Program: RFEM 5, RSTAB 8

Category: Geometrically Linear Analysis, Isotropic Linear Elasticity, Temperature Dependency, Member, Plate, Solid

Verification Example: 0076 – Truss Structure with Thermal Loading

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Description

A truss structure consists of three rods - one steel (no. 2) and two copper (no. 1, 3) joined by a rigid member. The structure is loaded by a concentrated force F and by a temperature difference T_c according to **Figure 1**, see also [1].

Material	Steel	Modulus of Elasticity	E_s	210000.000	MPa
		Poisson's Ratio	ν_s	0.300	–
		Coefficient of Thermal Expansion	α_s	1.200×10^{-5}	$°C^{-1}$
	Copper	Modulus of Elasticity	E_c	130000.000	MPa
		Poisson's Ratio	ν_c	0.354	–
		Coefficient of Thermal Expansion	α_c	2.000×10^{-5}	$°C^{-1}$
Geometry		Cross-section Width	a	10.000	mm
		Length	L	500.000	mm
Load		Concentrated Force	F	1000.000	N
		Thermal Loading	T_c	30.000	°C

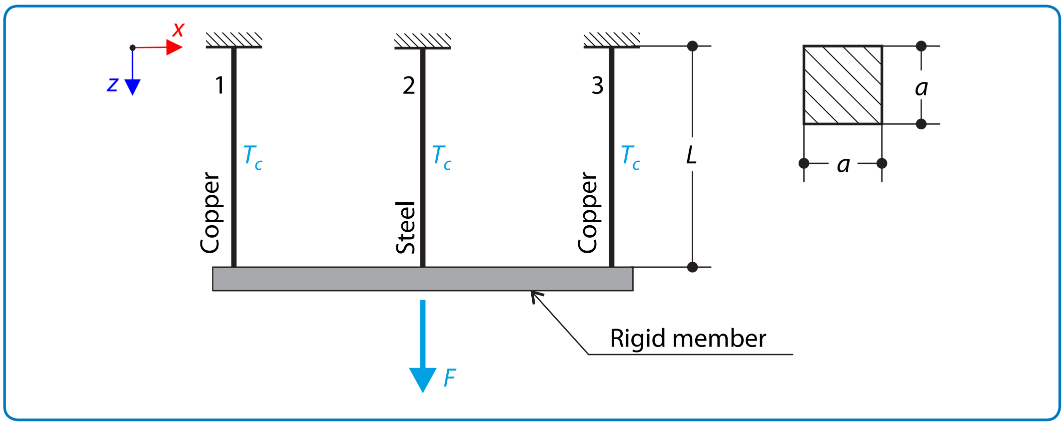

Figure 1: Problem Sketch

While neglecting self-weight, determine the total deflection u_z of the structure.

Analytical Solution

The total deflection u_z of the structure consists of the deflection u_{zF} caused by the concentrated force and the deflection u_{zT} by the thermal loading. Due to small-deformation linear analysis, the total deflection can be written as the sum of these deflections

$$u_z = u_{zF} + u_{zT} \tag{76 – 1}$$

Concentrated-Force Loading

The extension ΔL of the rod with length L and cross-section area A due to the axial force N can generally be expressed by the following formula

$$\Delta L = \frac{NL}{EA} \tag{76 – 2}$$

This applies to both steel and copper rods

$$\Delta L_s = \frac{N_s L}{E_s A} \tag{76 – 3}$$

$$\Delta L_c = \frac{N_c L}{E_c A} \tag{76 – 4}$$

Due to the rigid-beam coupling and symmetry, the extension of both steel and copper rods has to be identical

$$\Delta L_s = \Delta L_c \tag{76 – 5}$$

The force equilibrium can be derived from the free body diagram

$$F = N_s + 2N_c \tag{76 – 6}$$

Using equations **(76 – 5)** and **(76 – 6)**, the inner forces in steel and cooper rod are determined

$$N_s = \frac{FE_s}{(2E_c + E_s)} \tag{76 – 7}$$

$$N_c = \frac{FE_c}{(2E_c + E_s)} \tag{76 – 8}$$

The deflection of the truss structure due to the force loading is then

$$u_{zF} = \frac{N_s L}{E_s A} \approx 0.106 \text{ mm} \tag{76 – 9}$$

Thermal Loading

The extension ΔL of the rod with length L due to the temperature difference T_c can generally be expressed by the following formula:

$$\Delta L = \alpha L T_c \qquad (76 - 10)$$

Due to the different coefficient of the thermal expansion of steel and copper, the rods will be further pressed or tensioned by additional axial forces.

$$\Delta L_s = \alpha_s L T_c + \frac{N_s L}{E_s A} \qquad (76 - 11)$$

$$\Delta L_c = \alpha_c L T_c - \frac{N_c L}{E_c A} \qquad (76 - 12)$$

And as before, due to the rigid beam coupling and symmetry, the extensions of both steel and copper rods have to be identical

$$\Delta L_s = \Delta L_c \qquad (76 - 13)$$

From the force equilibrium

$$2N_c - N_s = 0 \qquad (76 - 14)$$

the inner axial force in the copper rod can be determined

$$N_c = \frac{(\alpha_c - \alpha_s) T_c A}{\frac{1}{E_c} + \frac{2}{E_s}} \qquad (76 - 15)$$

The deflection of the structure is then calculated according to **(76 – 11)** and **(76 – 12)**

$$u_{zT} \approx 0.246 \text{ mm} \qquad (76 - 16)$$

Finally, the total deflection of the structure is

$$u_z = u_{zF} + u_{zT} \approx 0.353 \text{ mm} \qquad (76 - 17)$$

RFEM 5 and RSTAB 8 Settings

- Modeled in RFEM 5.16.01 and RSTAB 8.16.01
- The element size is $l_{FE} = 0.050$ m, for Solid model $l_{FE} = 0.010$ m
- Isotropic linear elastic model is used

Results

Structure Files	Program	Entity
0076.01	RFEM 5	Member
0076.02	RSTAB 8	Member
0076.03	RFEM 5	Plate
0076.04	RFEM 5	Solid

Model	Analytical Solution	RFEM 5 / RSTAB 8 Solution	
	u_z [mm]	u_z [mm]	Ratio [-]
RFEM 5, Member	0.353	0.353	1.000
RSTAB 8, Member		0.353	1.000
RFEM 5, Plate		0.354	1.003
RFEM 5, Solid		0.354	1.003

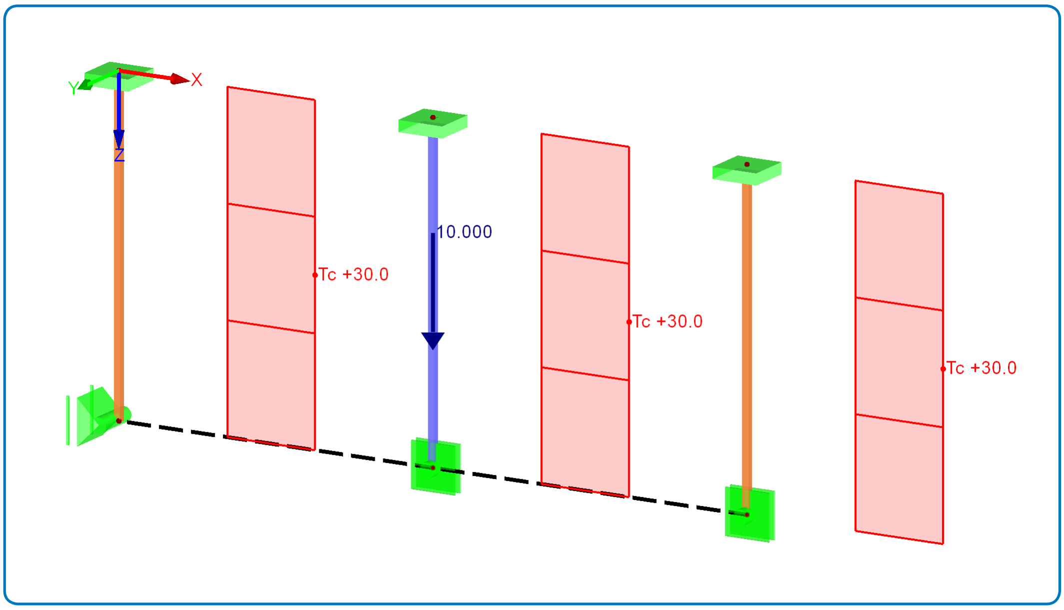

Figure 2: The truss structure modeled in RFEM 5

References

[1] TIMOSHENKO, S. *Strength of Material, Part I, Elementary Theory and Problems, 3rd Edition*. D. Van Nostrand Co., Inc., New York, NY, 1955.