Program: RFEM 5, RFEM 6

Category: Geometrically Linear Analysis, Isotropic Nonlinear Elasticity, Isotropic Plasticity, Member, Plate

Verification Example: 0017 – Plastic Bending - Continuous Load

0017 – Plastic Bending - Continuous Load

Description

A thin plate is fully fixed on the left end ($x = 0$) and subjected to a uniform pressure p according to the **Figure 1**. The problem is described by the following set of parameters.

Material	Elastic-Plastic	Modulus of Elasticity	E	210000.000	MPa
		Poisson's Ratio	ν	0.000	–
		Shear Modulus	G	105000.000	MPa
		Plastic Strength	f_y	240.000	MPa
Geometry	Plate	Length	L	1.000	m
		Width	w	0.050	m
		Thickness	t	0.005	m
Load		Pressure	p	2750.000	Pa

Small deformations are considered and the self-weight is neglected in this example. Determine the maximum deflection $u_{z,\max}$.

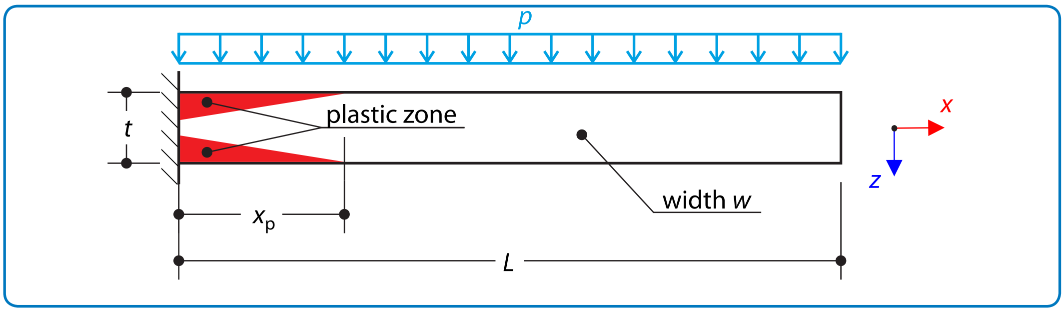

Figure 1: Problem sketch

Analytical Solution

The bending moment M for the plate under the continuous load $q = pw$ is defined as

$$M = -\frac{q(L-x)^2}{2} \tag{17 – 1}$$

Linear Analysis

Considering linear analysis (only elasticity) the maximum deflection of the structure can be calculated as follows:

$$u_{z,\max} = \frac{qL^4}{8EI_y} = 157.144 \text{ mm} \tag{17 – 2}$$

Nonlinear Analysis

The quantities of the load are discussed at first. The moment M_e when the first yield is occurred and the ultimate moment M_p when the structure becomes plastic hinge are calculated as follows

$$M_e = 2\int_0^{t/2} \sigma(z)zw\,dz = 2\int_0^{t/2} \frac{2f_y}{t}z^2w\,dz = \frac{f_y wt^2}{6} = 50.000 \text{ Nm} \tag{17 – 3}$$

$$M_p = 2\int_0^{t/2} \sigma(z)zw\,dz = 2\int_0^{t/2} f_y zw\,dz = \frac{f_y wt^2}{4} = 75.000 \text{ Nm} \tag{17 – 4}$$

The corresponding pressure p_e and p_p then results

$$p_e = \frac{2M_e}{L^2w} = 2000.000 \text{ Pa} \tag{17 – 5}$$

$$p_p = \frac{2M_p}{L^2w} = 3000.000 \text{ Pa} \tag{17 – 6}$$

It is obvious that the plate is brought into the elastic-plastic state by the pressure p according to the **Figure 1**. The bending stress is defined according to the following formula

$$\sigma_x(x,z) = -\kappa(x)Ez \tag{17 – 7}$$

where $\kappa(x)$ is the curvature defined as $\kappa(x) = d^2u_z/dx^2$ [1]. The elastic-plastic zone length is described by the parameter x_p according to the **Figure 1**. The bending stress quantity on the surface ($z = -t/2$) is equal to the plastic strength f_y at the point $x = x_p$, see the **Figure 2**. The curvature at this point can be calculated according to the formula

$$\kappa(x_p) = \frac{2f_y}{Et} \tag{17 – 8}$$

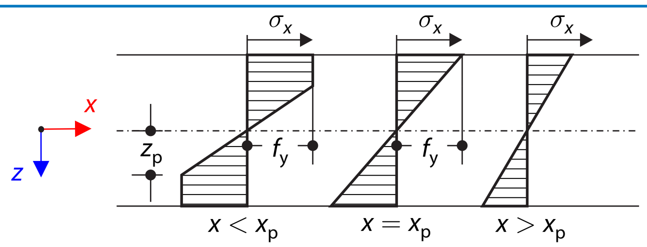

Figure 2: Bending stress distribution

The elastic-plastic moment at the point $x = x_p$ is then

$$M_{ep}(x_p) = \int_{-t/2}^{t/2} \sigma_x(x_p, z) z w \, dz = 2\int_{0}^{t/2} -\frac{2f_y}{t} z^2 w \, dz = -\frac{f_y t^2 w}{6} \tag{17 – 9}$$

The elastic-plastic moment $M_{ep}(x_p)$ (internal force) has to equal to the bending moment $M(x_p)$ (external force).

$$-\frac{f_y t^2 w}{6} = -\frac{q(L - x_p)^2}{2} \tag{17 – 10}$$

The elastic-plastic zone length x_p results from this equality as follows

$$x_p = L - t\sqrt{\frac{f_y w}{3q}} = 147.197 \text{ mm} \tag{17 – 11}$$

The curvature κ_e in the elastic zone ($x > x_p$) is described by the Bernoulli-Euler formula

$$\kappa_e = -\frac{M}{EI_y} = \frac{q(L - x)^2}{2EI_y} \tag{17 – 12}$$

where I_y is the quadratic moment of the cross-section to the y-axis[1]. The cross-section in the elastic-plastic state is divided into the elastic core and the plastic surface, which is described by the parameter z_p according to the **Figure 2**. This can be calculated using formula **(17 – 7)**.

$$z_p = \frac{f_y}{\kappa_p(x)E} \tag{17 – 13}$$

The elastic-plastic moment M_{ep} of the cross-section in the elastic-plastic state has to equal to the bending moment M.

$$M_{ep}(x) = 2\int_{0}^{z_p} -\kappa_p(x) E z^2 w \, dz + 2\int_{z_p}^{t/2} -f_y z w \, dz = -\frac{q(L - x)^2}{2} \tag{17 – 14}$$

The curvature κ_p in the elastic-plastic zone ($x < x_p$) results from this equality.

$$\kappa_p = \frac{1}{E}\sqrt{\frac{\frac{f_y^3 w}{3}}{-\frac{q(L - x)^2}{2} + \frac{f_y t^2 w}{4}}} \tag{17 – 15}$$

[1] $I_y = \frac{1}{12} w h^3 = 520.8\bar{3} \text{ mm}^4$

The total deflection $u_{z,\max}$ of the structure is defined as a superposition of the elastic-plastic and the elastic contribution using the Mohr's integral

$$u_{z,\max} = \int_0^{x_p} \kappa_p (L - x) \mathrm{d}x + \int_{x_p}^{L} \kappa_e (L - x) \mathrm{d}x = 83.117 + 83.117 = 166.234 \text{ mm} \qquad (17-16)$$

RFEM Settings

- Modeled in RFEM 5.26 and RFEM 6.01
- The element size is $l_{FE} = 0.020$ m
- In case of solid models mesh refinement across the thickness is used (6 elements per thickness)
- Geometrically linear analysis is considered
- The number of increments is 5
- Shear stiffness of the members is neglected

Results

Structure File	Entity	Material model	Hypothesis
0017.01	Member	Isotropic Plastic 1D	-
0017.02	Plate	Isotropic Plastic 2D/3D	von Mises
0017.03	Plate	Isotropic Nonlinear Elastic 2D/3D	von Mises
0017.04	Plate	Isotropic Nonlinear Elastic 2D/3D	Tresca
0017.05	Solid	Isotropic Plastic 2D/3D	von Mises
0017.06	Solid	Isotropic Nonlinear Elastic 2D/3D	von Mises
0017.07	Solid	Isotropic Nonlinear Elastic 2D/3D	Tresca
0017.08	Member	Isotropic Nonlinear Elastic 1D	-

Model	Theory	RFEM 5		RFEM 6	
	$u_{z,max}$ [mm]	$u_{z,max}$ [mm]	Ratio [-]	$u_{z,max}$ [mm]	Ratio [-]
Isotropic Plastic 1D	166.234	166.214	1.000	166.018	0.999
Isotropic Plastic 2D/3D, Plate		162.987	0.980	162.960	0.980
Isotropic Nonlinear Elastic 2D/3D, Plate, von Mises		165.730	0.997	165.700	0.997
Isotropic Nonlinear Elastic 2D/3D, Plate, Tresca		166.998	1.005	166.969	1.004
Isotropic Plastic 2D/3D, Solid		160.601	0.966	162.429	0.977
Isotropic Nonlinear Elastic 2D/3D, Solid, von Mises		163.003	0.981	165.593	0.996
Isotropic Nonlinear Elastic 2D/3D, Solid, Tresca		168.725	1.015	169.691	1.021
Isotropic Nonlinear Elastic 1D		166.214	1.000	166.018	0.999

References

[1] LUBLINER, J. *Plasticity theory*. Berkeley: University of California, 1990.