Program:	RFEM 5, RSTAB 8
Category:	Large Deformation Analysis, Isotropic Linear Elasticity, Member
Verification Example:	0043 – Cantilever Bend to Form a Circle

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Description

Determine the bending moment M, which acts at the free end of the cantilever and which bends the member to a circular shape. Neglecting beam's self weight, assuming the large deformation theory and loading the cantilever with this particular M, check the maximum deflections $u_{X,\max}$ and $u_{Z,\max}$.

Material	Steel	Modulus of Elasticity	E	210.000	GPa
		Shear Modulus	G	81.000	GPa
Geometry	Beam	Length	L	4000.000	mm
		Diameter	d	42.400	mm
		Wall Thickness	t	4.000	mm

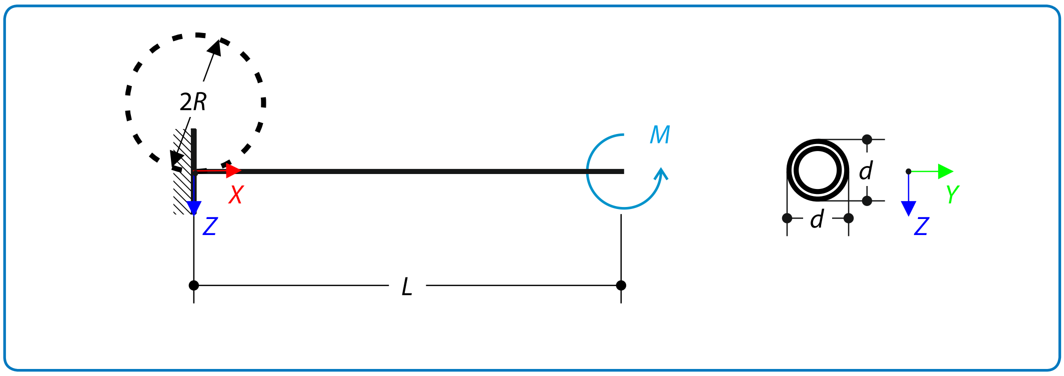

Figure 1: Problem sketch

Analytical Solution

The second moment of inertia around y axis I_y equals to (see **Figure 1**):

$$I_y = \frac{\pi\left[d^4 - (d-2t)^4\right]}{64} \approx 89908.5\ \mathrm{mm}^4 \qquad (43-1)$$

A beam in the large deformation analysis is described by the nonlinear differential equation

$$\kappa(x) = \frac{u_z''(x)}{\left[1 + (u_z'(x))^2\right]^{\frac{3}{2}}} = -\frac{M}{EI_y} \qquad (43-2)$$

which is an equation difficult to solve in general. However, the term on the right-hand side is constant and consequently the left-hand side, which is nothing else then the beam curvature κ, is constant also. The only curve which has constant curvature is a circle, therefore, the solution to this problem is a circle arc of radius R. We get

$$u_{x,\max} = R \sin\alpha - L \quad (43-3)$$

$$u_{z,\max} = R(1 - \cos\alpha) \quad (43-4)$$

where the radius of the circular arc equals to

$$R = \left|\frac{1}{\kappa(x)}\right| = \frac{EI_y}{M} \quad (43-5)$$

The angle of the circular arc equals to $\alpha = \frac{L}{R}$. In our case $\alpha = 2\pi$, which yields

$$R = \frac{L}{2\pi} \approx 636.620 \text{ mm} \quad (43-6)$$

The equations **(43 – 5)** and **(43 – 6)** yield the required loading moment

$$M = 2\pi \frac{EI_y}{L} \approx 29657.585 \text{ Nm} \quad (43-7)$$

Moreover, equations **(43 – 3)** and **(43 – 4)** yield the unknown maximum displacements

$$u_{X,\max} = -L = -4000.0 \text{ mm} \quad (43-8)$$

$$u_{Z,\max} = -2R \approx -1273.2 \text{ mm} \quad (43-9)$$

RFEM 5 and RSTAB 8 Settings

- Modeled in version RFEM 5.05.0030 and RSTAB 8.05.0030
- The element size is $l_{FE} = 0.004$ m
- The number of increments is 1
- Isotropic linear elastic material model is used
- Member division for large deformation or post-critical analysis is activated

Results

Structure File	Program
0043.01	RFEM 5
0043.02	RSTAB 8

Good agreement of the numerical results with the analytical solution was achieved:

Displacement	Analytical Solution	RFEM 5		RSTAB 8	
	[mm]	[mm]	Ratio [-]	[mm]	Ratio [-]
$u_{X,\max}$	-4000.0	-3998.5	1.000	-4000.0	1.000
$u_{Z,\max}$	-1273.2	-1273.1	1.000	-1273.2	1.000