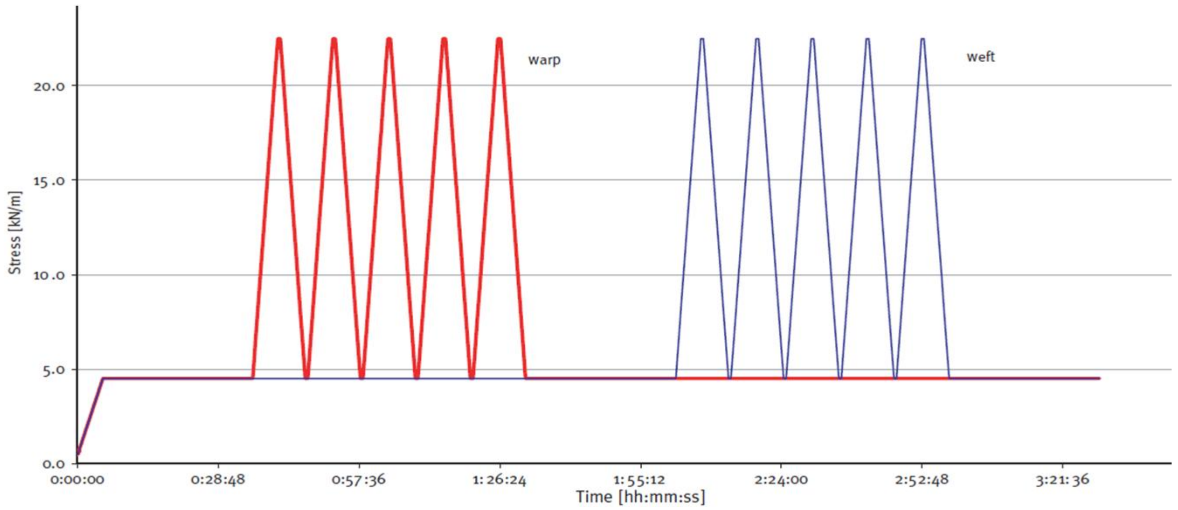
- Different materials can be taken into account for the FEM flattening



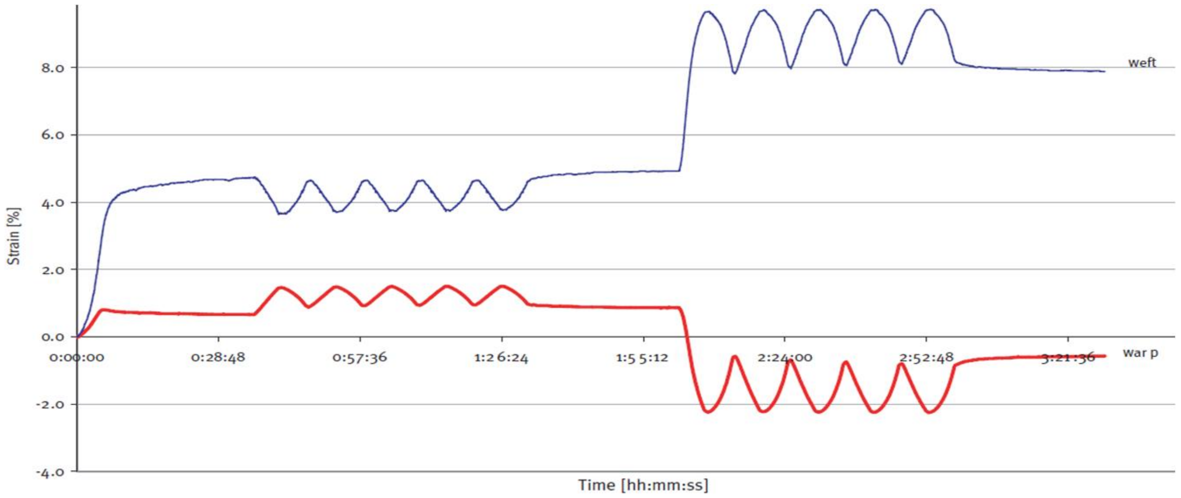
The flattening process ([20] with modifications)



Ensuring the same lengths of the boundary lines of the adjacent patterns ([20] with modifications)



Biaxial test: load history [35]

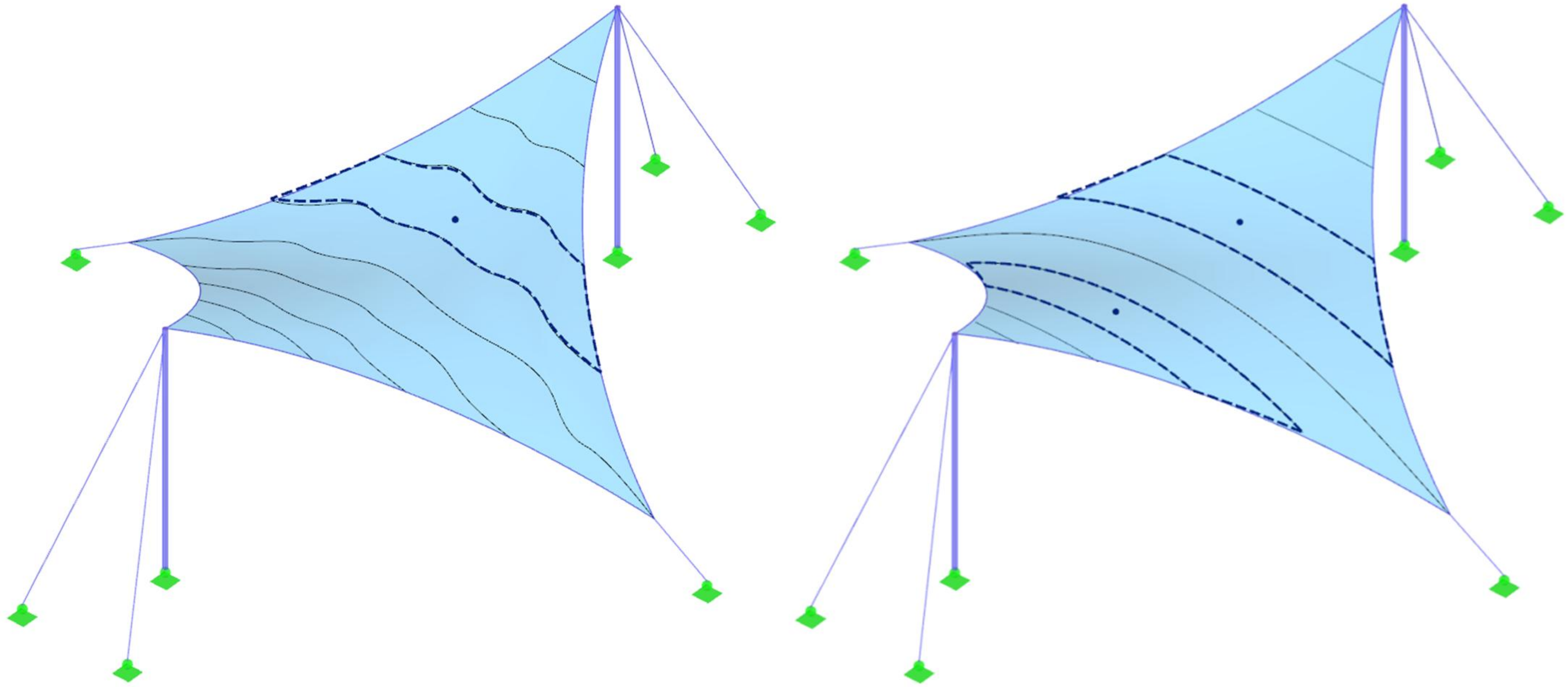


Biaxial test: measured strains [35]

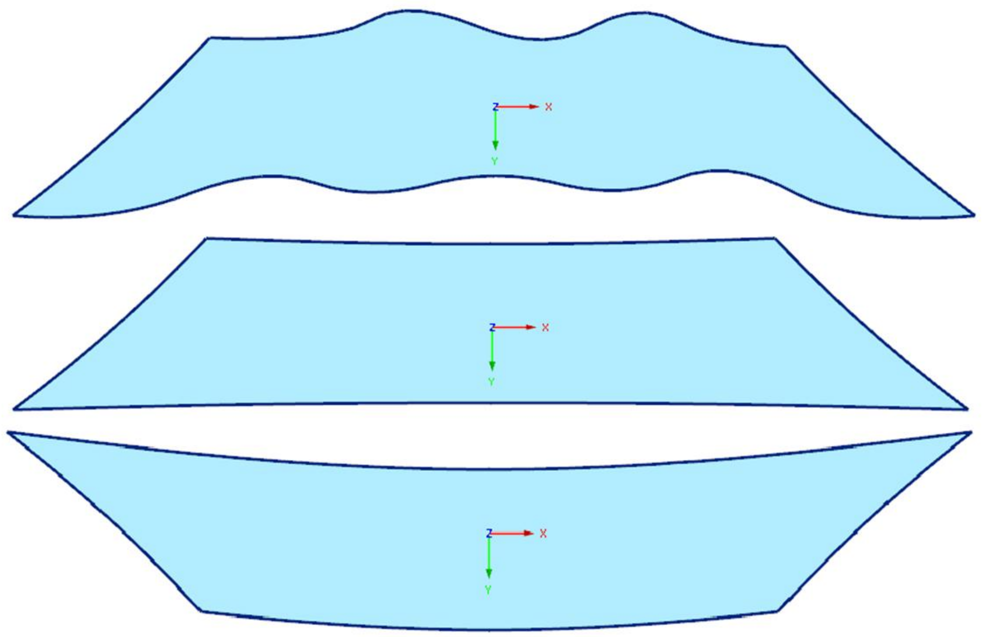
GENERATION OF CUTTING PATTERNS

Hypar Structure

- Different cutting lines



Using different cutting lines to split the membrane; arbitrary lines (left), geodesic lines (right top) and planar sections (right bellow)



Resulting patterns using the arbitrary lines (top), geodesic lines (middle) and planar sections (bellow)

GENERATION OF CUTTING PATTERNS

Hypar Structure

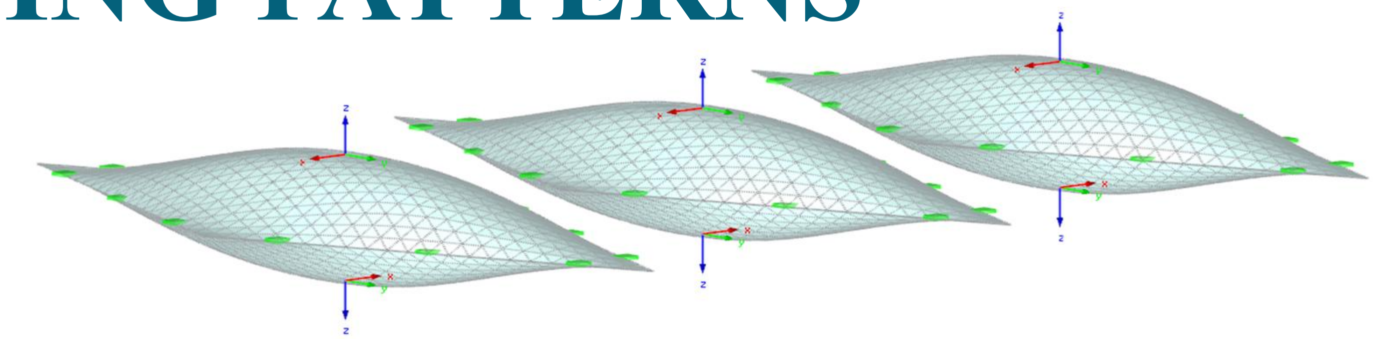
- Different cutting lines

ETFE Cushion

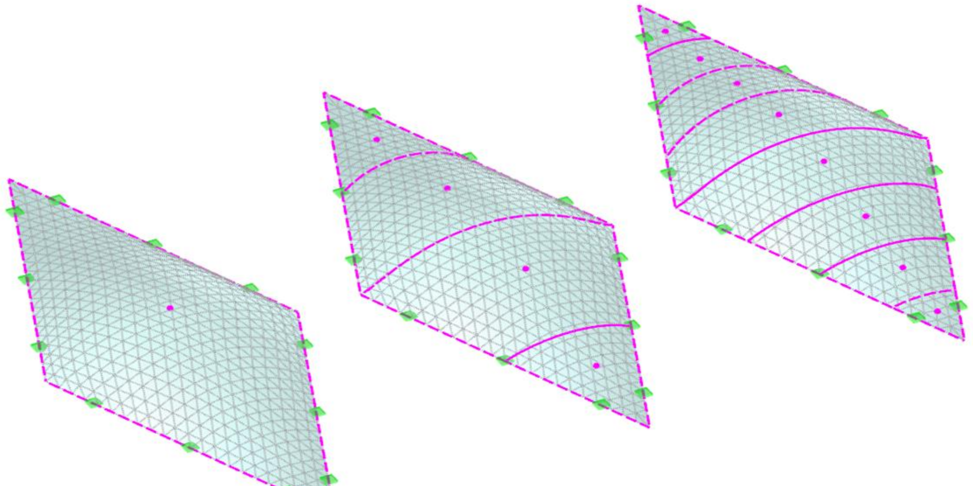
- Three cases of the same cushion
- Patterns evaluation
- Specified material

$E = 900.0\ MPa, G = 310.0\ MPa, \nu = 0.452$

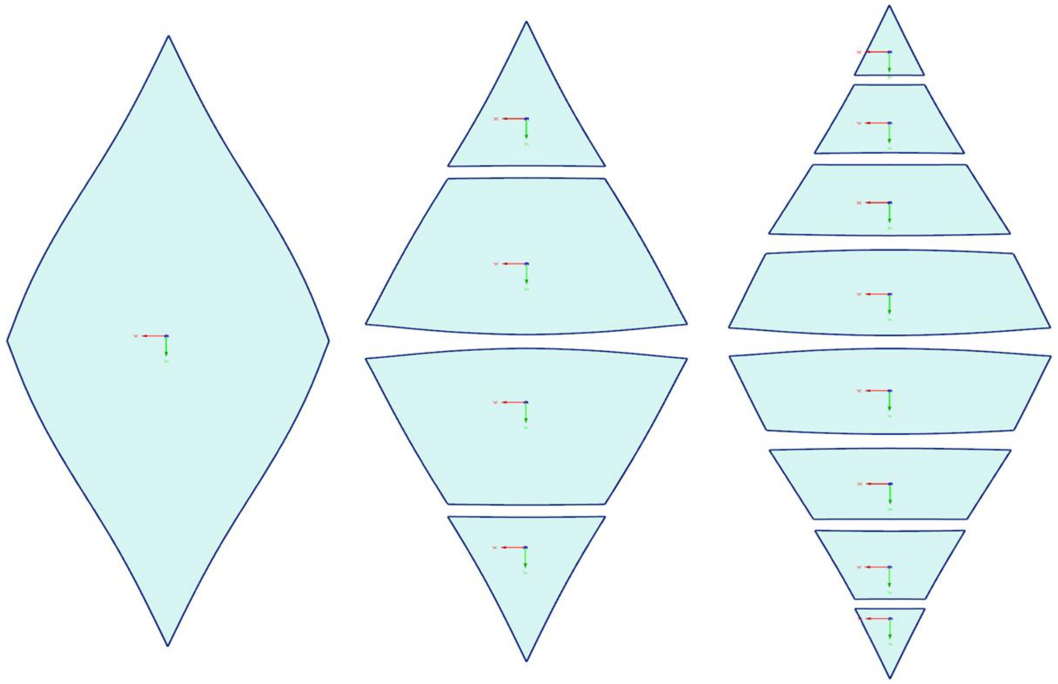
$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ & C_{22} & C_{23} \\ sym. & & C_{33} \end{bmatrix} = \begin{bmatrix} C_{xxxx} & C_{xxyy} & C_{xxxy} \\ & C_{yyyy} & C_{yyxy} \\ sym. & & C_{xyxy} \end{bmatrix} = \begin{bmatrix} 339.18 & 153.18 & 0.00 \\ & 339.18 & 0.00 \\ sym. & & 93.00 \end{bmatrix} \frac{kN}{m}$$



ETFE cushions with the x/y (warp/weft) orientation displaying, FE mesh

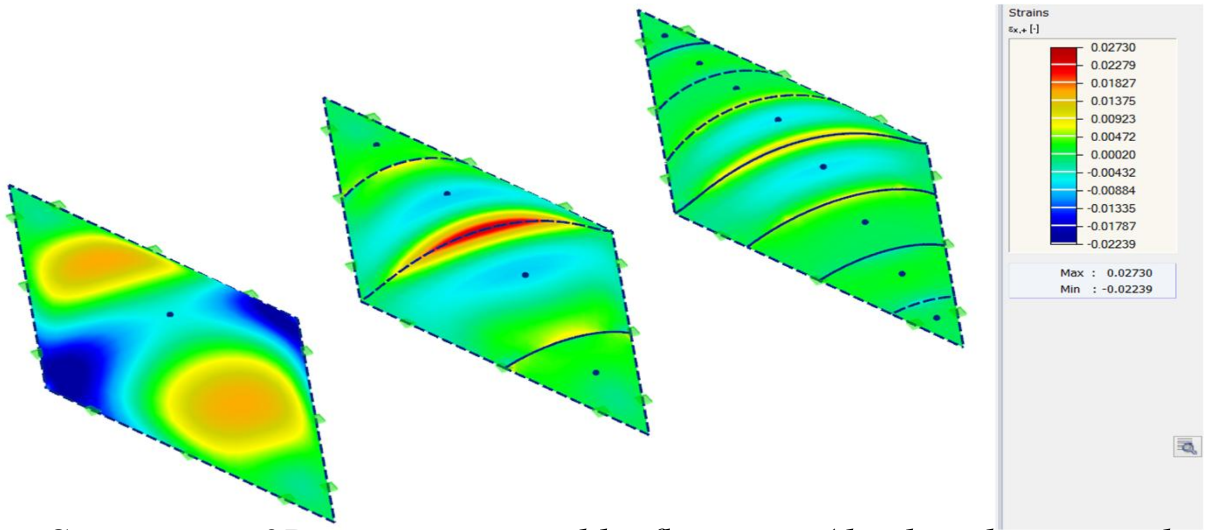


Spatial patterns (3D) with the information that the mathematical squashing was performed

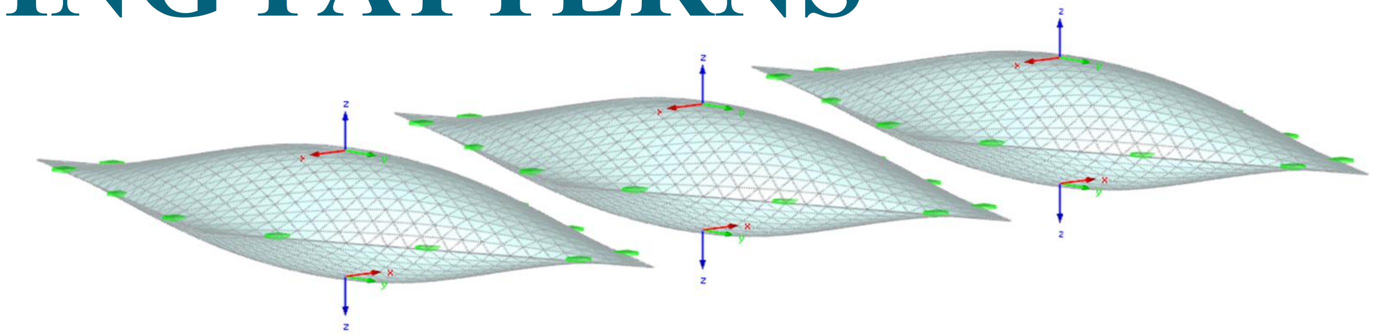


Flattened (2D) patterns with the information that the distortion energy minimization was performed

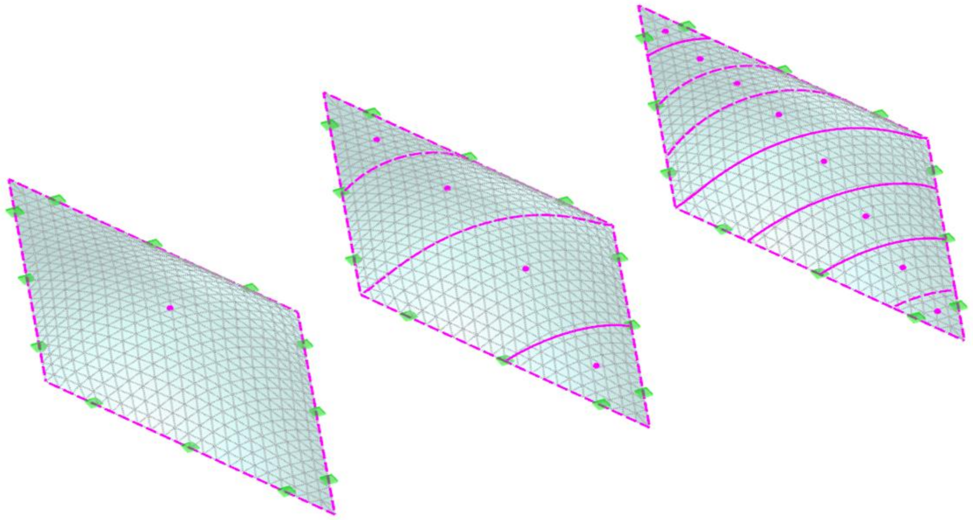
GENERATION OF CUTTING PATTERNS



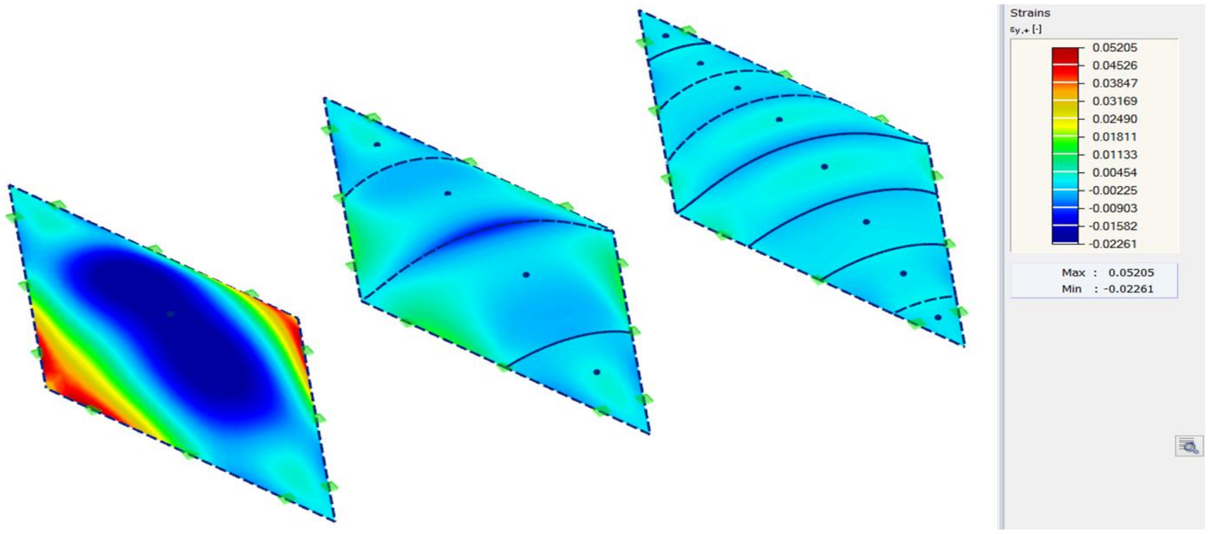
Strains ε_x in 2D patterns caused by flattening (displayed on spatial (3D) patterns for having compact model of all patterns)



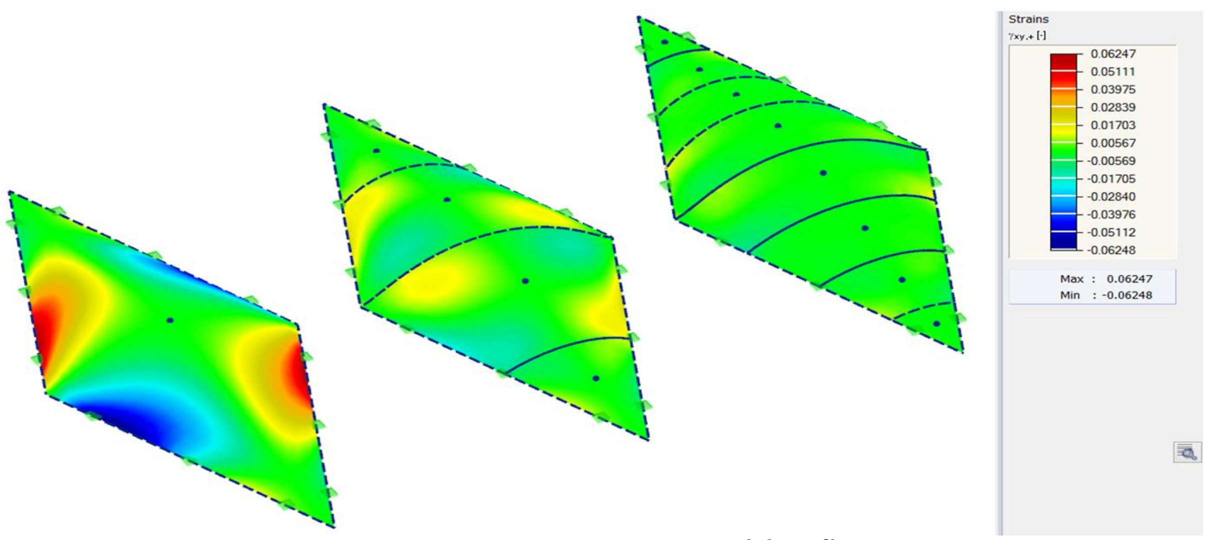
ETFE cushions with the x/y (warp/weft) orientation displaying, FE mesh



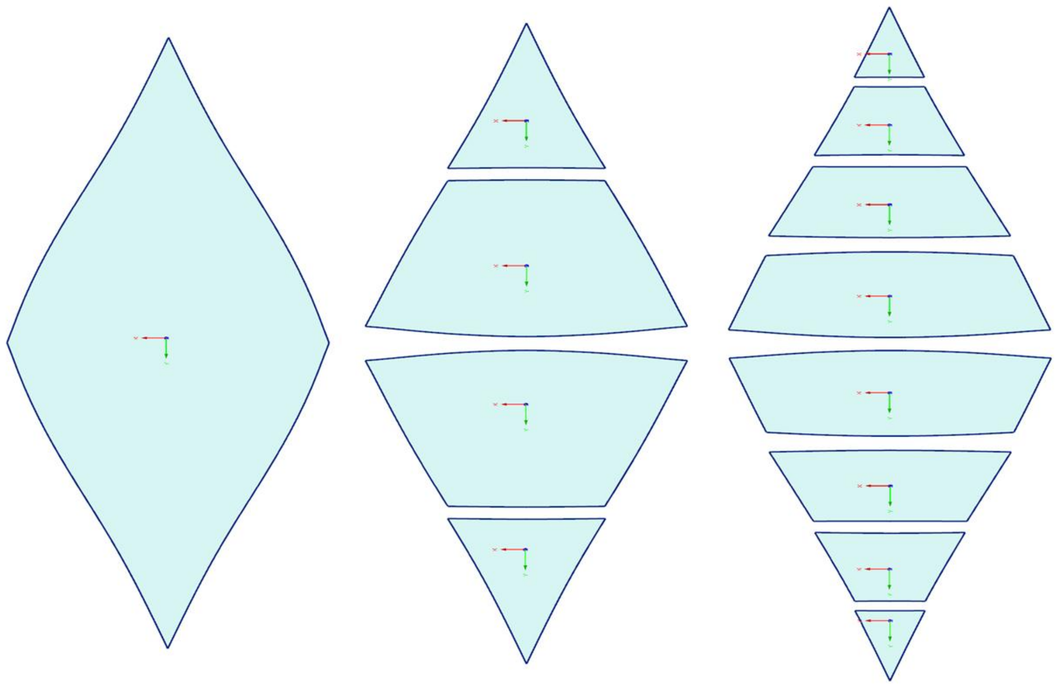
Spatial patterns (3D) with the information that the mathematical squashing was performed



Strains ε_y in 2D patterns caused by flattening

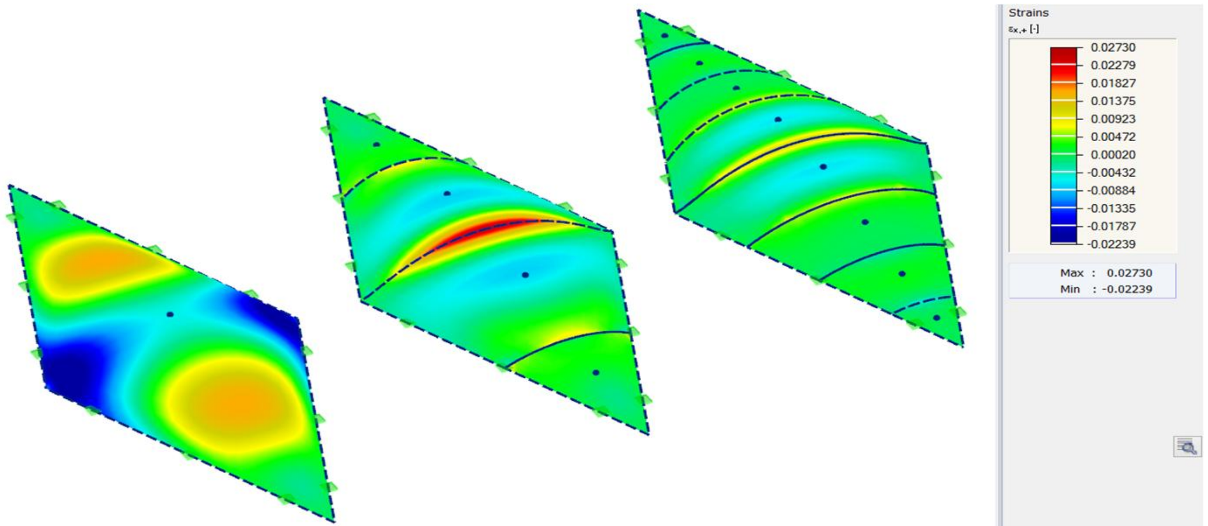


Strains ε_{xy} in 2D patterns caused by flattening

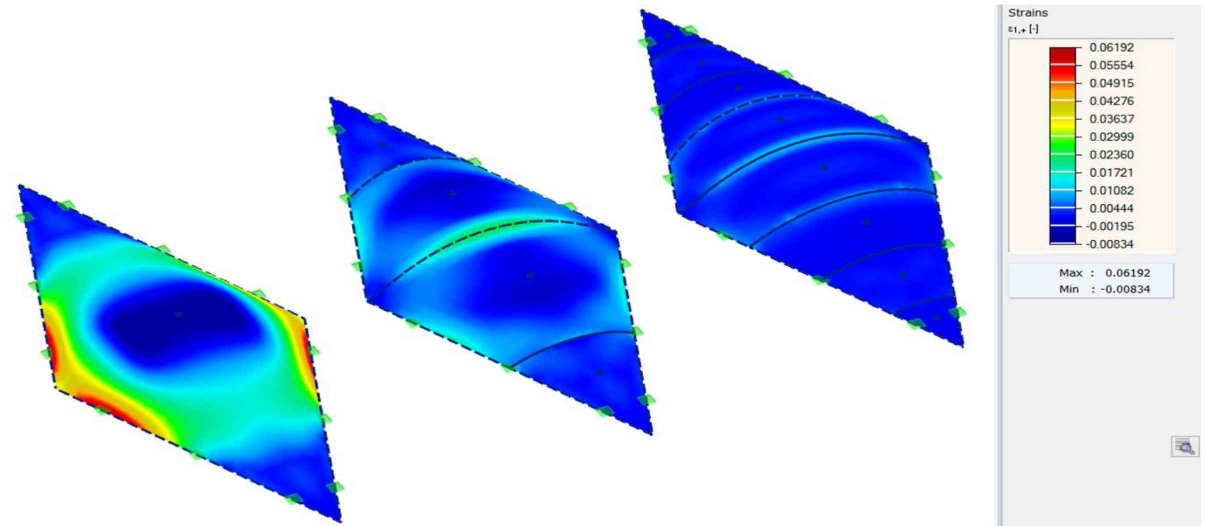


Flattened (2D) patterns with the information that the distortion energy minimization was performed

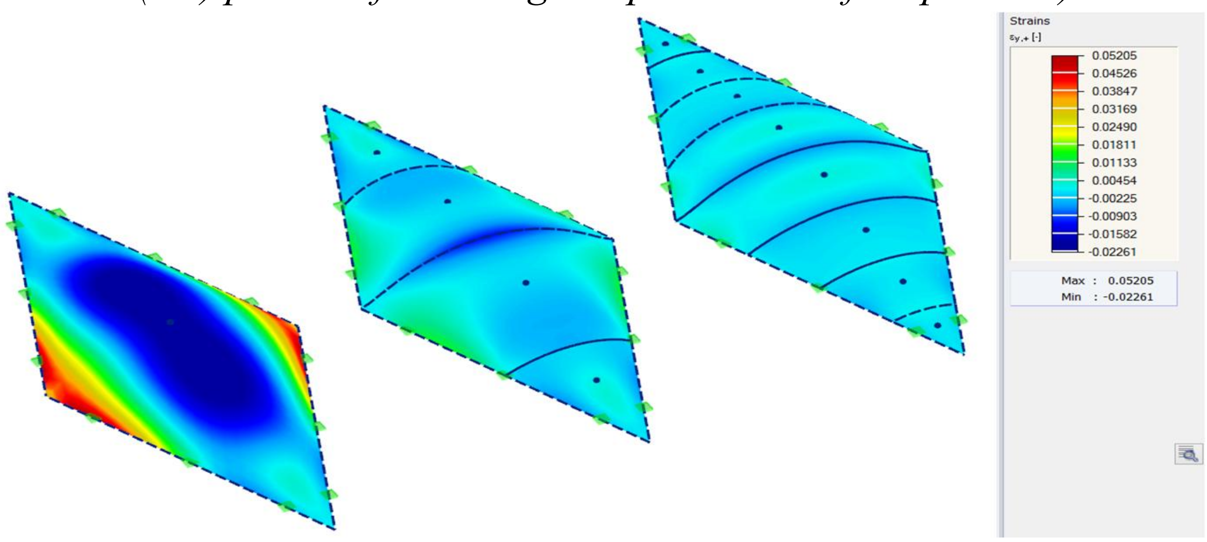
GENERATION OF CUTTING PATTERNS



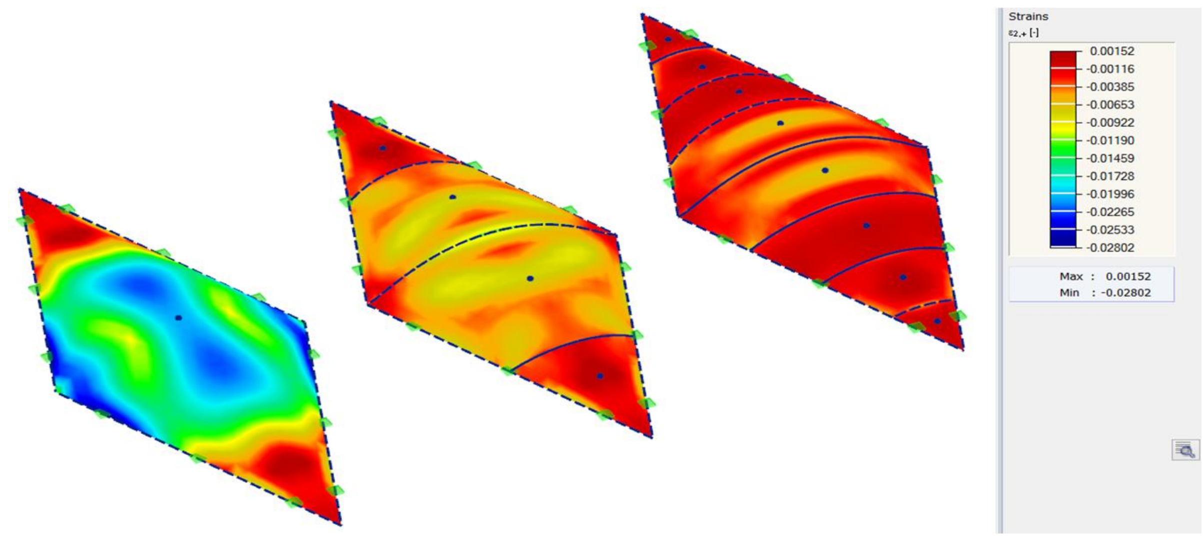
Strains ε_x in 2D patterns caused by flattening (displayed on spatial (3D) patterns for having compact model of all patterns)



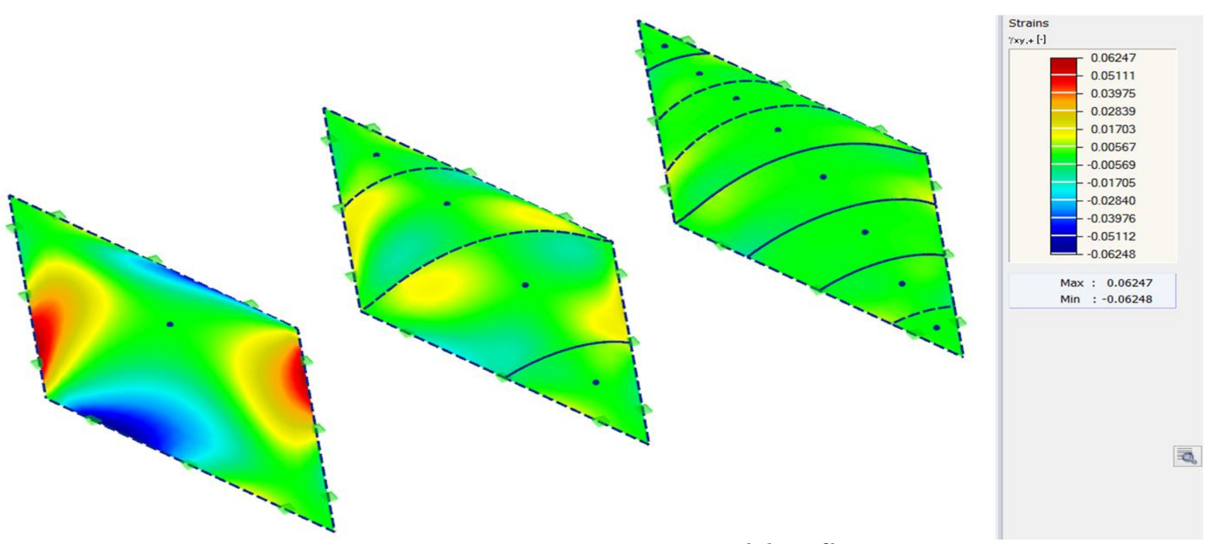
Strains ε_1 in 2D patterns caused by flattening



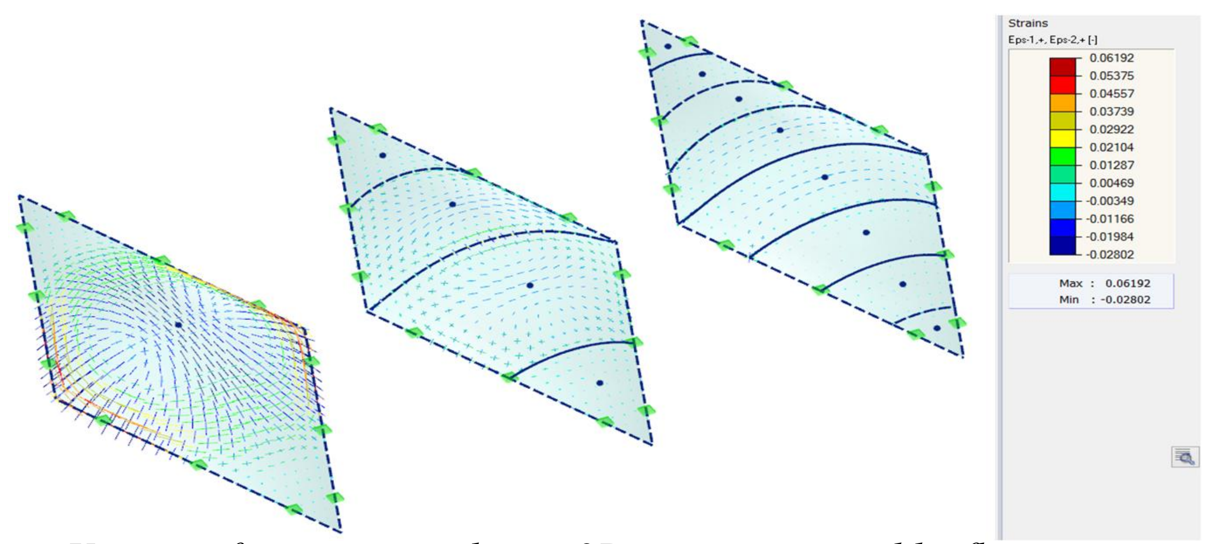
Strains ε_y in 2D patterns caused by flattening



Strains ε_2 in 2D patterns caused by flattening



Strains ε_{xy} in 2D patterns caused by flattening



Vectors of strains ε_1 and ε_2 in 2D patterns caused by flattening

GENERATION OF CUTTING PATTERNS

Hypar Structure

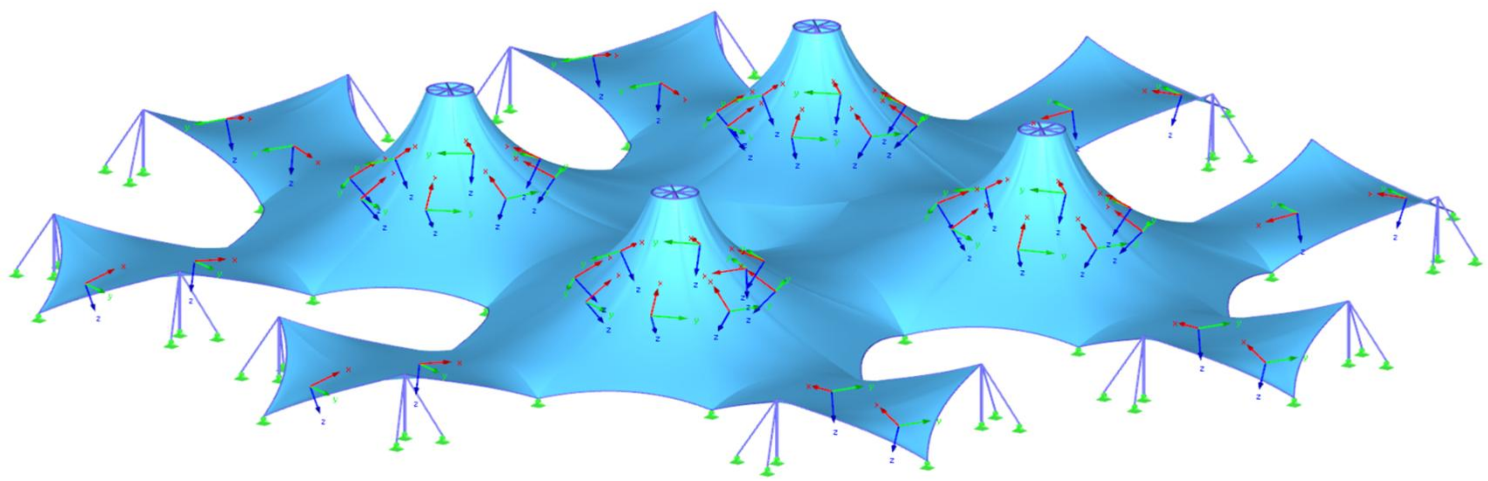
- Different cutting lines

ETFE Cushion

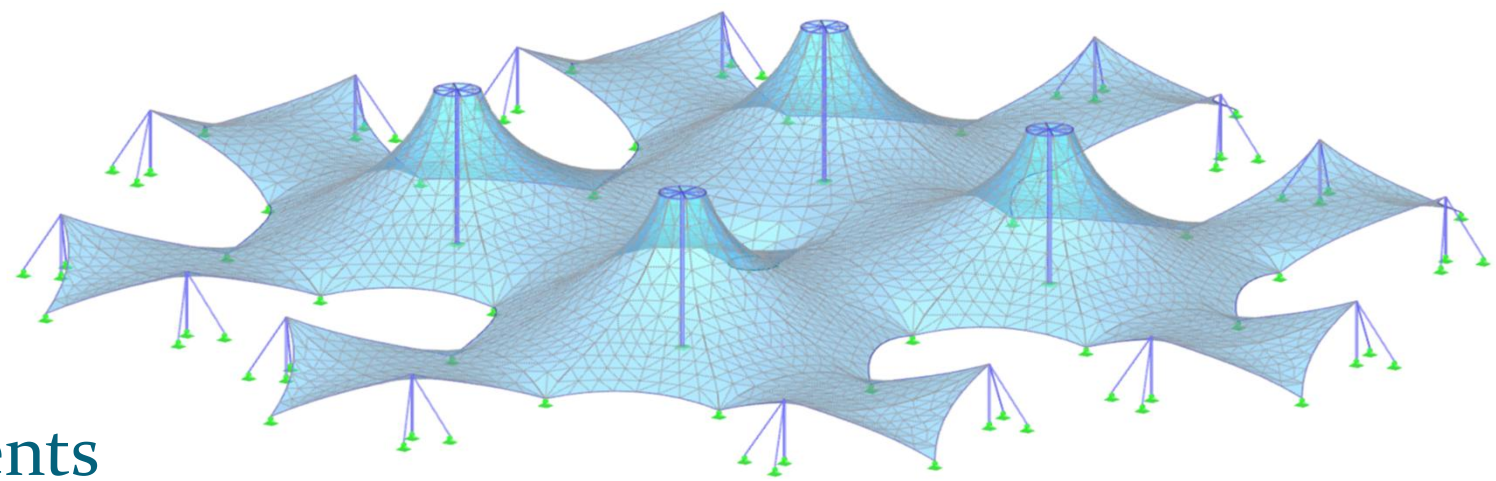
- Three cases of the same cushion
- Patterns evaluation
- Specified material

Membrane structure

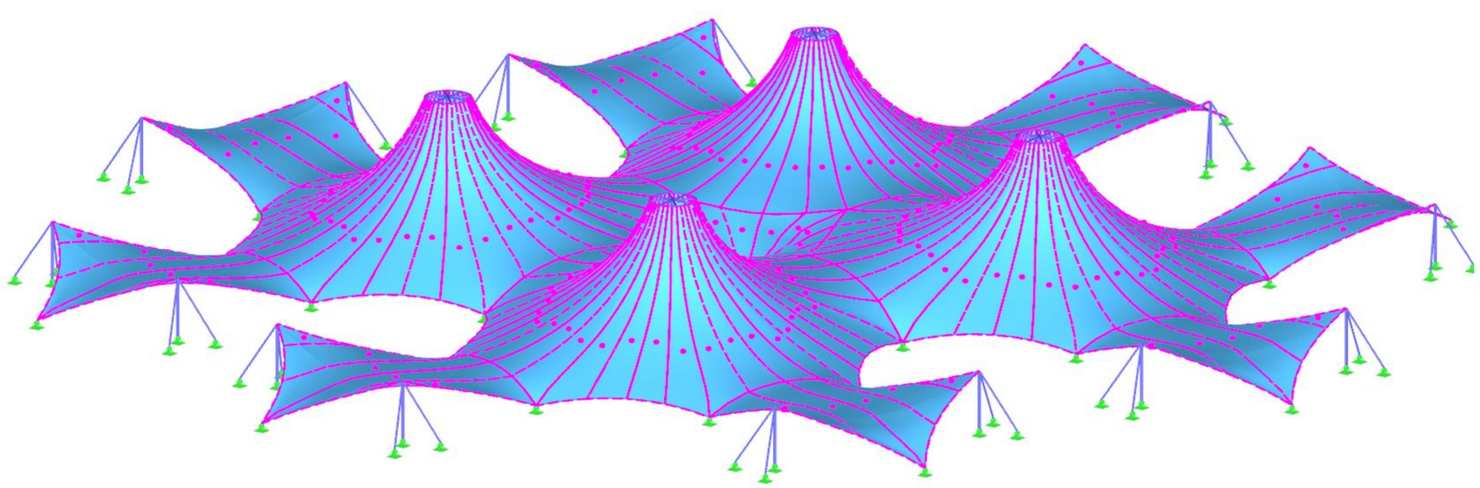
- Ensulign of construction requirements



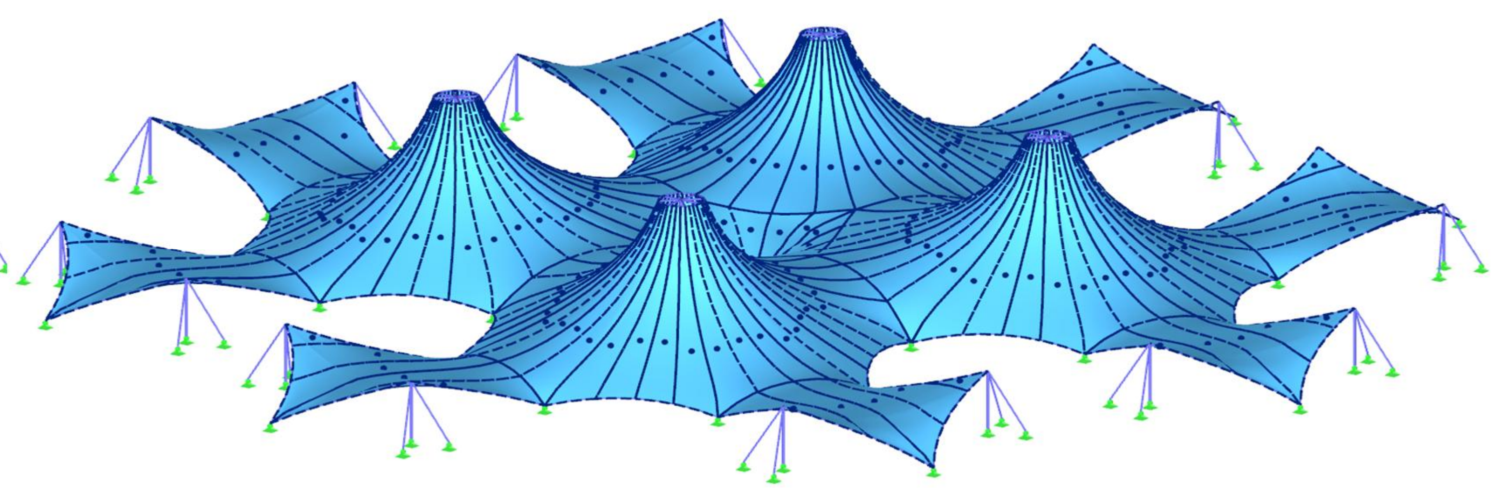
Membrane structure



FE mesh of the membrane structure



Spatial patterns with the information that the mathematical squashing was performed



Spatial patterns with the information that the distortion energy minimization was performed

THANK YOU FOR YOUR ATTENTION

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