**Program:** RFEM 5, RF-LOAD-HISTORY

**Category:** Geometrically Linear Analysis, Contact, Friction, Member

**Verification Example:** 0061 – Cyclically Loaded System with Friction

# 0061 – Cyclically Loaded System with Friction

## Description

Consider a node, restricted to move in the $X$ direction only, held by a spring with friction described by the frictional coefficient $\mu_X$ (realized by the nodal frictional support). The node is loaded in two steps: in the first step forces $F_Z$ and $F_X = 3\mu_X F_Z$ are applied, while in the second step force $F_X$ is removed. The aim of this example is to demonstrate irreversible process caused by friction. The end-point ends up, after loading and unloading, in the different position than at the beginning. Determine the movement of the node in the $X$ direction $u_X$ after the first and the second load step.

| | | | | | |
|---|---|---|---|---|---|
| Support | | Friction Coefficient | $\mu_X$ | 0.300 | – |
| Spring | | Spring Stiffness | $k$ | 0.600 | kN/m |
| | | Initial Length | $L$ | 1.000 | m |
| Load | Force | Vertical | $F_Z$ | 1.000 | kN |
| | | Horizontal | $F_X$ | 0.900 | kN |

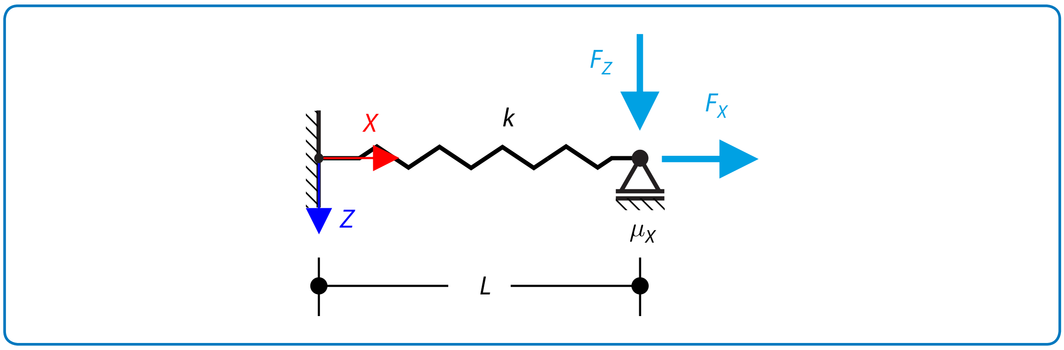


**Figure 1:** Problem sketch

## Analytical Solution

### First Load Step

In the first load step, when both forces $F_X$ and $F_Z$ are applied, force equilibrium in the $X$ direction at the considered node can be written as:

$$F_X - R = k u_{X,1} \qquad (61 – 1)$$

where $R$ is a frictional force, which can be expressed according to the vertical force $F_Z$ as:

$$R = \mu_X F_Z \qquad (61 – 2)$$

Combining equations **(61 – 1)** and **(61 – 2)** and knowing that $F_X = 3\mu_X F_Z$ formula for displacement $u_{X1}$ can be derived:

$$u_{X,1} = \frac{F_X - R}{k} = \frac{3\mu_X F_Z - \mu_X F_Z}{k} = \frac{2\mu_X F_Z}{k} = 0.100 \text{ m} \qquad (61-3)$$

## Second Load Step

In the second load step the horizontal force $F_X$ is removed, the spring starts to shrink and frictional force $R$ acts against its movement. Force equilibrium in the $X$ direction at the considered node can be then expressed as:

$$R = ku_{X,2} \qquad (61-4)$$

So the movement of the node in the second load step $u_{X,2}$ can be evaluated as:

$$u_{X,2} = \frac{R}{k} = \frac{\mu_X F_Z}{k} = 0.050 \text{ m} \qquad (61-5)$$

## RFEM 5 Settings

- Modeled in version RFEM 5.04.0108
- Geometrically linear analysis is considered

## Results

| Structure File | Program |
|---|---|
| 0061.01 | RF-LOAD-HISTORY |

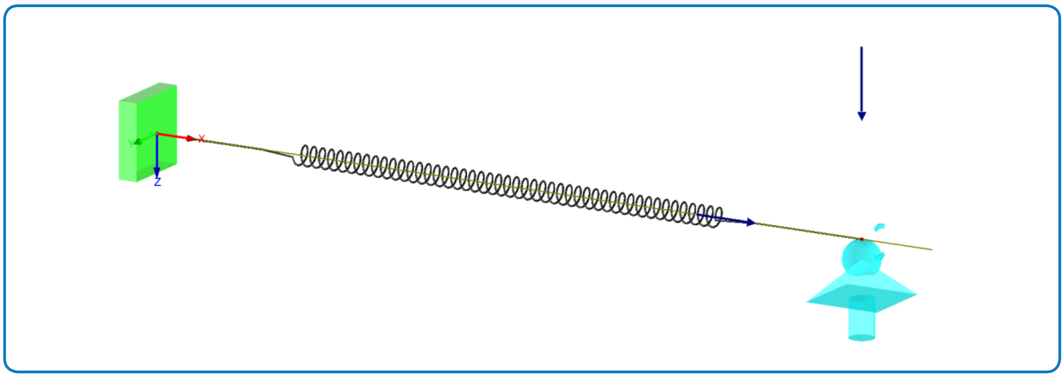


**Figure 2:** RFEM 5 Model

As can be seen from the following comparisons, an excellent agreement of analytical solution with numerical output was achieved:

| Load Step | Analytical Solution | RF-LOAD-HISTORY | |
|---|---|---|---|
| | $u_{X,i}$ [m] | $u_{X,i}$ [m] | Ratio [-] |
| The First ($i = 1$) | 0.100 | 0.100 | 1.000 |
| The Second ($i = 2$) | 0.050 | 0.050 | 1.000 |