Program: RFEM 5

Category: Geometrically Linear Analysis, Orthotropic Plasticity, Plate, Solid

Verification Example: 0009 – One-Dimensional Plasticity - Orthotropic Case

0009 – One-Dimensional Plasticity - Orthotropic Case

Description

A three-dimensional block is fixed at the both ends (end planes are fixed in the Z-direction and always one edge at each plane is restricted to move perpendicularly to its orientation). To prevent horizontal movement of the block one vertical edge is fixed in the X and Y directions. Block is divided in the middle: The upper half is made of an elastic material and the lower part is made of an elasto-plastic othotropic material with the yield surface described according to the Tsai-Wu plasticity theory. Material fibers are oriented by an angle $\beta = 45°$ (**Figure 1**). The block's middle plane is subjected to the vertical pressure. Assuming only the small deformation theory and neglecting block's self-weight, determine its maximum deflection.

Material	Elastic	Modulus of Elasticity	E	11000.000	MPa
		Poisson's Ratio	ν	0.000	–
	Plastic	Modulus of Elasticity	$E_x = E_y$	3000.000	MPa
			E_z	11000.000	MPa
		Poisson's Ratio	$\nu_{xy} = \nu_{yz} = \nu_{xz}$	0.000	–
		Shear Modulus	$G_{xy} = G_{yz} = G_{xz}$	5500.000	MPa
		Tensile Plastic Strength	$f_{t,x} = f_{t,z}$	7.000	MPa
			$f_{t,y}$	4.949	MPa
		Compressive Plastic Strength	$f_{c,x} = f_{c,z}$	7.000	MPa
			$f_{c,y}$	4.949	MPa
		Shear Plastic Strength	$f_{v,xy} = f_{v,zy} = f_{v,xz}$	99999.999	MPa
		Fiber Angle	β	45	°
Geometry	Beam	Side Length	d	0.050	m
		Height	h	2.000	m
Load		Pressure	p	32.000	MPa

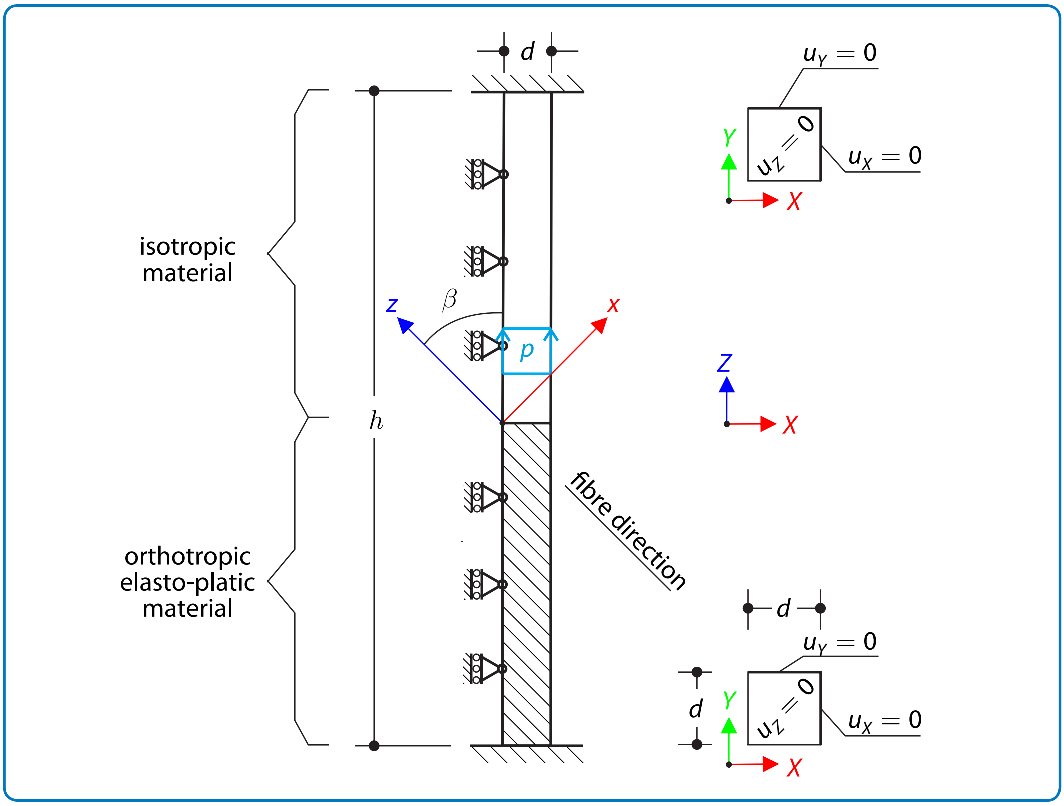

Figure 1: Problem sketch

Analytical Solution

Linear Analysis

Acting pressure can be divided between both halves of the block:

$$p = \sigma_{\text{iso}} + \sigma_{\text{ortho}} \tag{9 – 1}$$

where σ_{iso} is stress in the isotropic part of the block:

$$\sigma_{\text{iso}} = \varepsilon_{\text{iso}} E = \frac{2u_{Z,\text{max}}}{h} E \tag{9 – 2}$$

and σ_{ortho} is stress in the orthotropic part:

$$\sigma_{\text{ortho}} = \varepsilon_{\text{ortho}} E_Z = \frac{2u_{Z,\text{max}}}{h} E_Z \tag{9 – 3}$$

where E_Z is modulus of elasticity of the orthotropic part in the Z direction and can be derived according to the example 0007:

$$E_Z = \frac{1}{\frac{\sin^4 \beta}{E_x} + \frac{\cos^4 \beta}{E_z} + \frac{\sin^2 \beta \sin^2 \beta}{G_{xz}}} \tag{9 – 4}$$

Combining equation **(9 – 1)**, **(9 – 2)** and **(9 – 3)** formula for maximum deflection $u_{Z,\max}$ can be given:

$$u_{Z,\max} = \frac{ph}{2(E_Z + E)} = 1.818 \text{ mm} \tag{9 – 5}$$

Nonlinear Analysis

Considering $f_{c,x}; f_{c,y}; f_{c,z} \geq 0$, the Tsai-Wu surface of plasticity can be described by the following equation:

$$F = \sigma_x\left(\frac{1}{f_{t,x}} - \frac{1}{f_{c,x}}\right) + \sigma_y\left(\frac{1}{f_{t,y}} - \frac{1}{f_{c,y}}\right) + \sigma_z\left(\frac{1}{f_{t,z}} - \frac{1}{f_{c,z}}\right) + \frac{\sigma_x^2}{f_{t,x}f_{c,x}} + \frac{\sigma_y^2}{f_{t,y}f_{c,y}} + \frac{\sigma_z^2}{f_{t,z}f_{c,z}} + \frac{\tau_{yz}^2}{f_{v,yz}} + \frac{\tau_{xz}^2}{f_{v,xz}} + \frac{\tau_{xy}^2}{f_{v,xy}} - 1 = 0 \tag{9 – 6}$$

It is necessary to transform coordinates of the elasto-plastic material into the fiber direction, Because the loading pressure is applied in the Z-direction of the global axis system:

$$\begin{bmatrix} \sigma_x \\ \sigma_z \\ \tau_{xz} \end{bmatrix} = \begin{bmatrix} c^2 & s^2 & -2sc \\ s^2 & c^2 & 2sc \\ sc & -sc & c^2 - s^2 \end{bmatrix} \begin{bmatrix} \sigma_X \\ \sigma_Z \\ \tau_{XZ} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -1 \\ \frac{1}{2} & \frac{1}{2} & 1 \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} \sigma_X \\ \sigma_Z \\ \tau_{XZ} \end{bmatrix} \tag{9 – 7}$$

There is no pressure acting in the X-direction and applied pressure acts on the whole cross-section area in the Z-direction, stresses σ_X and τ_{XZ} are equal to zero and previous formula yields to:

$$\sigma_x = \sigma_z = \frac{\sigma_Z}{2} \tag{9 – 8}$$

The shear stresses are not important due to the high shear strength and the Tsai-Wu yield surface definition takes the following form:

$$\frac{\sigma_x^2}{f_{t,x}f_{c,x}} + \frac{\sigma_z^2}{f_{t,z}f_{c,z}} = 1 \tag{9 – 9}$$

Combining equations **(9 – 8)** and **(9 – 9)**, the expression for the stress in the plastic part can be obtained:

$$\sigma_Z = \sqrt{2f_{t,z}f_{c,z}} \tag{9 – 10}$$

The stress in the elastic part is then:

$$\sigma_{Z,\text{elastic}} = -p + \sigma_Z \tag{9 – 11}$$

The maximum displacement can be then simply evaluated as follows:

$$u_{Z,\text{max}} = \frac{|\sigma_{Z,\text{elastic}}|}{E}\frac{h}{2} = 2.009 \text{ mm} \qquad (9-12)$$

RFEM 5 Settings

- Modeled in version RFEM 5.05.0030
- The element size is $l_{\text{FE}} = 0.050$ m
- Geometrically linear analysis is considered
- The number of increments is 5
- The Mindlin plate theory is used

Results

Structure File	Entity	Material Model
0009.01	Solid	Orthotropic Plastic 3D
0009.02	Plate	Orthotropic Plastic 2D
0009.03	Solid	Orthotropic Elastic 3D
0009.04	Plate	Orthotropic Elastic 2D

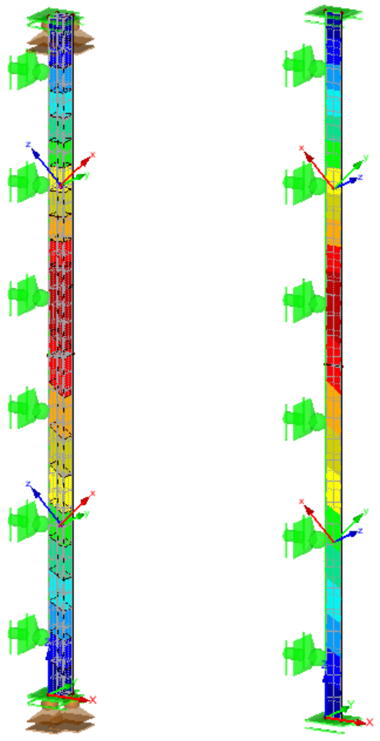

Figure 2: RFEM 5 results for 3D model on the left and 2D model on the right

As can be seen from the table below, good agreement of numerical outputs with analytical results was achieved.

Linear Analysis

Analytical Solution	RFEM 5 Orthotropic Elastic 3D		RFEM 5 Orthotropic Elastic 2D	
$u_{Z,\max}$ [mm]	$u_{Z,\max}$ [mm]	Ratio [-]	$u_{Z,\max}$ [mm]	Ratio [-]
1.818	1.838	1.011	1.834	1.009

Nonlinear Analysis

Analytical Solution	RFEM 5 Orthotropic Plastic 3D		RFEM 5 Orthotropic Plastic 2D	
$u_{Z,\max}$ [mm]	$u_{Z,\max}$ [mm]	Ratio [-]	$u_{Z,\max}$ [mm]	Ratio [-]
2.009	2.026	1.008	2.023	1.007