0019 – Plastic Bending - Moment Load

Description

A cantilever is fully fixed on the left end ($x = 0$) and subjected to a bending moment M according to the **Figure 1**. The problem is described by the following set of parameters.

Material	Elastic-Plastic	Modulus of Elasticity	E	210000.000	MPa
		Poisson's Ratio	ν	0.000	–
		Shear Modulus	G	105000.000	MPa
		Plastic Strength	f_y	240.000	MPa
Geometry	Cantilever	Length	L	2.000	m
		Width	w	0.005	m
		Thickness	t	0.005	m
Load		Bending Moment	M	6.000	Nm

Small deformations are considered and the self-weight is neglected in this example. Determine the maximum deflection $u_{z,\max}$.

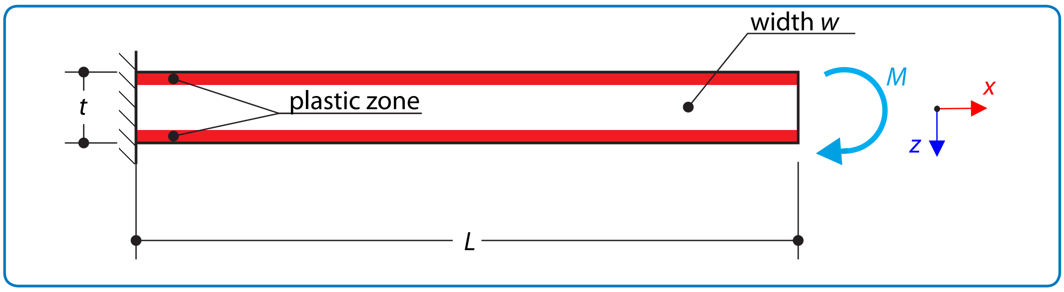

Figure 1: Problem sketch

Analytical Solution

Linear Analysis

Considering linear analysis (only elasticity) the maximum deflection of the structure can be calculated as follows:

$$u_{z,\max} = \frac{ML^2}{2EI_y} = 1.097 \text{ m} \qquad (19-1)$$

Nonlinear Analysis

The cantilever is loaded by the bending moment M. The quantities of this load are discussed at first. The moment M_e when the first yield occurs and the ultimate moment M_p when the structure becomes plastic hinge are calculated as follows

$$M_\mathrm{e} = 2\int_0^{t/2} \sigma(z) z w \,\mathrm{d}z = 2\int_0^{t/2} \frac{2f_\mathrm{y}}{t} z^2 w \,\mathrm{d}z = \frac{f_\mathrm{y} w t^2}{6} = 5.000 \text{ Nm} \tag{19 – 2}$$

$$M_\mathrm{p} = 2\int_0^{t/2} \sigma(z) z w \,\mathrm{d}z = 2\int_0^{t/2} f_\mathrm{y} z w \,\mathrm{d}z = \frac{f_\mathrm{y} w t^2}{4} = 7.500 \text{ Nm} \tag{19 – 3}$$

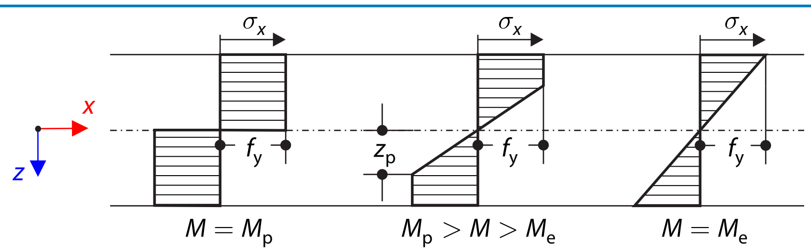

Figure 2: Bending stress distribution

It is obvious that the bending moment M causes the elastic-plastic state. The cross-section in the elastic-plastic state is divided into the elastic core and the plastic surface, which is described by the parameter z_p according to the **Figure 2**.

$$z_\mathrm{p} = \frac{f_\mathrm{y}}{\kappa(x) E} \tag{19 – 4}$$

where $\kappa(x)$ is the curvature defined as $\kappa(x) = \mathrm{d}^2 u_z/\mathrm{d}x^2$ [1]. The elastic-plastic moment M_ep in the cross-section (internal force) has to equal to the bending moment M (external force).

$$M_\mathrm{ep}(x) = 2\int_0^{z_\mathrm{p}} -\kappa(x) E z^2 w \,\mathrm{d}z + 2\int_{z_\mathrm{p}}^{t/2} -f_\mathrm{y} z w \,\mathrm{d}z = -M \tag{19 – 5}$$

The curvature κ results from this equality

$$\kappa = \frac{f_\mathrm{y}}{E}\sqrt{\frac{1}{3\left(\frac{t^2}{4} - \frac{M}{f_\mathrm{y} w}\right)}} \tag{19 – 6}$$

The total deflection of the structure $u_{z,\mathrm{max}}$ is calculated using the Mohr's integral

$$u_{z,\mathrm{max}} = \int_0^{L} \kappa(x)(L - x) \,\mathrm{d}x = 1.180 \text{ m} \tag{19 – 7}$$

RFEM Settings

- Modeled in RFEM 5.26 abd RFEM 6.01
- The element size is $l_{FE} = 0.020$ m
- Geometrically linear analysis is considered
- The number of increments is 5
- Shear stiffness of the members is neglected

Results

Structure File	Entity	Material mode	Hypothesis
0019.01	Plate	Orthotropic Plastic 2D	Tsai-Wu
0019.02	Plate	Isotropic Plastic 2D/3D	von Mises
0019.03	Member	Isotropic Plastic 1D	-
0019.04	Plate	Isotropic Nonlinear Elastic 2D/3D	von Mises
0019.05	Plate	Isotropic Nonlinear Elastic 2D/3D	Tresca
0019.06	Member	Isotropic Nonlinear Elastic 1D	-

Model	Analytical Solution	RFEM 5		RFEM 6	
	$u_{z,\text{max}}$ [m]	$u_{z,\text{max}}$ [m]	Ratio [-]	$u_{z,\text{max}}$ [m]	Ratio [-]
Orthotropic Plastic 2D	1.180	1.190	1.008	1.190	1.008
Isotropic Plastic 2D/3D, Plate		1.173	0.994	1.173	0.994
Isotropic Plastic 1D		1.180	1.000	1.180	1.000
Isotropic Nonlinear Elastic 2D/3D, Plate, Mises		1.190	1.008	1.190	1.008
Isotropic Nonlinear Elastic 2D/3D, Plate, Tresca		1.190	1.008	1.190	1.008
Isotropic Nonlinear Elastic 1D		1.180	1.000	1.180	1.000

References

[1] LUBLINER, J. *Plasticity theory*. Berkeley: University of California, 1990.