Program: RFEM 5, RSTAB 8

Category: Geometrically Linear Analysis, Second-Order Analysis, Large Deformation Analysis, Isotropic Linear Elasticity, Member

Verification Example: 0042 – Bending Cantilever with Axial Force

0042 – Bending Cantilever with Axial Force

Description

A cantilever is loaded by a transversal and an axial force on the right end and it is fully fixed on the left end ($u_x = u_y = u_z = \varphi_x = \varphi_y = \varphi_z = 0$) according to the **Figure 1**. The problem is described by the following set of parameters.

Material	Steel	Modulus of Elasticity	E	210000.000	MPa
		Poisson's Ratio	ν	0.300	–
Geometry	Cantilever	Length	L	10.000	m
		Cross-section width	w	0.080	m
Load		Transversal Force	F_z	7.650	kN
		Axial Force	F_x	1.600	kN

The self-weight is neglected in this example. Determine the maximum deformations of the structure $u_{z,\max}$ and $u_{x,\max}$ by means of the geometrically linear analysis, second order analysis and large deformation analysis.

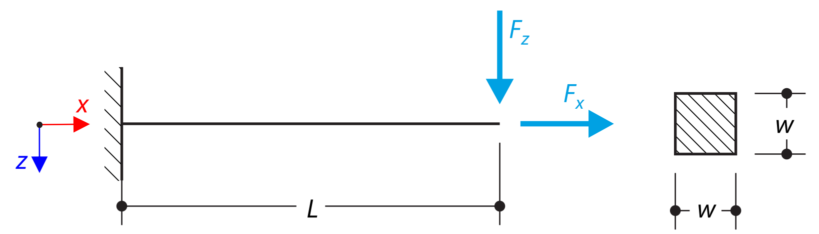

Figure 1: Problem Sketch

Analytical Solution

Linear Analysis

Desired deformation can be found by means of Geometrically linear analysis very simply. The transversal force and the axial force act independently and the force equilibrium is determined from the original geometry, because the expected deformations are small. The axial force causes only the axial deformation. Both deformations are defined by well-known formulae.

$$u_{x,\max} = \frac{F_x L}{EA} = 0.012 \text{ mm} \qquad (42-1)$$

$$u_{z,\max} = \frac{F_z L^3}{3EI_y} = 3557.478 \text{ mm} \quad (42-2)$$

Where A is the cross-section area and I_y is the quadratic moment of the cross-section to the y-axis[1].

Second-Order Analysis

Consider the second order analysis the axial force affects both the axial deformation and the transversal deformation. On the other hand the axial deformation is influenced only by the axial force and it is defined by the formula **(42 – 1)** as in case of Geometrically linear analysis. The bending moment is defined as follows:

$$M_y = -F_z(L-x) + F_x\left(u_{z,\max} - u_z(x)\right) \quad (42-3)$$

where $u_{z,\max}$ is the deflection at the point $x = L$. The solution can be found by the Euler-Bernoulli differential equation.

$$\frac{\mathrm{d}^2 u_z}{\mathrm{d}x^2} = -\frac{M_y}{EI_y} \quad (42-4)$$

It can be rewritten into the form:

$$\frac{\mathrm{d}^2 u_z}{\mathrm{d}x^2} - \alpha^2 u_z = -\frac{1}{EI_y}F_z x + \frac{1}{EI_y}\left(F_z L - F_x u_{z,\max}\right) \quad (42-5)$$

where α is defined as:

$$\alpha = \sqrt{\frac{F_x}{EI_y}} \quad (42-6)$$

The total solution consists of the homogeneous and the particular solution:

$$u_z = C_1 e^{\alpha x} + C_2 e^{-\alpha x} + u_{zP} \quad (42-7)$$

where C_1 and C_2 are the unknown constants, which can be obtained from the boundary conditions. The particular solution u_{zP} can be found in the form of the linear function.

$$u_{zP} = C_3 x + C_4 \quad (42-8)$$

where constants C_3 and C_4 can be calculated by substituting the particular solution and its derivatives into the differential equation **(42 – 5)**. The constants then results

[1] $I_y = \frac{1}{12}w^4$

$$C_3 = \frac{F_z}{F_x} \tag{42 – 9}$$

$$C_4 = -\frac{F_z L}{F_x} + u_{z,\text{max}} \tag{42 – 10}$$

The boundary conditions are following:

$$u_z(0) = 0 \tag{42 – 11}$$

$$u_z'(0) = 0 \tag{42 – 12}$$

$$u_z(L) = u_{z,\text{max}} \tag{42 – 13}$$

From conditions **(42 – 11)** and **(42 – 12)** results constants C_1, C_2.

$$C_1 = -\frac{C_4\alpha + C_3}{2\alpha} \tag{42 – 14}$$

$$C_2 = -\frac{C_4\alpha - C_3}{2\alpha} \tag{42 – 15}$$

The constant $u_{z,\text{max}}$, which is the desired solution, results from the condition **(42 – 13)**

$$u_{z,\text{max}} = \frac{F_z\left(L\alpha e^{\alpha L} + L\alpha e^{-\alpha L} - e^{\alpha L} + e^{-\alpha L}\right)}{F_x\alpha\left(e^{\alpha L} + e^{-\alpha L}\right)} = 3266.136 \text{ mm} \tag{42 – 16}$$

Large Deformation Analysis

Large deformation analysis is carried out in ANSYS Mechanical APDL.

RFEM 5 and RSTAB 8 Settings

- Modeled in RFEM 5.05.0030 and RSTAB 8.05.0030
- The element size is $l_{\text{FE}} = 0.100$ m
- The number of increments is 5
- Isotropic linear elastic material model is used
- Shear stiffness of the member is neglected

Results

Structure Files	Program	Method of Analysis
0042.01	RSTAB 8	Geometrically Linear Analysis
0042.02	RSTAB 8	Second-Order Analysis
0042.03	RSTAB 8	Large Deformation Analysis
0042.04	RFEM 5	Geometrically Linear Analysis
0042.05	RFEM 5	Second-Order Analysis
0042.06	RFEM 5	Large Deformation Analysis

Method of Analysis	Analytical Solution	RSTAB 8		RFEM 5	
	$u_{x,max}$ [mm]	$u_{x,max}$ [mm]	Ratio [-]	$u_{x,max}$ [mm]	Ratio [-]
Geometrically Linear Analysis	0.012	0.012	1.000	0.012	1.000
Second-Order Analysis	0.012	0.012	1.000	0.012	1.000

Method of Analysis	ANSYS 15 (BEAM188)*	RSTAB 8		RFEM 5	
	$u_{x,max}$ [mm]	$u_{x,max}$ [mm]	Ratio [-]	$u_{x,max}$ [mm]	Ratio [-]
Large Deformation Analysis	-546.214	-546.429	1.000	-545.419	0.999

Method of Analysis	Analytical Solution	RSTAB 8		RFEM 5	
	$u_{z,max}$ [mm]	$u_{z,max}$ [mm]	Ratio [-]	$u_{z,max}$ [mm]	Ratio [-]
Geometrically Linear Analysis	3557.478	3557.478	1.000	3557.515	1.000
Second-Order Analysis	3266.136	3266.163	1.000	3266.036	1.000

Method of Analysis	ANSYS 15 (BEAM188)*	RSTAB 8		RFEM 5	
	$u_{z,\max}$ [mm]	$u_{z,\max}$ [mm]	Ratio [-]	$u_{z,\max}$ [mm]	Ratio [-]
Large Deformation Analysis	2973.405	2973.965	1.000	2974.432	1.000

* Remark: Numerical solution in ANSYS 15 was carried out by the company Designtec s.r.o.