Program:	RFEM 5, RFEM 6
Category:	Geometrically Linear Analysis, Isotropic Linear Elasticity, Plate
Verification Example:	0064 – Thick–Walled Vessel

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Description

A thick–walled vessel is loaded by inner and outer pressure. The vessel is open–ended, thus there is no axial stress. The problem is modeled as a quarter model (see **Figure 1**) and described by the following set of parameters.

Material	Modulus of Elasticity	E	1.000	MPa
	Poisson's Ratio	ν	0.250	–
Geometry	Inner Radius	r_1	200.000	mm
	Outer Radius	r_2	300.000	mm
Load	Inner Pressure	p_1	60.000	kPa
	Outer Pressure	p_2	10.000	kPa

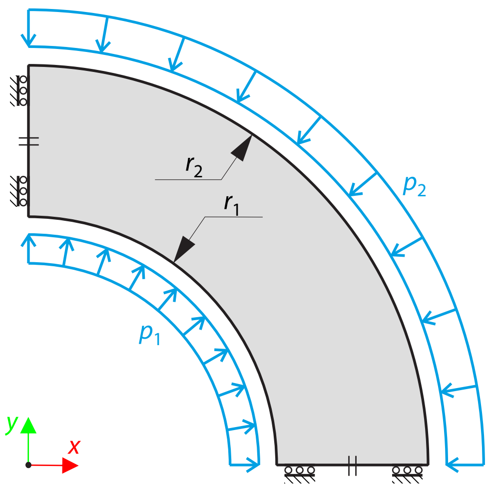

Figure 1: Problem Sketch

Determine the radial deflection of the inner and outer radius $u_r(r_1)$, $u_r(r_2)$. The self-weight is neglected.

Analytical Solution

The stress state of the thick–walled vessel is described by the equation of equilibrium

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_t}{r} = 0 \qquad (64 – 1)$$

where σ_r, σ_t and r is the radial stress, tangential stress and radius respectively. The relation between strains and deflections is described as follows:

$$\varepsilon_r(r) = \frac{du_r(r)}{dr} \tag{64 – 2}$$

$$\varepsilon_t(r) = \frac{u_r(r)}{r} \tag{64 – 3}$$

$$\sigma_t(r) = \frac{E}{1-\nu^2}\left[\varepsilon_t(r) + \nu\varepsilon_r(r)\right] \tag{64 – 4}$$

$$\sigma_r(r) = \frac{E}{1-\nu^2}\left[\varepsilon_r(r) + \nu\varepsilon_t(r)\right] \tag{64 – 5}$$

Using these equations and Hooke's Law, equations **(64 – 4)** and **(64 – 5)**, the second–order differential equation is obtained.

$$\frac{d^2u_r(r)}{dr^2} + \frac{du_r(r)}{dr} - \frac{u_r(r)}{r} = 0 \tag{64 – 6}$$

The solution of this differential equation is supposed in the following form:

$$u_r(r) = r^n \tag{64 – 7}$$

Using this assumption, the coefficient n yields $n = \pm 1$. Thus the solution can be writen as a linear combination:

$$u_r(r) = C_1 r + \frac{C_2}{r} \tag{64 – 8}$$

This solution can be substituted into Hooke's Law and simplified into the following form:

$$\sigma_t(r) = K + \frac{C}{r^2} \tag{64 – 9}$$

$$\sigma_r(r) = K - \frac{C}{r^2} \tag{64 – 10}$$

where K and C are real constants, which could be obtained using the following boundary conditions.

$$\sigma_r(r_1) = -p_1 \tag{64 – 11}$$

$$\sigma_r(r_2) = -p_2 \tag{64 – 12}$$

$$K = \frac{p_1 r_1^2 - p_2 r_2^2}{r_2^2 - r_1^2} \tag{64 – 13}$$

$$C = (p_1 - p_2)\frac{r_1^2 r_2^2}{r_2^2 - r_1^2} \tag{64 – 14}$$

The radial deflection of the inner and outer radius of the open–ended vessel $u_r(r_1)$, $u_r(r_2)$ can be then determined using the above mentioned equations and Hooke's Law again.

$$u_r(r_1) = \frac{r_1}{E}\left[\sigma_t(r_1) - \nu\sigma_r(r_1)\right] = 27.000\text{mm} \qquad (64-15)$$

$$u_r(r_2) = \frac{r_2}{E}\left[\sigma_t(r_2) - \nu\sigma_r(r_2)\right] = 21.750\text{mm} \qquad (64-16)$$

RFEM Settings

- Modeled in RFEM 5.06 and RFEM 6.01
- The element size is $l_{FE} = 0.002$ m
- Isotropic linear elastic material model is used

Results

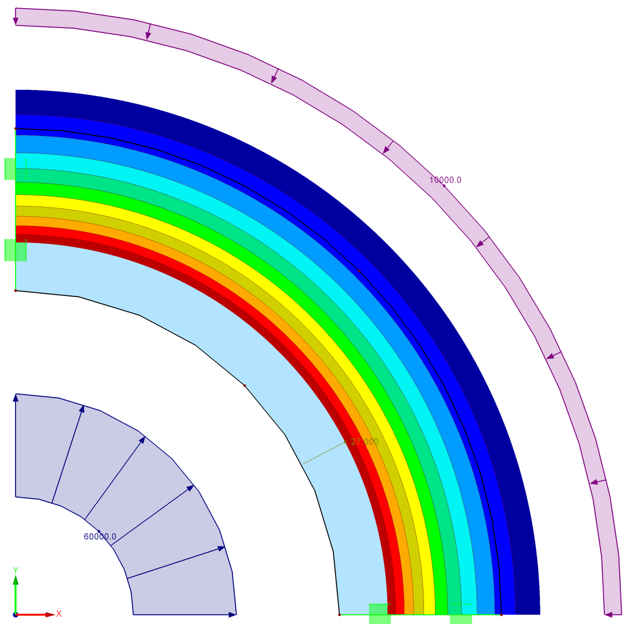

Figure 2: Results in RFEM - total deflection

Structure Files	Program
0064.01	RFEM 5, RFEM 6

Quantity	Analytical Solution	RFEM 5	Ratio	RFEM 6	Ratio
$u_r(r_1)$ [mm]	27.000	27.000	1.000	27.000	1.000
$u_r(r_2)$ [mm]	21.750	21.750	1.000	21.747	1.000